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Mariner Mars 9 Stereophotogrammetry*

Orbit characteristics of the Mariner Mars 9 spacecraft made possible a practically 100% photographic coverage of the Martian surface, while topographic features were covered by convergent stereopairs.

Evaluation of these stereopairs presented quite a few unconventional problems. The standard relative orientation procedure was not applicable, due to lack of passpoints and also due to peculiarities of TV imaging techniques, such as transmission quality, low resolution, small vidicon format, electronic distortion, etc. One way to circumvent this problem is to use the orbital and spacecraft parameters for an external relative orientation, which requires a number of rotations and counter-rotations between various coordinate systems (camera, spacecraft, orbit, Mars, Sun, and Canopus). The most difficult influence is that of Mars spin axis rotation, because the time difference of a stereopair may be anywhere between 84 seconds and several months.

Abstract: *The Mariner Mars 9 spacecraft performed primarily a mapping mission. The spacecraft was inserted into an elliptical orbit around Mars on November 14, 1971, with a nominal 12-hour period, a nominal periapsis of 1300 km, and a nominal apoapsis of 18,000 km, and it is expected to stay in this orbit for approximately 50 years, although communications with it were turned off October 27, 1972. These orbit characteristics enabled a photographic coverage of practically 100% of the Martian surface, with many of the outstanding topographic features being covered by convergent stereopairs.*

Method

The scientific productivity of the Mariner Mars 1971 Project has surpassed everyone's dreams. In the area of surface mapping alone, the Mariner 9 spacecraft has mapped the whole planet, in contrast to the original NASA objective of approximately 70% coverage. Many of the outstanding Martian topographic features were covered by convergent stereopairs; this paper describes one method used for their solution.

Analytical evaluation of a stereopair is based on the principle of relative orientation, i.e., the position (rotation and translation) of the right camera has to be determined in the image coordinate system of the left camera, in which the space coordinates of individual points are computed. Relative orientation is a routine affair in conventional photogrammetry, but in extraterrestrial cases its direct solution is usually not possible, due to lack of passpoints and also due to peculiarities of TV imaging techniques. These problems were discussed in detail in Ref. 1.

However, it is possible to perform an external relative orientation, whereby the rotational and translational parameters are determined indirectly from orbital and spacecraft data supplied by the navigation team.

Let us first discuss the necessary rotations from the right to the left camera system, whose sequence is shown in Fig. 1, where

RC = image coordinate system of the right camera

RS = RC rotated into the

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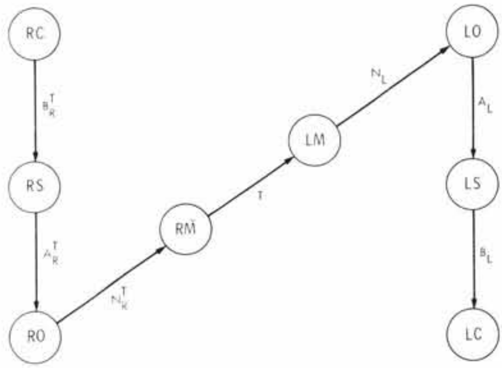

FIG. 1. Sequence of rotations.

spacecraft coordinate system

RO = RS rotated into the orbit coordinate system

RM = RS rotated into the Mars equatorial coordinate system

LC, LS, LO, and LM = identical rotations for the left camera

The coordinate systems themselves and their mutual relations are schematically shown in Figs. 2-5.

In matrix notation, the rotation from the right to the left camera can be written as

$$X_L = MX_R \tag{1}$$

where

$$M = B_L A_L N_L T N^t_R A^t_R B^t_R \tag{2}$$

It is now necessary to determine the elements of the individual matrices that form the final matrix M:

1. *Matrix B*–rotation from spacecraft into camera system.

$$b_{1,1} = \cos\gamma \cos\varepsilon - \sin\gamma \cos\delta \sin\varepsilon$$
$$b_{1,2} = \sin\gamma \cos\varepsilon + \cos\gamma \cos\delta \sin\varepsilon$$
$$b_{1,3} = \sin\delta \sin\varepsilon$$
$$b_{2,1} = -\cos\gamma \sin\varepsilon - \sin\gamma \cos\delta \cos\varepsilon$$
$$b_{2,2} = -\sin\gamma \sin\varepsilon + \cos\gamma \cos\delta \cos\varepsilon$$
$$b_{2,3} = \sin\delta \cos\varepsilon$$
$$b_{3,1} = \sin\gamma \sin\delta$$
$$b_{3,2} = -\cos\gamma \sin\delta$$
$$b_{3,3} = \cos\delta$$

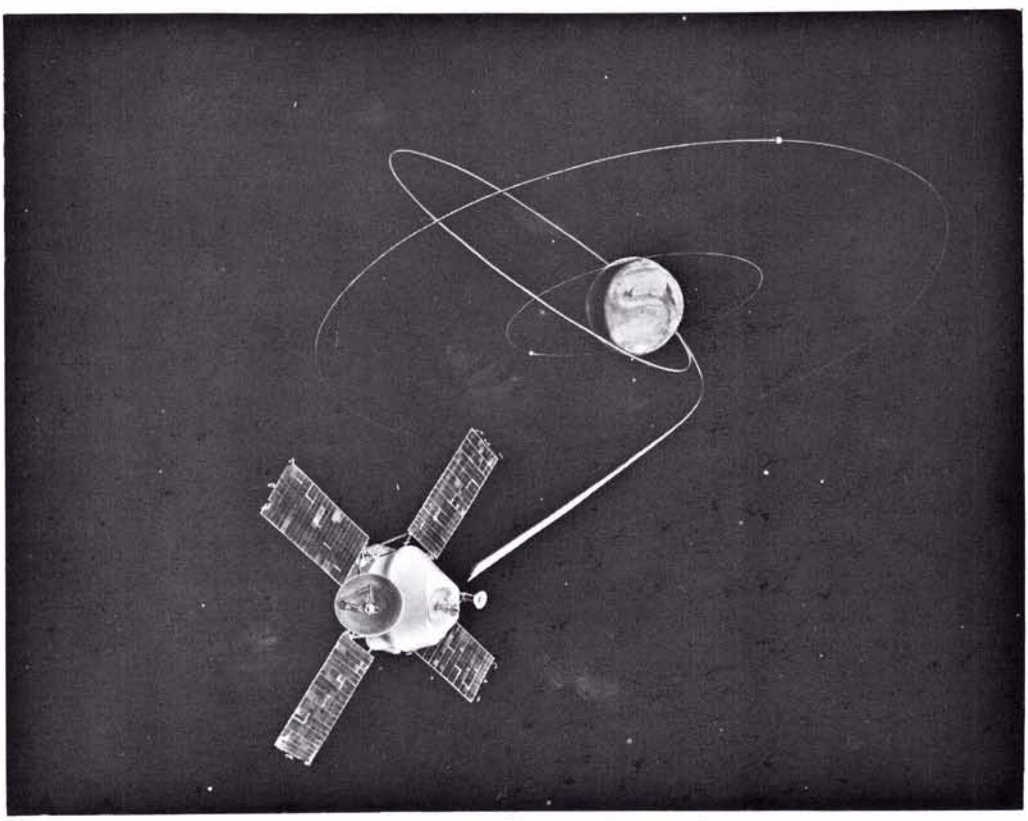

FIG. 2. Insertion of spacecraft into orbit.

where

γ = clock angle $-(57.8° -$ Canopus orientation)

δ = cone angle $- 180°$

ε = rotation of the vidicon reseau grid with respect to the scan platform

Clock and cone angles are pointing directions of the camera mounted on the spacecraft scan platform and "Canopus orientation" indicates the angular direction to the star Canopus. This parameter was calculated from available navigational data and tabulated as a function of orbit revolutions in Ref. 2 (a sample of these tables is shown in Table 1).

2. *Matrix A*–rotation from orbit into spacecraft system

$$a_{1,1} = \cos\alpha$$
$$a_{1,2} = \sin\alpha$$
$$a_{1,3} = 0$$
$$a_{2,1} = -\sin\alpha\cos\beta$$
$$a_{2,2} = \cos\alpha\cos\beta$$
$$a_{2,3} = \sin\beta$$
$$a_{3,1} = \sin\alpha\sin\beta$$
$$a_{3,2} = -\cos\alpha\sin\beta$$
$$a_{3,3} = \cos\beta$$

where

a

$\alpha = 90° -$ Sun horizontal

$\beta = 90° -$ Sun vertical

The terms "Sun horizontal" and "Sun vertical," which denote the horizontal and vertical components of the Sun vector, were calculated and tabulated similarly to the Canopus orientation above (see Table 1 and Ref. 2).

3. *Matrix N*–rotation from the Mars equatorial system into the orbit system.

$$n_{1,1} = \cos\omega\cos\Omega - \sin\omega\sin\Omega\cos i$$
$$n_{1,2} = \cos\omega\sin\Omega + \sin\omega\cos\Omega\cos i$$
$$n_{1,3} = \sin\omega\sin i$$

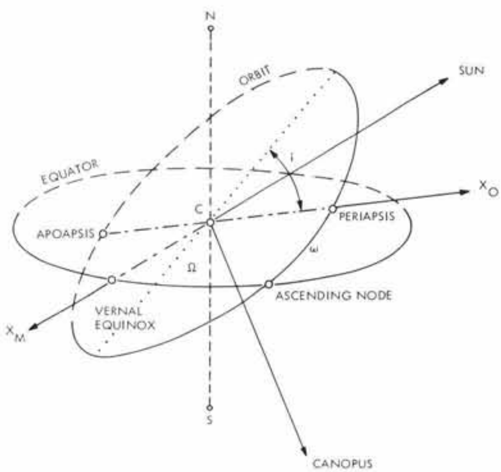

FIG. 3. Mars equatorial and orbit coordinate

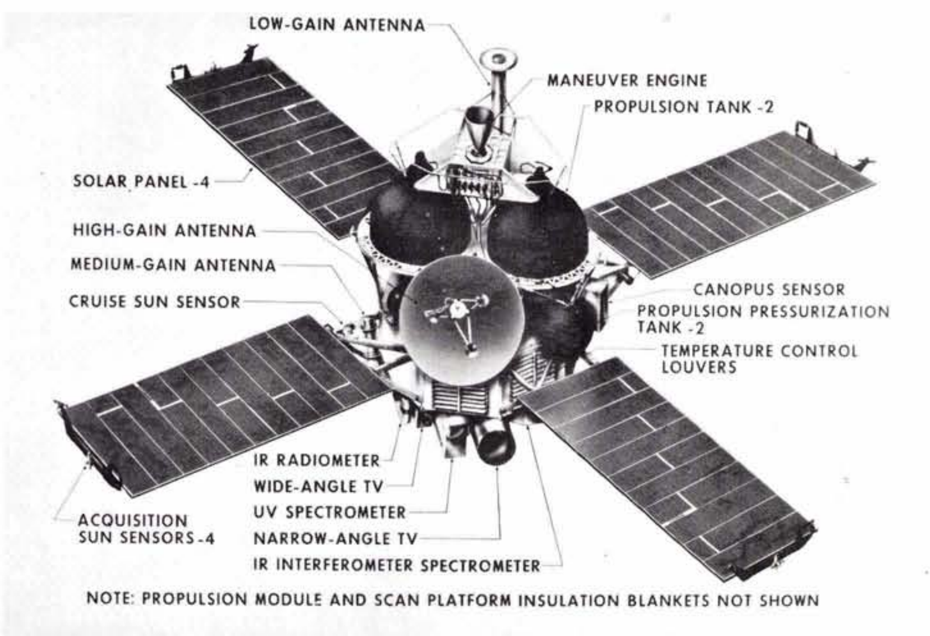

FIG. 4. Top view of the spacecraft.

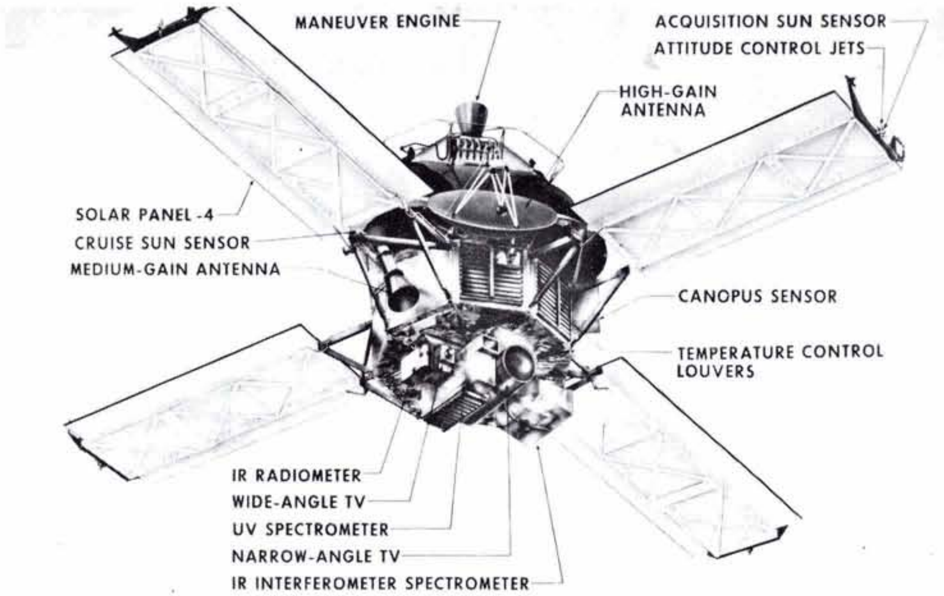

FIG. 5. Bottom view of the spacecraft.

$$
\begin{aligned}
n_{2,1} &= -\sin\omega\cos\Omega - \cos\omega\sin\Omega\cos i\\
n_{2,2} &= -\sin\omega\sin\Omega + \cos\omega\cos\Omega\cos i\\
n_{2,3} &= \cos\omega\sin i\\
n_{3,1} &= \sin\Omega\sin i\\
n_{3,2} &= -\cos\Omega\sin i\\
n_{3,3} &= \cos i
\end{aligned}
$$

where

i = inclination of the orbit plane with respect to the Mars equator (approximately 65°).

Ω = angular distance of the orbit ascending node from the Mars vernal equinox

ω = angular distance of the periapsis from the orbit ascending node

All these parameters are navigational data that are tabulated in Ref. 2 and Table 1.

4. *Matrix T*—Mars spin axis rotation.

This is the most interesting and important of all the rotations involved in this procedure. It should be realized that many overlapping photographs were taken in different revolutions, sometimes separated by several months, which means that the photographed features cannot be treated as stationary objects. The basic idea is to rotate the right camera into a hypothetical position that would have to be attained, had no spin axis rotation occurred:

$$
\begin{aligned}
t_{1,1} &= \cos\theta\\
t_{2,1} &= -\sin\theta\\
t_{3,1} &= 0\\
t_{1,2} &= \sin\theta\\
t_{2,2} &= \cos\theta\\
t_{3,2} &= 0\\
t_{1,3} &= 0\\
t_{2,3} &= 0\\
t_{3,3} &= 1
\end{aligned}
$$

where

θ = Mars spin axis rotation (which must be determined for each individual case)

It should also be noted that the transposed matrices in equation (2) must be calculated independently from matrices B, A, and N as a function of different navigational parameters, especially if the right and left photographs were not taken in the same revolution.

The product of matrices B, A, N, and T determines the final rotational matrix M as indicated in relations (1) and (2), but this solves only the first half of the whole problem. It is still necessary to calculate the stereobase in the coordinate system of the left camera, which is relatively simple to do.

Figure 6 explains the layout of the orbit coordinate system. The position of the spacecraft can be then written as

$$
\begin{aligned}
X_Q &= d\cos\psi\\
Y_Q &= d\sin\psi\\
Z_Q &= 0
\end{aligned}
\qquad (3)
$$

TABLE 1. MARINER 9 NAVIGATIONAL DATA (IN DEGREES)

Revolution number	*Periapsis omega*	*Sun horizontal*	*Sun vertical*	*Canopus orientation*	*Periapsis latitude*	*Sun latitude*	*Periapsis longitude*	*Sun longitude*	*Mars orientation*
326	332.75	324.13	.96	18.83	−24.31	8.07	48.15	32.51	125.44
327	332.75	323.91	.73	18.90	−24.31	8.17	223.55	207.60	300.84
328	332.74	323.69	.51	18.96	−24.31	8.26	38.94	22.70	116.23
329	332.74	323.47	.28	19.02	−24.32	8.35	214.34	197.79	291.62
330	332.74	323.25	.06	19.09	−24.32	8.44	29.73	12.88	107.01
331	332.73	323.03	359.83	19.15	−24.32	8.54	205.12	187.97	282.40
332	332.73	322.81	359.61	19.22	−24.33	8.63	20.52	3.06	97.80
333	332.72	322.59	359.38	19.28	−24.33	8.72	195.91	178.15	273.19
334	332.72	322.37	359.16	19.35	−24.33	8.82	11.30	353.24	88.58
335	332.72	322.15	358.93	19.41	−24.34	8.91	186.70	168.33	263.97
336	332.71	321.93	358.71	19.48	−24.34	9.00	2.09	343.43	79.36
337	332.71	321.71	358.48	19.54	−24.34	9.09	177.49	158.52	254.76
338	332.71	321.49	358.26	19.61	−24.35	9.19	352.88	333.61	70.15

where

d = spacecraft distance from Mars center

ψ = true anomaly

Both these parameters are a function of time

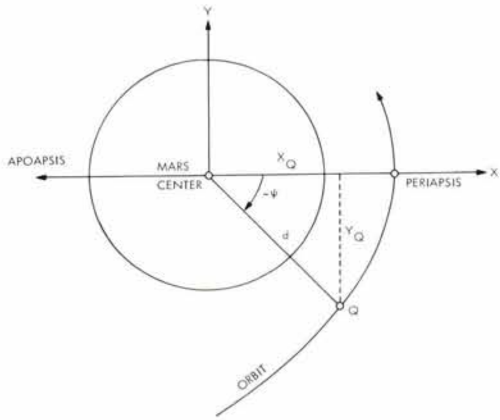

FIG. 6. Spacecraft position in the orbit.

and are tabulated in the navigation logbook.

The components of the stereobase S are then equal to

$$\begin{aligned} S_X &= X_{QR} - X_{QL} \\ S_Y &= Y_{QR} - Y_{QL} \\ S_Z &= 0 \end{aligned} \tag{4}$$

It is now necessary to rotate the stereobase S from the orbit system into the left camera system:

$$S' = B_L A_L N_L T N^t_R S \tag{5}$$

where B, A, N, and T are the same matrices as in equation (2).

It is also convenient to reduce the base components so that the X-component is equal to 1; i.e.,

$$\begin{aligned} S''_X &= 1 \\ S''_Y &= \frac{S'_Y}{S'_X} \\ S''_Z &= \frac{S'_Z}{S'_X} \end{aligned} \tag{6}$$

Space coordinates of any point P can now be computed in the coordinate system of the left camera, if both its left and right image coordinates are measured:

$$Z_P = \frac{S''_Z x^{\circ\prime\prime} - z^{\circ\prime\prime}}{x^{\circ\prime\prime} - \frac{x'}{z'} z^{\circ\prime\prime}} S'_X \tag{7}$$

$$X_P = Z_P \frac{x'}{z'}$$

$$Y_P = Z_P \frac{y'}{z'}$$

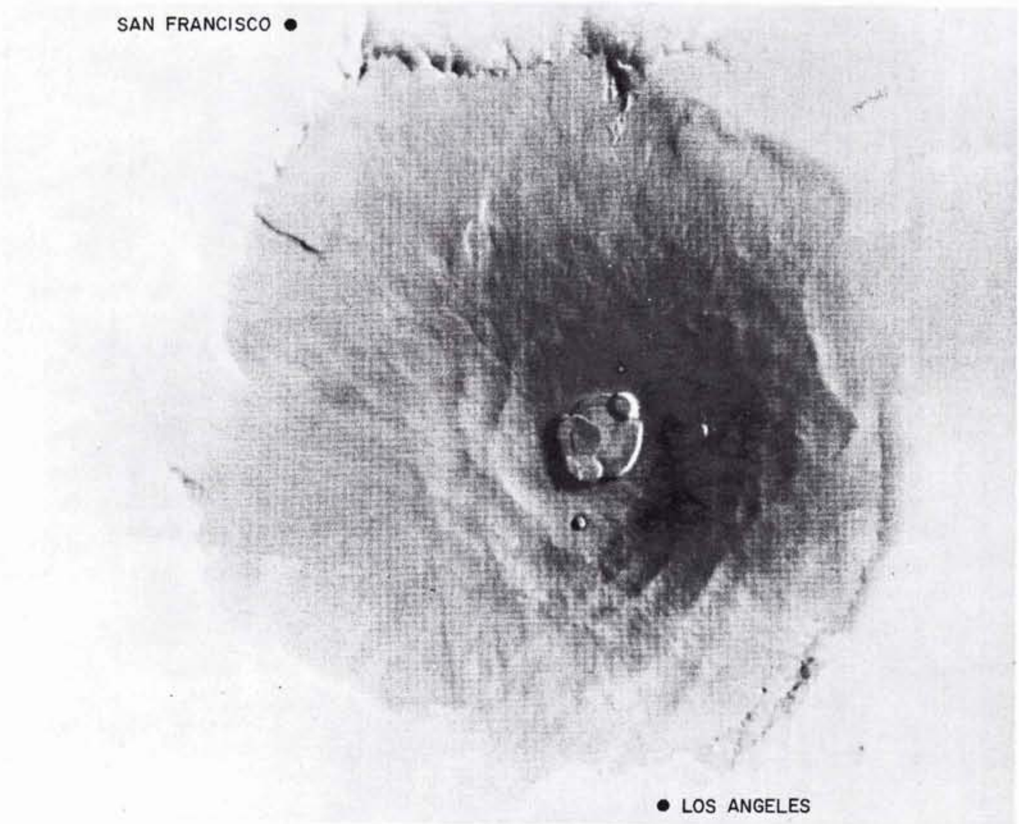

FIG. 7. Nix Olympica volcano.

where

$$\begin{bmatrix} x^{\circ\prime\prime} \\ y^{\circ\prime\prime} \\ z^{\circ\prime\prime} \end{bmatrix} = M \begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix} \qquad (8)$$

and also

x', y', x'', y'' = left and right camera image coordinates

z', z'' = negative focal length of the left and right camera (they are usually the same, either -52.267 mm or -500.636 mm).

The image coordinates naturally have to be corrected for optical and electronic distortion, and the influence of the principal point eccentricity must also be considered. These problems are discussed in detail in Refs. 3 and 4. The layer of atmosphere on Mars is so thin that atmospheric refraction is negligible. On the other hand, Mars curvature may or may not be included, depending on local conditions and the specific purpose of evaluation.

Many photographs were taken in a nearly vertical position, which means that coordinates X_P, Y_P, Z_P are close to an orthographic projection, but they can easily be rotated into any other desired projection or transformed into ground control points by using the method of absolute orientation.

It is not the purpose of this paper to discuss the results obtained, because the reduction and evaluation of available data have not yet been completed. It is also necessary to compare the results with results obtained by other agencies and individuals, and a thorough statistical error analysis likewise remains to be done. However, the method has undoubtedly proved its worth, especially for evaluation of outstanding Martian features like the mammoth volcano in the Nix Olympica region (Fig. 7), and the results may be viewed as another proof that analytical photogrammetry is a useful and powerful tool.

REFERENCES

1. Benesh, M., *Extraterrestrial Convergent Photogrammetric Mapping System—An Error Analysis*, Technical Memorandum 33-455, Jet

Propulsion Laboratory, Pasadena, California, December 1, 1970.
2. *Mariner Mars 1971 Science Operation Handbook,* Jet Propulsion Laboratory, Pasadena, California, November 9, 1971 (JPL internal document).
3. Snyder, L. M., *Mariner 9 TV Subsystem Calibration Report,* Publication 610-202, Jet Propulsion Laboratory, Pasadena, California, November 15, 1971 (JPL internal document).
4. Benesh, M., *Mariner Mars 1971 Photogrammetric Calibrations,* paper presented at the XIIth Congress of the International Society of Photogrammetry in Ottawa, Canada, July 23 to August 4, 1972. Jet Propulsion Laboratory, Pasadena, California.