WOLFGANG FAIG*
HIDEYA MONIWA
University of New Brunswick
Fredericton, N.B., Canada

Convergent Photos for Close Range

The increased computational effort may well be compensated by the improved accuracy of the results.

ABSTRACT: *Depending on the object which is to be photographed at close range, the convergent mode can be applied in order to strengthen the geometrical configuration of the solution. An object rotation with respect to a fixed camera station will provide this mode. Besides a description of the method, the results of a thorough investigation of this approach are compared with the results of the normal case applied for the same object. Recommendations are made concerning efficiency, feasibility, accuracy and adjustment procedure.*

INTRODUCTION

CONVERGENT photographs have been used sporadically in photogrammetric practice during the last 50 years. Although it yields a significant theoretical accuracy increase[5] over vertical photographs, the latter are generally used because it is less awkward for practical stereo measurements. Practical tests[6, 7, 9] indicate that for one model an accuracy improvement of approximately 30 percent in both position and elevation can be obtained; however, they also show that for aerial triangulation the error propagation is less favorable than with vertical photographs. After approximately 15 models the vertical photographs yield better results. This is an additional limitation for the use of convergent photography.

In close-range photogrammetry, however, we are usually concerned with single models and are therefore more interested in the achievable accuracy. As most objects are small, they can often be moved with respect to a fixed camera thus permitting stereophotography. In [4] the various possibilities are discussed. Unless the object is instable or moving, one camera is often used for stereophotogrammetric measurements.

The analogue background of photogrammetry has led users to try to create the normal case of photography because of its simplicity, namely the theoretically perfect vertical photograph. In the present age of analytical photogrammetry and computer oriented compilation, this is no longer a valid argument. The emphasis is rather shifted towards achievable accuracy. For nonmetric cameras it is necessary in this respect to use analytical methods for many projects, because several distortions have to be corrected. It is therefore logical to consider convergent photographs because of their higher accuracy, more favorable base-height ratios and difficulties in stereo vision can be tolerated, especially with well identified object points.

Konecny explains in [8] the discrepancy between theoretical and practical accuracy gain in using convergent photography. Because the remaining errors in the interior orientation parameters have a strong direct influence on the models resulting from convergent photographs, some of the advantage is cancelled. In convergent photography as applied by the authors, this is eliminated by the fact that the parameters of the interior orientation are determined during the evaluation. In fact, this approach was used in several applications for calibration purposes only.

THE MATHEMATICAL MODEL

In conventional photogrammetry (aerial or terrestrial) convergent photographs are obtained either with two cameras or with one

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camera that is shifted *and* rotated. In close-range photogrammetry, the same effect can be achieved by *only rotating* the object in respect to a fixed camera for successive photography (Figure 1). The result is a pair of photographs which produces a stereomodel as if the object *AB* was photographed from the two imaginary positions E_1 *and* E_2.

This convergent system is equivalent to the conventional convergent photography with all its inherent advantages (e.g., up to 100 percent overlap, better base-height ratio and higher accuracy) and disadvantages (e.g., scale change within a photograph).

For the evaluation of this approach, a computer program was written in Fortran IV which performs first a space resection solution for each imaginary camera station using collinearity equations and carrying the parameters of interior orientation as unknowns, then a space intersection solution for determining the unknown object points. The lack of intersection of rays (coplanarity condition) is termed *residual parallax* and used as an indication of the accuracy of the solution. Furthermore, for a number of object points the photogrammetrically determined coordinates are compared with their known values in order to obtain an accuracy measure for the coordinates. The main equations of the approach are summarized below.

According to Reference 1, the collinearity equations are:

$$(x_p - x_0) = -c \times \left[\frac{m_{11}(X_p - X_c) + m_{12}(Y_p - Y_c) + m_{13}(Z_p - Z_c)}{m_{31}(X_p - X_c) + m_{32}(Y_p - Y_c) + m_{33}(Z_p - Z_c)} \right] \tag{1}$$

$$(y_p - y_0) = -c \times \left[\frac{m_{21}(X_p - X_c) + m_{22}(Y_p - Y_c) + m_{23}(Z_p - Z_c)}{m_{31}(X_p - X_c) + m_{32}(Y_p - Y_c) + m_{33}(Z_p - Z_c)} \right]$$

where

$$\begin{aligned} M_1^* &= m_{11}(X_p - X_c) + m_{12}(Y_p - Y_c) + m_{13}(Z_p - Z_c) \\ M_2^* &= m_{21}(X_p - X_c) + m_{22}(Y_p - Y_c) + m_{23}(Z_p - Z_c) \\ M_3^* &= m_{31}(X_p - X_c) + m_{32}(Y_p - Y_c) + m_{33}(Z_p - Z_c). \end{aligned} \tag{2}$$

Equations 1 are transformed into:

$$\begin{aligned} F_x &= (x_p - x_0) M_3^* + c M_1^* = 0 \\ F_y &= (y_p - y_0) M_3^* + c M_2^* = 0. \end{aligned} \tag{3}$$

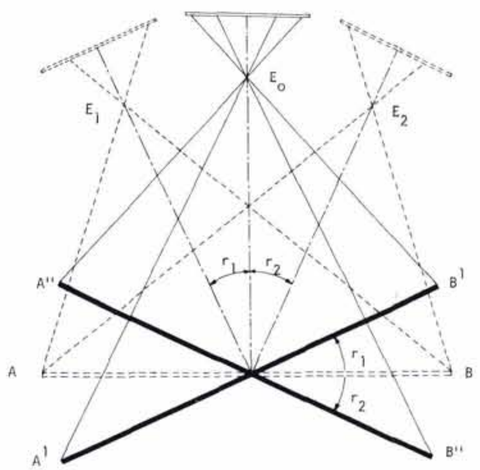

FIG. 1. Convergent photographs obtained by object rotation.

At this stage radial-symmetric and decentering lens distortions, as well as film shrinkage and nonperpendicularity of the comparator axes are considered.

By introducing the latter, the image-coordinate system is solely based on the comparator. Therefore, fiducial marks are not necessary. Radial-symmetric and decentering lens distortions were modelled according to Brown [2, 3] and presented as corrections in the *x*- and *y*-directions.

For radial-symmetric lens distortion the corrections are:

$$\begin{aligned} dr_x &= (x_p - x_0)(k_1 r^2 + k_2 r^4 + k_3 r^6 + \cdots) \\ dr_y &= (y_p - y_0)(k_1 r^2 + k_2 r^4 + k_3 r^6 + \cdots) \end{aligned} \tag{4}$$

where

$$r = \sqrt{[(x_p - x_0)^2 + (y_p - y_0)^2]}$$

and x_P, y_P being the measured comparator coordinates of the image point *P*. For decentering lens distortion, the following corrections are used:

$$\begin{aligned} dp_x &= (1 + p_3 r^2 + p_4 r^4 + \cdots) \cdot [p_1(r^2 + 2(x_p - x_0)^2 + 2p_2(x_p - x_0)(y_p - y_0)] \\ dp_y &= (1 + p_3 r^2 + p_4 r^4 + \cdots) \cdot [p_2(r^2 + 2(y_p - y_0)^2 + 2p_1(x_p - x_0)(y_p - y_0)]. \end{aligned} \tag{5}$$

Scale differences along the principal axes and their possible nonperpendicularity were modelled together as:

$$dg_x = A(y_p - y_0) \qquad (6)$$
$$dg_y = B(y_p - y_0)$$

with

$$A = (1 + ds)\sin d\beta \qquad (7)$$
$$B = (1 + ds)\cos d\beta - 1$$

where ds is the scale change along y-axis with respect to scale 1 along x-axis, and $d\beta$ the angle difference to orthogonality (Figure 2).

Because

$$(x_p - x_0) = (x_p - x_0) + dr_x + dp_x + dg_x \qquad (8)$$
$$(y_p - y_0) = (y_p - y_0) + dr_y + dp_y + dg_y,$$

the collinearity Equations 3 were modified to

$$F_x = [(x_p - x_0) + dr_x + dp_x + dg_x]M_3^* + cM_1^* = 0 \qquad (9)$$
$$F_y = [(y_p - y_0) + dr_y + dp_y + dg_y]M_3^* + cM_2^* = 0$$

In order to solve for the unknowns, the radial lens distortion parameters are limited to three terms, k_1, k_2 and k_3, whereas for the decentering lens distortion two terms p_1 and p_2 are considered. Accordingly, Equations 9 are solved for the 16 unknown parameters of interior and exterior orientation. In solving for the 16 unknowns per camera, at least 8 object control points are required. With more points, a least-square adjustment is performed in the following manner.

Consider Equations 9 being a set of the measured quantities L and the unknowns X:

$$F(X, L) = 0 \qquad (10)$$

with

$$L = (X_p, Y_p, Z_p, x_p, y_p)$$
$$X = (k_1, k_2, k_3, p_1, p_2, A, B, c, x_0, y_0, X_c, Y_c, Z_c, \omega, \phi, \kappa).$$

As Equation 10 is non-linear, the equations are *linearized using* a Taylor *expansion:*

$$F(X, L) = F(X^0, L) + \left(\frac{\partial F}{\partial X}\bigg|_{X^0, L}\right) \cdot dX + \left(\frac{\partial F}{\partial L}\bigg|_{X^0, L}\right) \cdot V = 0 \qquad (11)$$

where X° is an approximation of X. V is the vector of residuals of the measured quantities L, the square sum of which is to be minimized. This constitutes the general case of least-squares adjustment.

If the object control points X_P, Y_P, Z_P are assumed as error-free or known to a very high accuracy, the mathematical model includes only residuals V_X or V_Y for the image coordinates. This is expressed as

$$F(X) - L' = 0 \qquad (12)$$

with

$$L' = (x_p, y_p).$$

The linearized form will then be

$$F(X) - L' = F(X^0) + \left(\frac{\partial F}{\partial X}\bigg|_{X^0}\right) \cdot dX - V = 0. \qquad (13)$$

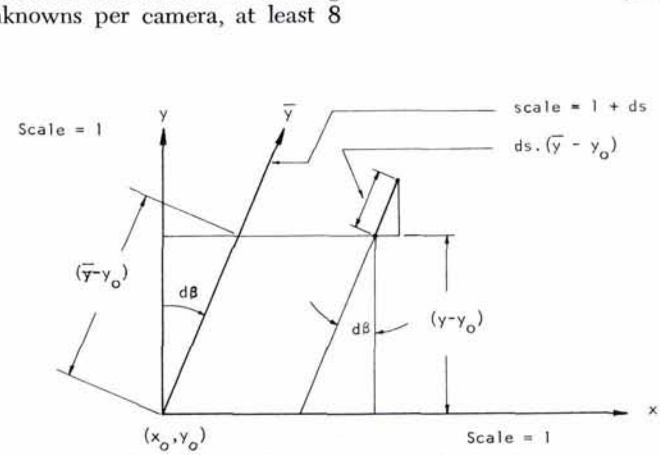

FIG. 2. Scale change and non-perpendicularity of comparator axes.

This simplified version of the general case of least squares adjustment is valid only if the ratio of the projection distance Z_c to the camera constant c is such that the measuring errors of the object control points is insignificant on the image plane. For the following investigation, the general case of least-squares adjustment was employed and compared with the simplified version (Table 2).

The program is arranged such that different weights for all measured quantities (object-space control and comparator measurements) can be assigned. Due to generally weak initial approximations X°, the solution is obtained after several iterations. The convergence criterion is

$$\left| \frac{\sigma_{0_{new}} - \sigma_{0_{old}}}{\sigma_{0_{new}}} \right| < 10^{-8}.$$

Space intersection and residual parallax computations follow standard photogrammetric formulations which do not need to be discussed here.

Practical Investigations

Two different types of problems were handled with this approach, namely calibration of nonmetric cameras and object evaluation. The latter was also performed using vertical stereophotography.

Camera Calibrations

For the purpose of camera calibration, a three-dimensional test field was constructed (see Reference 10 and Figure 3).

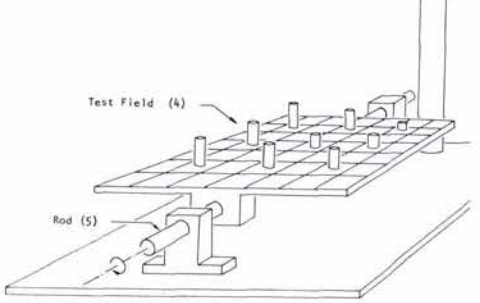

Fig. 3. Rotatable test field with camera support.

The camera (1) is fastened to a supporting bar (2), which can be moved along a vertical metal rod (3) in order to set the desired camera height. The test field consists of a grid plate (4) of sheet aluminum which is supported by a horizontal steel rod (5) which acts as rotational axis.

The test field consists of 61 points, namely 36 grid intersections engraved every 25 millimeters (mm) on the aluminum surface and 25 brass bolts with known but different heights, fixed perpendicularly to the plate.

The test field was coordinated precisely to an accuracy of less than 0.03 mm. The centering of the bolts proved to be most critical and any discrepancy exceeding this value was eliminated by recentering. This could be detected by measuring the points located in different planes separately for each plane on the comparator and then mathematically transforming them into the grid system. A series of 30 observations for all control points resulted in the following standard deviations for the test field:

$$\sigma_X = 0.021 \text{ mm}$$
$$\sigma_Y = 0.026 \text{ mm}$$
$$\sigma_Z = 0.003 \text{ mm}$$

This test field was used to calibrate the cameras as shown in Table 1.

The measured values were processed several times using different parameters of interior orientation in order to detect the critical ones for a particular camera. This was done in order to save computer time and object-space control for subsequent use of these cameras. For each camera, the case of carrying all parameters was also adjusted with the simplified case using Equation 13 instead of Equation 11 as before. The interesting results are indicated in Tables 2 and 3.

These tables indicate some very interesting facts, namely that a simple calibration is sufficient for Camera 1, radial lens distortion is dominant for Camera 2 and film shrinkage and related distortions dominate the calibration of Camera 3. For practical work, there-

Table 1. Cameras Calibrated by the Convergent Procedure

Camera	*Name*	*Lens*	*Approx. Focal Length*
1	Nikomat FT	Nikkor – H	50 mm
2	Nikon F	Nikkor – S	55 mm
3	Asahi Pentax	Takumar	35 mm

TABLE 2. CALIBRATION RESULTS WITH DIFFERENT PARAMETERS

Camera	Photo Scale	Distortion Parameters	Standard Deviations of Coordinates in [mm] σ_X	σ_Y	σ_Z	Mean Residual Parallax [mm]
1	1:13	none	0.080	0.064	0.102	0.052
1	1:13	all	0.079	0.064	0.100	0.051
2	1:8	none	0.085	0.072	0.274	0.031
2	1:8	radial only	0.049	0.064	0.272	0.030
2	1:8	radial + decentering	0.046	0.062	0.265	0.031
2	1:8	all	0.047	0.061	0.266	0.028
3	1.14	none	0.025	0.010	0.089	0.040
3	1:14	radial only	0.026	0.012	0.087	0.039
3	1:14	radial + decentering	0.025	0.011	0.075	0.039
3	1:14	all	0.005	0.009	0.067	0.035
3	1:14	decentering only	0.025	0.011	0.076	0.039
3	1:14	affinity only	0.005	0.008	0.082	0.036

TABLE 3. COMPARISON OF ACCURACIES OBTAINED WITH GENERAL AND SIMPLIFED ADJUSTMENT VERSIONS USING THE SAME DATA

Adjustment based on equation	Camera	Distortion Parameters	Standard Deviations of Coordinates in [mm] σ_X	σ_Y	σ_Z	Mean Residual Parallax [mm]
11	1	all	0.079	0.064	0.100	0.051
13			0.094	0.088	0.109	0.093
11	2	all	0.047	0.061	0.266	0.028
13			0.077	0.071	0.270	0.029
11	3	all	0.005	0.009	0.067	0.035
13			0.053	0.068	0.095	0.108

fore, all other parameters can be neglected as their influence is insignificant.

OBJECT EVALUATION

A semi-cylinder (20 cm long, radius 2 cm) with 49 well-defined points was photographed together with 40 three-dimensional control points. The camera, a Hasselblad 500C, was used in the quasi-convergent mode with ± 5° object rotations as well as in the vertical mode with a 5-cm parallel shift of the object. In Table 4 the resulting accuracies for the two modes are compared.

TABLE 4. ACCURACIES OBTAINED IN MILLIMETERS AT OBJECT SCALE BY EVALUATING VERTICAL AND CONVERGENT PHOTOGRAPHS OF THE SAME OBJECT

Standard Deviation	Vertical Mode (5 cm-Base)	Convergent Mode, Rotation Angle ± 5°
σ_X	0.043 mm	0.032 mm
σ_Y	0.059 mm	0.030 mm
σ_Z	0.158 mm	0.097 mm
Mean Residual Parallax	0.076 mm	0.014 mm

The improvement in the accuracy of the coordinates is as expected for convergent photography. It is worth noting the sizable improvement in the fullfilment of the coplanarity condition (residual parallax).

CONCLUSIONS

The convergent condition in close-range photogrammetry can relatively easily be obtained by an object rotation in respect to a fixed camera. Because the parameters of interior orientation are treated as unknown in the numerical evaluation of the photographs, the full potential of accuracy improvement for convergent photography can be utilized. This, of course, can only be achieved by analytical evaluation which, for non-metric photographs, is necessary for most precision applications even in the vertical mode or normal case. As shown for some cameras, it is possible to reduce the number of distortion parameters by *precalibration*, thereby reducing computer time as well as control requirements.

Considering accuracy as the main factor, the convergent situation should be applied wherever practically feasible, which means for stable, small and well-controlled objects.

As the required accuracy of the object-space control points presents a geodetic problem in many instances and is sometimes not quite met, it is advisable to consider these points in the adjustment as *observed quantities* rather than fixed.

The increased computational effort for the general case is usually compensated by better results.

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