PROF. CHARLES I. MILLER
Lafayette, Indiana 45907
Purdue University

Stereo Models for Measuring the Space Scene

Valuable tool for foresters, geologists, wildlife managers and others for determining slope gradients, heights and contour locations on vertical aerial photos with a pocket stereoscope.

INTRODUCTION

THE EXISTENCE OF stereoscopic vision has been known for at least 135 years. During the first 70 or 80 years of this time, the use of stereographs was generally limited to nontechnical photographic work in which a high degree of accuracy of image formation was not vital. In the past 40 or 50 years much work has been done on measuring the space scene on stereoscopic pairs of metrical quality with average stereoscopic vision. It provides a valuable tool for foresters, geologists, wildlife managers, and others who frequently desire to determine slope gradients, heights, and contour locations on vertical aerial photographs, but who are not proficient in the use of floating dot devices.

Stereographs may be drawn to produce stereoscopic models as explained by Rule (1938) in a definitive article on stereoscopic

ABSTRACT: *The paper explains how to produce metrical quality stereoscopic models, and how to use them to measure the space scene in viewing them with a pocket stereoscope. The method, which requires only inexpensive stereographs, a pocket folding stereoscope, and common sense, can be used by anyone with average stereoscopic vision. It provides a valuable tool for all those who frequently desire to determine slope gradients, heights, and contour locations on vertical aerial photographs, but who are not proficient in the use of floating dot devices.*

aerial photographs by the introduction of floating dots, or pointers, whose separation can be made to correspond with that of some point in the stereograph.

One can also measure the space scene by using a stereograph, which is printed as a positive transparency, to introduce a stereoscopic model of known metrical qualities into the space scene. For example, for each of the inverted cones (Figure 1) the slope percent in the stereoscopic model is known. By comparison with the slopes of the cones, slope percentages in the space scene can be determined and converted to ground slope percentages. In addition, the crosses on the cones can be used to determine heights. This method, which requires only inexpensive stereographs, a pocket folding stereoscope, and common sense, can be used by anyone drawings. But if complex models are desired, drawings often do not produce the desired results. Images coming to our eyes are only an invitation to organize them through inner motility; our conception of three-dimensional space is conditioned in motoral as well as in optical terms. Then too, drawings must be prepared with great care to avoid violating the accomodation-convergence ratio normal to binocular vision. Consequently, it is preferable, and easier, to obtain stereoscopic models by taking stereoscopic photographs of real models. Generally speaking the methods that have been used to obtain such models, although they produce excellent illustrative material, do not produce metrical-quality stereoscopic models. Nevertheless, one gets the impression that Judge (1935), Nelson (1958), and Latham (1972) had the

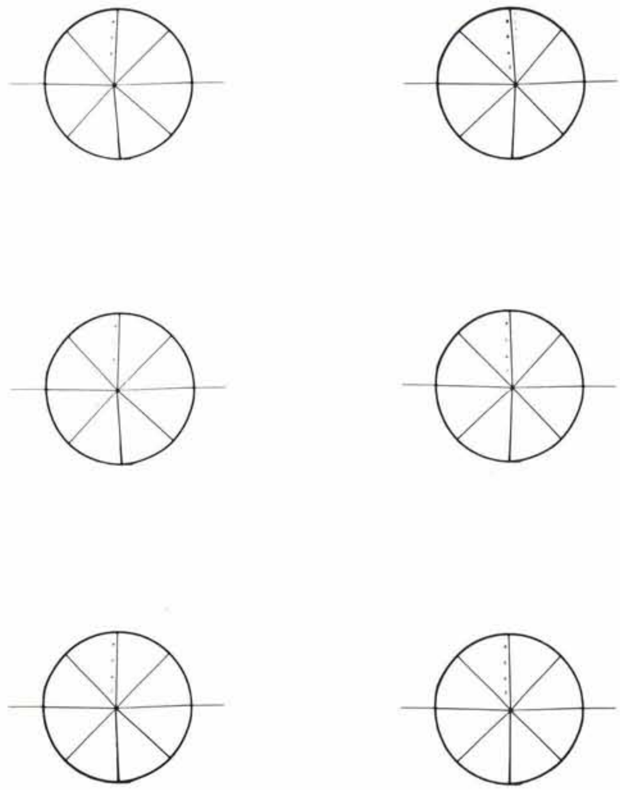

FIG. 1. Stereograph that produces stereoscopic models of inverted cones for measuring the space scene. (T_S, tangent of slope in space scene, is 0.9 for top model, 0.6 for center model and 0.3 for lower model. Stereograph is enlarged 2×.)

idea, although they did not spell it out. The purpose of this paper, then, is to explain how to produce metrical quality stereoscopic models, and how to use them to measure the space scene when viewing with a pocket stereoscope.

In this paper I shall work with vertical aerial photographs. However, the methods are applicable to any stereoscopic photograph, and even in the manufacture of stereoscopic rangefinders. Indeed, there is an analogy between the use of the combination of a camera and a stereoscope to produce the stereoscopic model in the space scene, and the use of a stereoscopic rangefinder to produce a stereoscopic model in space. The lens systems of the camera correspond to the objective systems of the rangefinder, and the lenses of the stereoscope correspond to the eyepiece systems of the rangefinder. With the stereoscope one places a stereoscopic pair of pictures at the focal distances of the lenses; with the rangefinder one locates a stereoscopic pair of reticles at the focal distances of the eyepieces.

BACKGROUND AND PRELIMINARIES

The formula to compute h, the actual height of an object imaged on a stereoscopic pair of vertical aerial photographs, may be written

$$h = \frac{H(dP)}{P + dP}$$

Reduced to the scale of the photographs, h_P, the scale-model height of the object, would be

$$h_P = \frac{H(dP)}{P + dP}\left(\frac{f}{H}\right) = \frac{f(dP)}{P + dP}$$

where H is the perpendicular distance from the camera base line to the horizontal plane at the top (or base) of the object, P is the parallax of a point at distance H, f is the focal length of the camera lens, and dP is the difference in parallax between the top and base of the object. (If h, or h_P, extends downward from the plane at distance H, dP is minus; if h, or h_P, extends upward from the plane at distance H, dP is plus.)

The height h_P is the height an object must be in the space scene to obtain a scale model. But as I have shown (1960), if photographs are viewed stereoscopically, the height visualized is rarely equal to h_P, because h_S, the apparent height of an object in the space scene, depends on S, the apparent distance from the eyes to the horizontal plane at the top (or base) of the object, e, the interpupillary distance, and dP, which is as defined above. Specifically,

$$h_s = \frac{S(dP)}{e + dP}$$

Assuming 60 percent overlap for stereoscopic pairs of vertical aerial photographs, if the focal length of the camera lens is less than 24 inches, the ratio of h_S to h_P will be greater than 1; if the focal length of the camera lens is greater than 24 inches, the ratio of h_S to h_P will be less than 1. This stretching or compressing of the space scene, which is called vertical exaggeration, E, may be quantified as follows:

$$E = \frac{h_s m}{h_P m} = \frac{\left(\frac{s(dP)}{e + dP}\right)}{\left(\frac{f(dP)}{P + dP}\right)} = \frac{S(P + dP)}{f(e + dP)}$$

The effect of introducing m, linear magnification, is simply to increase h_P and h_S m times. Note that m cancels.

It should be understood that E is an estimate of vertical exaggeration only. It does not appraise the slight modification in the direction of lines extending depthwise. Distortion, as this modification is called, results in large part, from incorrect positioning of the eyes and the stereoscope relative to the photographs of the stereograph. Distortion can, for all practical purposes, be eliminated by careful alignment of the stereograph along the base line, by setting the stereoscope so the spacing of the lenses equals the interocular distance, and with a little trial and error in positioning the stereoscope.

By definition, T_S, the tangent of a slope in the space scene, and T , the tangent of a slope in a scale model (tangent of scale-model slope = tangent of ground slope), would be

$$T_s = h_s/d$$
$$T = h_P/d$$

which leads to

$$T_s = S(dP)/(e + dP)d$$
$$T = f(dP)/(P + dP)d$$

where d is the base of the right triangle forming the slope in the space scene or in a scale model, and the other terms are as previously defined.

Because $E = h_S/h_P$, it follows that

$$E = T_S/T$$
$$T = T_S/E$$

Thus, if we determine the tangent of a slope in the space scene by comparison with a known slope in the stereoscopic model, we can determine the tangent of the ground slope by dividing this value by vertical exageration, E. (As a slope percent is the tangent of the slope angle multiplied by 100, T_S or T can be converted to percent by multiplying by 100.)

It is illuminating to substitute the expressions for T_S and E in the equation $T = T_S/E$:

$$T = \frac{\frac{S(dP)}{(e+dP)d}}{\frac{S(P+dP)}{f(e+dP)}} = \frac{f(dP)}{(P+dP)d}$$

One sees that the expression $S/(e+dP)$ cancels. This is fortuitous, because the expression is difficult for most people to determine accurately, and because it varies for different individuals. However, because it cancels, an average value of $S/(e+dP)$ may be used without affecting the accuracy of the final determination of T . The importance of this fact will be seen in the next section.

I have found that average values for S and $(e+dP)$ for use with a pocket stereoscope are: S = 16 inches and $(e+dP)$ = 2.56 inches. Then $S/(e+dP) = 16/2.56 = 6.25$. Therefore, the formulas for T_S and E may be rewritten:

$$T_S = 6.25(dP)/d$$
$$E = 6.25(P+dP)/f$$

For good approximations of E, dP may be dropped from the equation to give

$$E = 6.25P/f$$

Or, because $P/f = B/H$, where B is length of camera base line,

$$E = 6.25B/H$$

In the next section I shall explain how the above information can be used to prepare stereoscopic models to measure the space scene by viewing with a pocket stereoscope. But note that the equation to compute vertical exaggeration, $E = 6.25P/f$, is useful by itself. One can interpret the space scene more intelligently if he knows how much it is stretched or compressed. Indeed, as I observed when working with Davison (1963), if E is less than 1, stereoscopic viewing is no better than nonstereoscopic viewing for interpreting vegetation. And other things being equal, as E increases the accuracy of height determinations with floating dot devices increases. To compute an average value of E for any stereoscopic pair, P is taken as the average of the photo base lines of the pair, that is, the average distance from principal point to conjugate principal point on the photographs.

Practical Details

To obtain several stereoscopic models to measure the space scene it is necessary to prepare only one model. To illustrate, let us prepare a paper model of an inverted right circular cone that has eight lines on the inside surface extending from the base to the point, that has ten crosses along one of the lines, and that has two *reference lines* on the circumference of the cone base. Then let us photograph this model to obtain a stereograph that produces stereoscopic models of inverted cones with slope tangents, T_s, of 0.30, 0.60, and 0.90 (Figure 1). (The reader may think of other models. If so, he can prepare them by the procedures explained here.)

A suitable model of the cone will be obtained if we let R, the cone radius, equal 2.144 inches and T , the tangent of the cone slope angle, equal 0.60. Then L, the cone slant height, will be 2.500 inches.

A suitable radius r for the cone in the completed stereograph is about 0.2 inches. Consequently, the photo scale reciprocal, PSR, in the stereograph should be

$$PSR = R/r = 2.144/0.2 = 10.72 \quad .$$

And because we shall photograph the model with a Nikon F camera with a focal length f of 50 mm., or 1.969 inches, H, the distance from the lens to the base of the cone, should be

$$H = f(PSR) = 1.969(10.72) = 21.1 \text{ inches.}$$

(This value of H will not give the calculated scale because, if the camera is focused at a finite distance, f will not equal the principal distance. But a change in scale, that is, a change in r, will not effect T_s in the stereoscopic model. However, the final value of r, which is determined after the photographs are developed, must be used to calculate dP if heights are desired in the model.)

Now we shall set the cone into a circular hole on a flat, white surface with the base of the cone flush with the surface. Then we shall take six photographs of the model. From these six photographs we shall prepare three stereograms that we shall assemble into a single stereograph that will give the three stereoscopic models.

To get the photographs for each stereoscopic model, one takes an exposure, makes a lateral displacement of the model, and takes another exposure. (One can, of course, move the camera instead of the model. But for our example it is more convenient to move the model.) The rules are:

- Prepare the model with black lines and place it on a flat, white surface. (To insure that the stereo depth of field is not exceeded, the height h of the model, should be less than $0.021\,H^2/(B + 0.021\,H)$, where H is the perpendicular distance from the camera base line to the horizontal plane at the top, or base, of the object, and B is the length of the camera base line.)
- Use a high-contrast copy film.
- Use two photoflood lamps, one on either side of the model. Arrange them so the light strikes the model at about 45 degrees.
- Use the same stop, shutter speed, and lighting conditions for all exposures.
- Determine the camera setting as for any photograph, but be certain a stop is used that will give sufficient depth of field.
- Mount the camera rigidly with the camera axis perpendicular to the surface, or plane, containing the model. (A macrostat stand for close-up and copying work makes it easy to attain these conditions and to locate the photoflood lamps properly.)
- Move the entire surface in a lateral direction perpendicular to the camera axis so the model moves along a line through the photo center and parallel to the picture margin as seen in the view finder. (Be certain that the camera angle of view is great enough so the model will be completely within the frame of the picture in all views.)
- To obtain the pictures for any given stereogram, locate the model at the same distance from the lens in both views.

Figure 2 shows the position of the camera and the positions of the model for our example. The calculations to determine the amount to move the model, that is, B, the

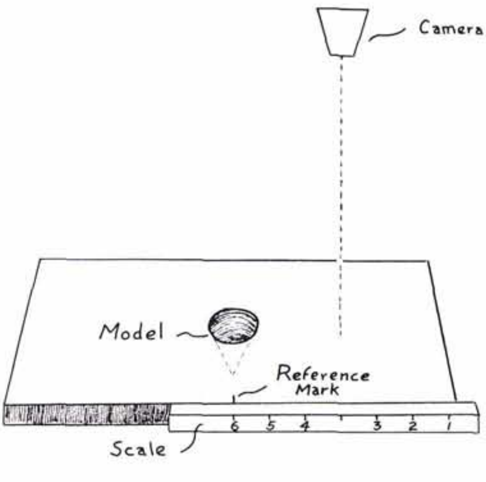

FIG. 2. Location of the camera relative to the model (an inverted cone) where the model is in position No. 6. Other positions of the model are obtained by sliding the surface containing the model so that the reference mark on the surface is at the desired number on the scale. For example the given in the text, marks on the scale are spaced 0.895 inches apart.

length of the camera base line, are based on the formulas given in the previous section of this article. The calculations are summarized in Table 1.

After taking photographs for which the values of B given in Table 1 are used, they are developed. The negatives are then assembled to produce the desired stereogram. This is done as follows:

- ★ Lay out the negatives, glossy side down, on a white surface in the order given in the last column of Table 1. Trim the negatives as necessary.
- ★ Space the points of the cones in each set of negatives about 2.1 inches apart (e.g., point of cone in negative 3 should be 2.1 inches from point of cone in negative 4, etc.). Align each set of cones so their reference lines fall on a common line.
- ★ Space the three sets of cones so lines connecting their reference lines will be about one inch apart.
- ★ Check the alignment of the cones while viewing with a pocket stereoscope. Tape the outside edges of the negatives to the surface with masking tape. (In the space scene the points of all cones should be in the same horizontal plane.)
- ★ Fill in the open space between the two sets of negatives with opaque paper. Fasten in place with Scotch tape. Put Scotch tape over all joints, but do not put tape over the images. Remove the masking tape.
- ★ Print this assemblage as a single print on Dupont Cronar, or equivalent, to produce positive transparencies.

We now have a stereograph that will produce the desired stereoscopic models (Figure 1).

To compute the parallax difference, dP, between each of the ten crosses on the three inverted cones, use the formula

$$dP = (0.1)d(T_s)/6.25$$

(There is a slight error in the result obtained by this equation because dP between each pair of crosses is not exactly the same. However, the error is negligible.)

In the finished stereograph, which is a contact print, $r = 0.22$ inches. Thus, because $r = d$, dP between each pair of crosses on the three cones as shown in Table 2.

APPLICATIONS

In viewed with a pocket stereoscope, the transparent stereograph will produce stereoscopic models of three inverted cones of known slope gradients, and known values of dP between the crosses. The use of the stereoscopic models to determine the tangent of ground slopes from the space scene imaged on vertical aerial photographs is simple:

- ■ Properly line up the photographs for stereoscopic viewing. Space the conjugate images about 2.1 inches apart.

TABLE 1. CAMERA BASE LINE B REQUIRED TO OBTAIN A GIVEN SLOPE TANGENT T_S IN THE SPACE SCENE WHEN TANGENT OF SLOPE T IN SCALE MODEL EQUALS 0.6

T_S	*Value of E to obtain* T_S°	*Value of B to obtain* T_S† *(inches)*	*Positions of model for desired effect (See Fig. 2)*
0.3	0.50	1.79	3 and 4
0.6	1.00	3.58	2 and 5
0.9	1.50	5.37	1 and 6

° $E = T_S/T$

† Solve for P in formula: $E = 6.25(P+dP)/f$. Then, as $B = P(PSR)$, $B = (fE-6.25\,dP)PSR/6.25$. (Note that $dP = d(T_S)/6.25$, and is minus.)

TABLE 2. DISTANCE dP IN INCHES BETWEEN PAIRS OF CROSSES

Cone slope, T_s	*dP between each pair of crosses (inches)*
0.3	.0011
0.6	.0021
0.9	.0032

■ Lay the transparent stereograph over the photographs and view it stereoscopically with a pocket stereoscope. Adjust spacing of the photographs so that the points of the inverted cones fall in a horizontal plane slightly above a horizontal plane through the area of interest in the space scene.

■ While stereoscopically viewing both the terrain in the space scene and the inverted cones, select the inverted cone with the slope that most nearly matches the slope of interest in the space scene. (As the inverted cones have slope tangents that increase in increments of 0.30, one may desire to interpolate if an exact match cannot be found.)

■ Divide the tangent of the slope obtained in the third step by the vertical exaggeration, E, of the vertical aerial photographs being viewed. This will give the tangent of the ground slope. (To determine E, satisfactory results will be obtained from the formula, $E = 6.25P/f$.)

EXAMPLE

Assume that we match the inverted cone slope with tangent of 0.6 with the slope of interest in the space scene. Further, assume that the photographs were taken with a camera with focal length of 6 inches, and that the average photo base line (average distance between principal point and conjugate principal point for the two photographs of the stereoscopic pair) is 3.2 inches. Thus,

$$T = \frac{T_s}{E} = \frac{0.6}{6.25(3.2/6)} = 0.18 .$$

The use of the stereoscopic models to determine heights on vertical aerial photographs calls for experimentation on the part of the interpreter. However, for the beginner the following procedure is recommended:

▲ Properly line up the photographs for stereoscopic viewing. Space the conjugate images about 2.1 inches apart.

▲ Lay the transparent stereograph over the photographs and view it stereoscopically with a pocket stereoscope. Adjust spacing of the photographs so the points of the inverted cones fall in the horizontal plane through the base of the objects for which heights are desired.

▲ Move an appropriate cone into position by the object of interest and find the cross on the cone that is the same height as the object.

▲ Compute height between individual crosses for the photographs being used. Multiply this number by the number of cross intervals from base to top of object to obtain height of object.

EXAMPLE

Assume that we desire to measure heights above a horizontal plane for which $H = 5000$ feet and $P = 3.2$ inches. Using the cone for which $T_s = 0.6$, we obtain 5 cross intervals from base to top of object of interest. Thus,

$$h = \frac{H(dP)}{P+dP} \text{ (No. cross intervals) } =$$

$$\frac{5000(.0021)}{3.2 + .0021} \text{ (5) } = 16.4 \text{ feet}$$

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