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Triangulation With Santoni Stereosimplex 11c

Aero-triangulation examples of systematic instruments and correction are demonstrated for the Santoni-Stereosimplex IIc.

INTRODUCTION

SINCE THE INTRODUCTION of semi-analytical Aerotriangulation or Aerotriangulation by Independent Models (5) the general trend in photogrammetry has been to use stereoplotters, other than universal, for data acquisition. Numerous instruments belong to this group of plotters, among them the topographic plotters. The Galileo Santoni Stereosimplex IIc represents this type of topographic plotter.

ABSTRACT: *The systematic errors inherent in the Santoni Stereosimplex IIc are discussed and demonstrated. Two aero-triangulation examples are presented: (1) the semi-analytical method, which incorporates the systematic instrumental errors; and (2) the analytical aero-triangulation method, in which the systematic instrumental errors are corrected. It is concluded from these experiments that the major influence on the accuracy of an aero-triangulation is contributed by the perspective centers. Furthermore, the Santoni Stereosimplex IIc is able to perform accurate aero-triangulation, particularly if the systematic instrumental errors are numerically corrected.*

The Santoni IIc plotter was used to conduct the experimentation to be described in this paper; the purpose was to evaluate it as a data acquisition instrument for aerial triangulation. No attempt has been made to establish triangulation methods for this particular instrument. A summary study of methodology, however, has been done by Nasu (4), which will be published elsewhere.

It must be realized that the Santoni IIc Stereosimplex was not designed to be an aero-triangulator or a data acquisition instrument in general. In this respect, the instrument was used outside of its intended field, which means that this study puts emphasis on the consistency of performance rather than on absolute accuracy. Further, the correction of systematic instrumental errors is accented rather than their elimination by instrumental calibration.

INSTRUMENTAL ERRORS IN GENERAL

The instrumental errors which produce the maximum effect on the data obtained by any stereo plotting instrument, can be classified into two basic categories; the projection error in general and coordinatograph error in the model space. The projection errors may be due to various sources, but the majority are produced by the gimbal axes. The effect of the nonperpendicularity of the coordinatograph axes is so well known, that it will not be discussed here in detail.

Every instrument contains at least two gimbal axes, one in the projection center and the other one below the center representing the projected point. In some instruments, such as the Santoni Stereosimplex, a third gimbal axis lies above the projection center representing the photo point.

The errors generated by these gimbal axes are readily recognizable if one projects a grid from the photo-space to the model space and measures the grid intersections in the model coordinate system. Because of the significance of the systematic errors of the gimbal axes and their detection, it may be worthwhile to review these errors in general and then examine particularly the Stereosimplex errors to see their effect on aerial triangulation.

Figure 1.A and B illustrate the problem involved. Figure 1.A shows the ideal situation where the primary and secondary axis intersect each other at the point which also includes the projection ray. This point in the center of the gimbal axis is symbolized as G in the figure corresponding to the projection center (or to the projected point depending on the location of the gimbal axis). In practice the situation is different as shown by Figure 1.B. The projection ray or its substitute, the space rod, intersects the XY plane at G″ instead of at G, thus introducing an e_1 eccentricity error. Since the projection of point G″ does not coincide with G, an e_2 eccentricity is created along the primary axis. Further, if one assumes that the primary and secondary axes do not intersect each other at point G then an e_3 error results.

The individual effect of these eccentricity errors on the model coordinate can be defined mathematically and can be computed according to Hothmer (1) for example, or their total effect can be obtained by means of model deformation equations as given in reference (8). However, the prefered way to illustrate these errors is by means of the deformed grid since this is the most direct approach to detect and evaluate these errors. In addition the operator of the plotter in question can check and evaluate the instrument periodically when used for triangulation.

To do this evaluation, one must bring the plate holder parallel to the model coordinate system, i.e., by leveling it. In Figure 2 the distorted grids have been displaced due to the various eccentricity errors. Figure 2A shows the distorted grid in solid line along with the undistorted in dashed line. This is the case if e_1, a positive or negative eccentricity error, exists; thus producing a distortion similar in appearance in an ω tilt, otherwise considered as pseudo ω.

In a similar manner the effect of $\pm$ e_2 ec-

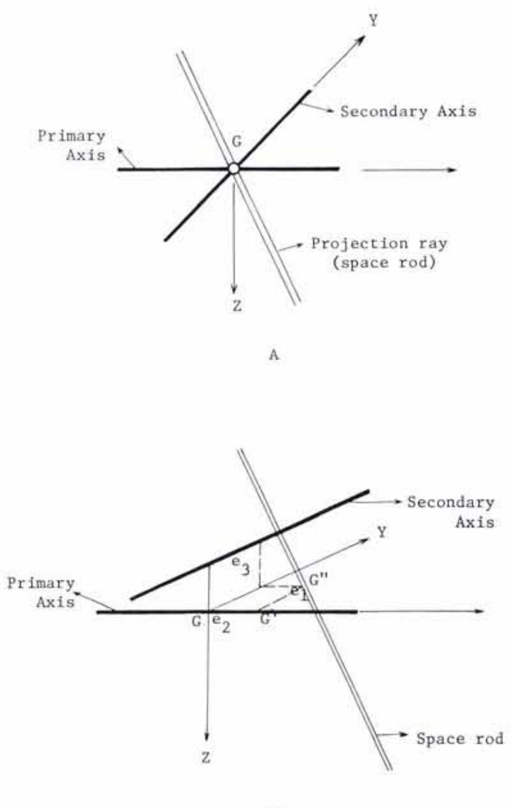

FIG. 1.

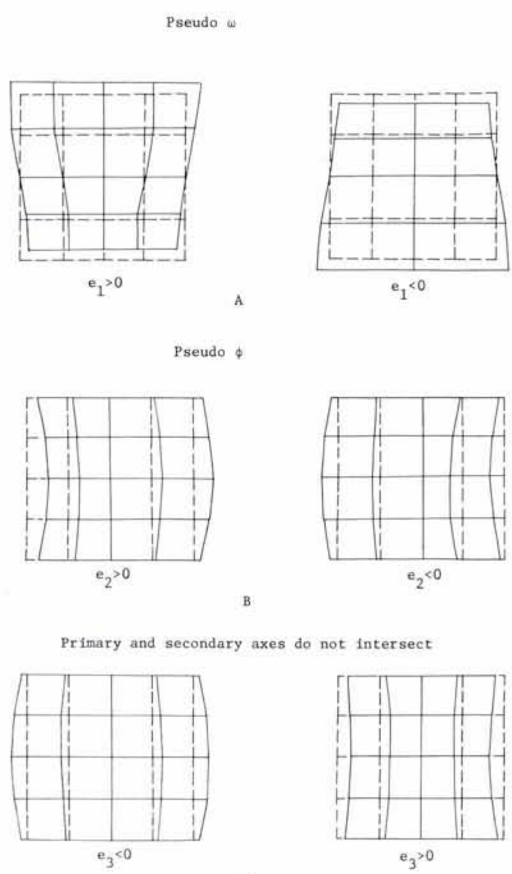

FIG. 2.

centricity error or pseudo φ is exhibited in Figure 2B. The effect of $\pm$ e_3 as shown in Figure 2C results from the primary and secondary axes not intersecting each other. In practice, the compound of these deformations usually occurs, and the largest of these eccentricity errors will dominate the shape of the deformed grid.

The projection ray in the Santoni Stereosimplex may also be displaced by the mechanical construction of the photo point. The photo point is materialized by ball and cam center C as shown in Figure 3A. To meet the geometric requirements of this construction the radius of the ball should be the same in any direction $r_1 = r_2$. Further, the radius of the ball must be the same as the cam radius $r_1 = r_2 = r_C$, and they must have a common center C. The projection ray must pass through the point C; if not, then a similar distortion occurs in this case as shown by Figure 2. In addition, a deformation results if the center of the cam is not located in C but varies eccentrically by the certain amount. Figure 3B exhibits this distortion in a form of a cushion shaped grid.

These distortions, as illustrated in Figures 2 and 3, show the full 9″ × 9″ size of the grid. In practice, however, only 60% of this can be measured in the instrument.

Instrumental Error in the Santoni IIc

The Galileo Corporation of America provided the instrument used in this experiment. It was precisely calibrated about a year before the experiment, and checked before experimentation.

This checking consisted of precise measurement of the grid and analysis of its distortion. The accuracy of the grid plate used for this purpose was $\pm$ 2 micrometers. The grid intersection was measured such that the instrument had f = 150 mm, and the observation level was at z = 300 mm. After several observations the standard error σ was found to be $\pm$ 10 micrometers. The results of these observations are exhibited in Figure 4 for the left and in Figure 5 for the right projector. Although the intervals of the grid intersections are 2 cm in these figures, only every second (4 cm intersection) is exhibited for reasons of clarity.

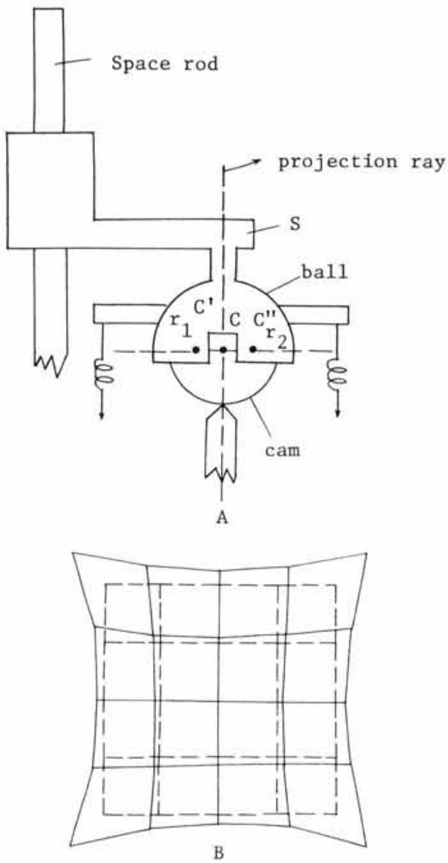

FIG. 3.

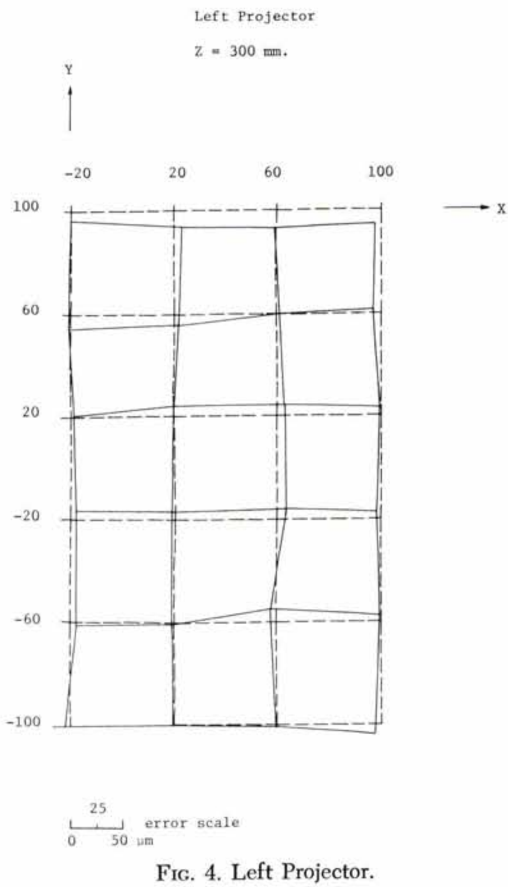

FIG. 4. Left Projector.

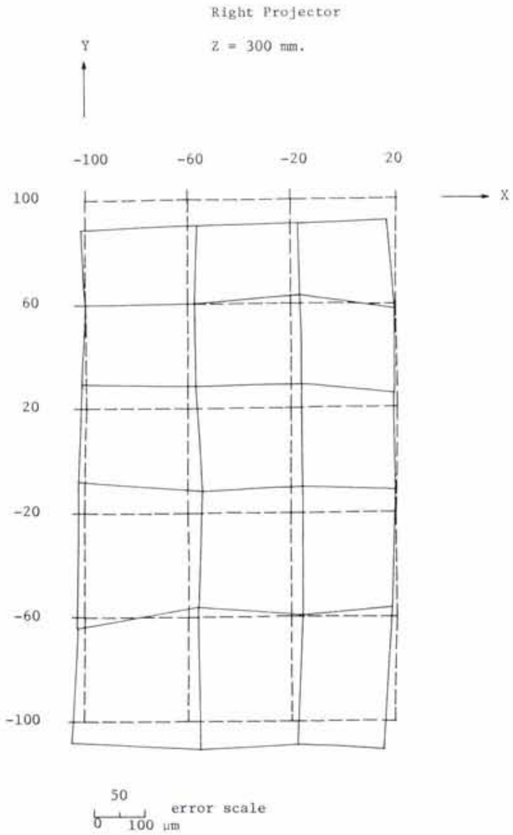

FIG. 5. Right Projector.

To identify the grid intersections, their millimeter values are used for the numbering system. However, as observed in the model space, their dimensions are not shown as millimeters. (It must be multiplied by 2 in Z = 300 mm level.)

It can be seen from Figure 4 that the left projector has an eccentricity of e_3, that is, the primary and secondary axes do not intersect. This e_3 eccentricity has the maximum influence on the distortion of grids. In addition to this, a small pseudo φ and ω exist. The right projector had considerable more distortion than was expected. (Note that the error scale is doubled as compared to Figure 4.) Analyzing the distortion pattern, there is a well pronounced e_3 eccentricity, along with e_1. That means the primary and secondary axes do not intersect and a pseudo ω is recognizable.

The calibration of the instrument by persons other than the factory trained specialist is not encouraged. Hence the effect of these systemmatic instrumental errors on aerotriangulation should be investigated.

The instrumental systematic errors affect the aerotriangulation in the form of distorted model coordinates and by the process of model connection. Because the semi-analytical method of aero-triangulation uses the projection centers for mathematical model connection, their coordinates must be determined before triangulation. This determination is rather problematic in an instrument whose primary and secondary axes do not intersect, as there is no single perspective center for such projectors. As a consequence, only an assumed "average" location of projection centers can be obtained.

Further, in an instrument where the projectors are not free from pseudo ω and pseudo φ, the location of the assumed or "average" projection centers is not constant but changing by the orientation elements of each photograph, i.e., by the amount of φ and ω. Thus this change takes place from model to model. This phenomenon can be readily seen in Figure 1B where the projection of e_1 and e_3 on the XY plane changes with the rotation of the secondary axis around the primary. Further, the φ tilt around the secondary axis will change the projection of e_3 on the XY plane. Besides these changes, the maximum effect will take place in the z direction.

Because of the foregoing, an experiment has been conducted to determine the most desirable method of determining the projection centers and to obtain numerical information for semi-analytical aero-triangulation.

SEMI ANALYTICAL AERO-TRIANGULATION

A strip of photograph obtained over the U.S. Coast and Geodetic Survey's Camera Test Area at McClure, Ohio, was used for practical experimentation. The strip consists of eight models flown with F-501, f = 152 mm. camera at the scale of 1″ = 1000′. These photographs have 60% overlap. Forty-nine premarked control points were distributed in the area in such a way that most of the points were located in two rows falling in the upper and lower edges of the photographic strip. Part of these points were considered as control points and others as points to be triangulated.

The first step, as usual, was to determine the model coordinates of the projection centers. There are several methods available to accomplish this; the best known is the physical whereby the X and Y coordinates of the perspective centers are determined by making the space rod vertical by the special level provided for calibration by the manufacturer, and determining the Z coordinates by reading grid intersections and using similar triangle concepts with known values for the instrumental focal length. There are numerous other numerical methods available to determine the coordinates of the projection centers, namely space resection (2), space intersection (3), ideal grid method (4), and others. Here, space intersection is used as the mathematical method. Two independent experiments have been made to determine the repeatability and compatibility of the methods. The standard errors of observations of coordinates were

$$\sigma_{XY} = \pm 18\,\mu m \text{ and } \sigma_z = \pm 25\,\mu m$$

The results of both experiments are given in Table 1 representing all coordinates in millimeter. It can be seen from the table that a rather large difference exists between the two methods and the repeated experiments. It can be concluded from the table that the accuracy of the projection centers will have a major influence on the obtainable accuracy of aero-triangulation.

The model connection was obtained by a three dimensional transformation. These transformations were based on the coordinates of projection centers and of the pass or transfer points. In this triangulation, six pairs of pass points were used between each neighboring model. This number of pass

TABLE 1.

	Projector			
Method		*Left*		*Right*
	X	88.520	X	268.650
		88.636		268.755
Physical	Y	301.725	Y	302.835
		301.500		302.980
	Z	382.422	Z	382.392
		382.454		382.360
	X	88.703	X	270.002
		88.687		269.988
Mathematical	Y	301.572	Y	303.809
		301.574		303.828
	Z	382.532	Z	382.477
		382.467		382.355

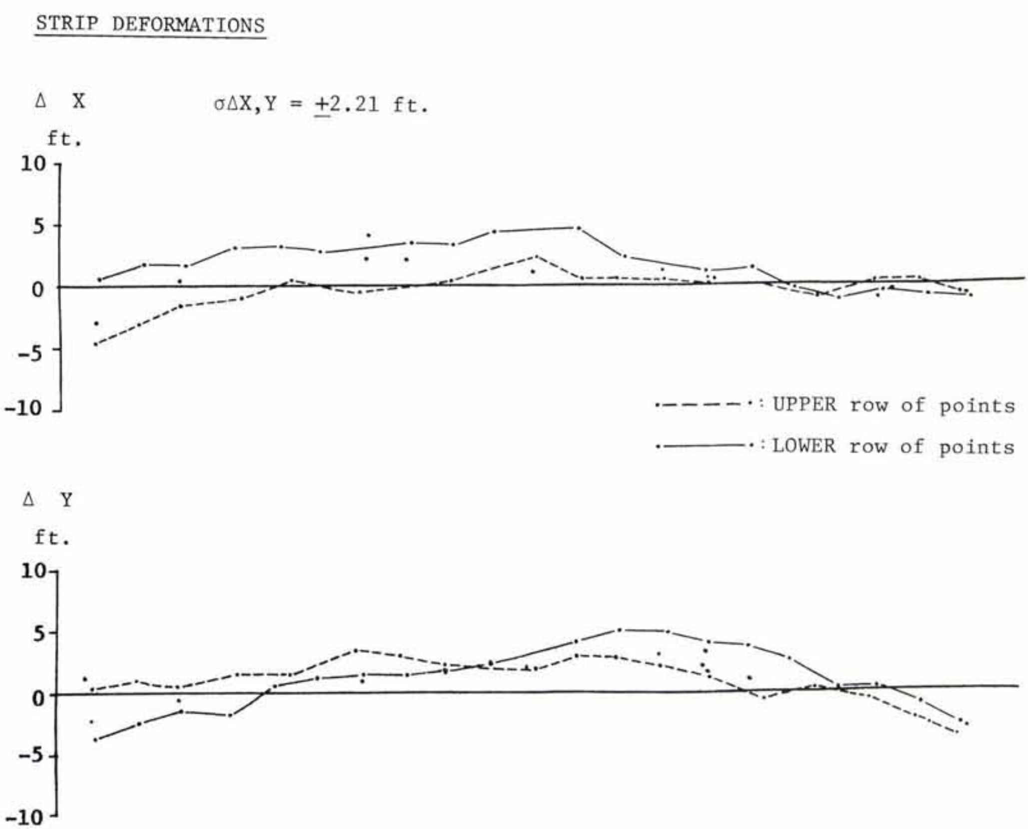

FIG. 6. Strip Deformations.

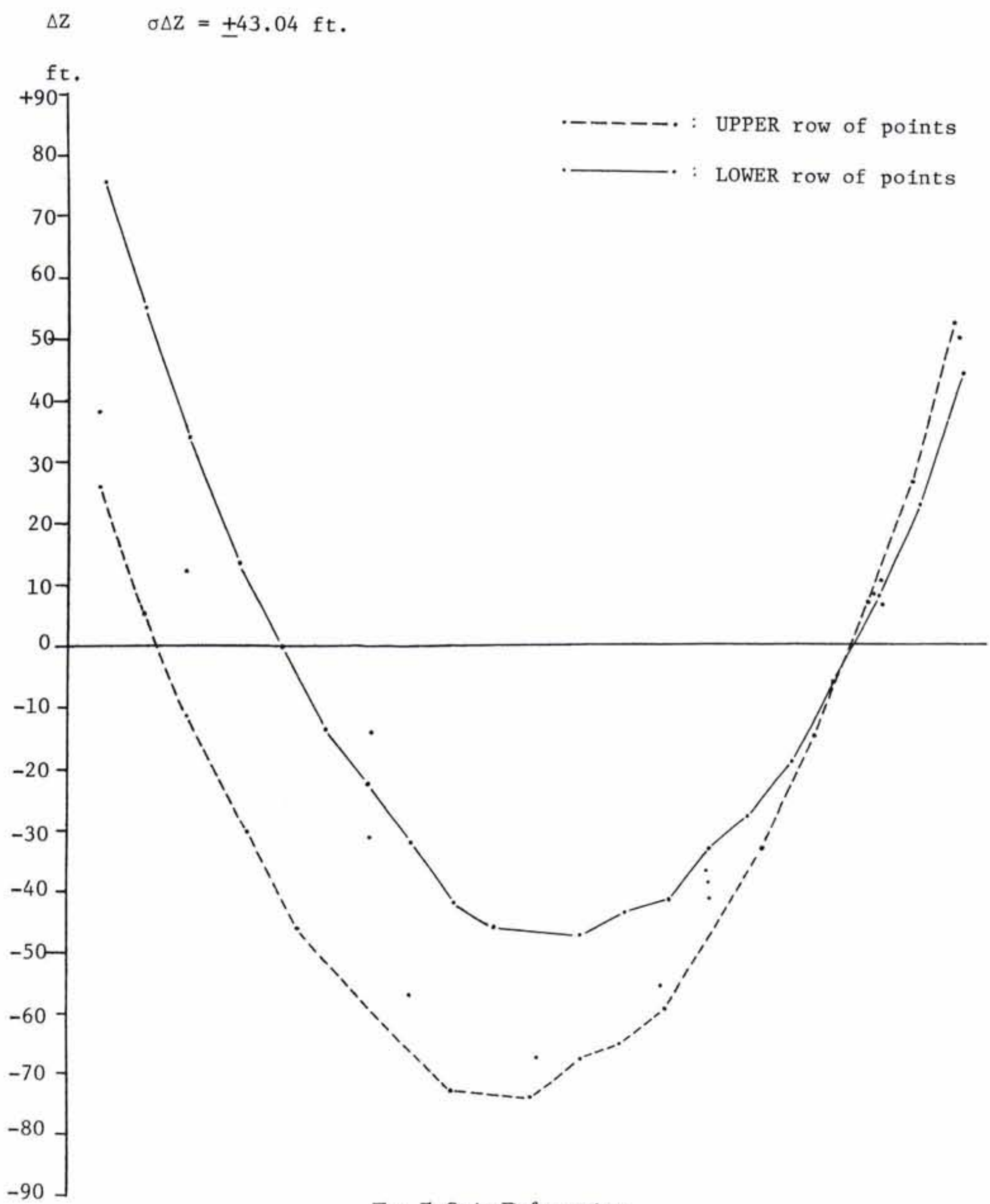

FIG. 7. Strip Deformations.

points necessitated using a least-squares adjustment.

The absolute orientation of the connected models was also performed by a three dimensional coordinate transformation. After the absolute orientation, the residual errors of the ground coordinates made by classical ground survey could then be computed.

Figures 6, 7 and 8 exhibit the obtained residual errors of strip deformation. In Figures 6 and 7 the residuals are computed from the strip where the projection centers were determined by the physical method. In Figure 8 these residual errors are shown as obtained from the strip where the coordinates of the projection centers were determined by mathematical means.

It can be seen from Figure 6 that the strip deformation in X and Y coordinates represents more or less second order curves incorporating also some scale errors. The residual standard error in X and Y computed from all points was found to be ± 2.21 ft., a good result considering that no adjustment

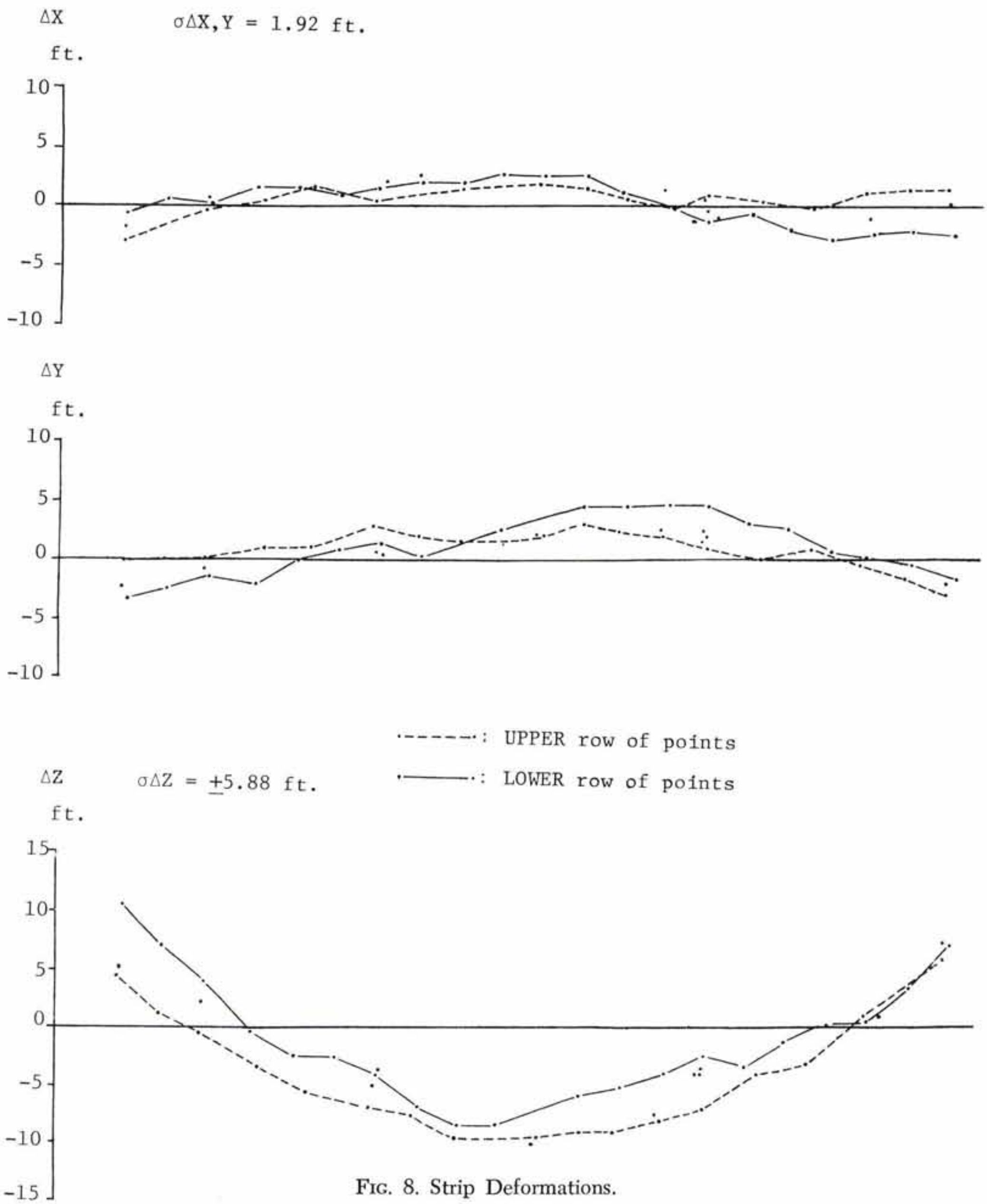

FIG. 8. Strip Deformations.

for strip deformation had yet been executed.

In Figure 7 the strip deformation in elevation is given. It can be seen from this deformation that the maximum effect of the systematic errors at the projection centers occurs in the Z direction. Because the residual ± 43.04 ft. standard error is far too large to be desirable, the use of the physical method to determine the coordinates of projection centers is discouraged.

Figure 8 gives the strip deformations obtained from the data in which the X, Y and Z coordinates of the projection centers were mathematically determined, resulting in a standard residual error of ± 1.92 ft. The deformation lines still exhibit the second order character, but scale error is somewhat less evident than in the previous method. The deformation of the elevations, however, shows a marked improvement; the standard residual error at +5.88 ft. is about one-tenth of the previous one. Since this amount can be accounted for by strip adjustment, the desirable degree of accuracy can then be real-

TABLE 2. RESIDUAL STANDARD ERRORS AFTER STRIP ADJUSTMENT IN SEMI-ANALYTICAL AERO-TRIANGULATION (INFT)

	Adjustment (XY)			*Adjustment (Z)*		
	Order of Polynomial			*Order of Polynomial*		
Method	*1st*	*2nd*	*3rd*	*1st*	*2nd*	*3rd*
Physical	2.09	0.85	0.71	43.04	9.87	1.54
Mathematical	2.25	0.86	0.69	5.88	1.12	1.01

ized.

The strips of both triangulation experiments have been adjusted by polynomial of various orders and the residual standard errors of the coordinates computed. The results are summarized in Table 2. It can be seen from the Table that a rather good result can be obtained by the mathematical method, adjusting the planimetric coordinates and the elevations with polynomials, up to third order.

This experiment shows that to a certain degree the higher order polynomial can correct the effect of systematic instrumental errors.

ANALYTICAL AERO-TRIANGULATION

It was reported in (7) that the Santoni Stereosimplex IIc can be used as a mono comparator with stereo observation because its design and construction enables this instrument to perform analytical aero-triangulation. The advantage in using analytical aero-triangulation is that the systematic instrumental errors can be determined by calibration and accounted for by numerical correction.

Table 3 shows a brief outline of the analytical aero-triangulation method.

It can be seen from the Table that the various steps in the triangulation program are separated more than in routine practice. This was required in order to obtain detailed data to compare with the semi-analytical method.

In the first step, the preparation consisted of mathematical calibration of the instrument. The method of calibration was described in detail in reference (7) and the procedure in (8); it will not be necessary to repeat it here. The calibration results in a correction matrix oriented to the collimation marks of the plate holder. The photo coordinates of any point can thus be corrected for systematic instrumental error. The photo-coordinates can then be observed the same way as in any comparator, and the reduction of the photo coordinates in Table 3 is self-explanatory.

The formation of models by triplets is similar in concept to that of the U.S.C. & G.S. method although different in mathematical detail. The strip was formed from the individual models in a manner similar to that

TABLE 3. ANALYTICAL AERO-TRIANGULATION

Preparation	Determination of Correction Matrices
	Calibration of reference system on plate holders
Observation of Photo Coordinates	
Reduction of Photo Coordinates	Correction of Instrumental Errors
	Correction for Film distortion
	Correction of Lens distortion
Formation of Models	Orientation of Triplets
Formation of Strip	Three dimensional transformations
Absolute Orientation	Three dimensional transformation
Strip Adjustment	Polynomial adjustment

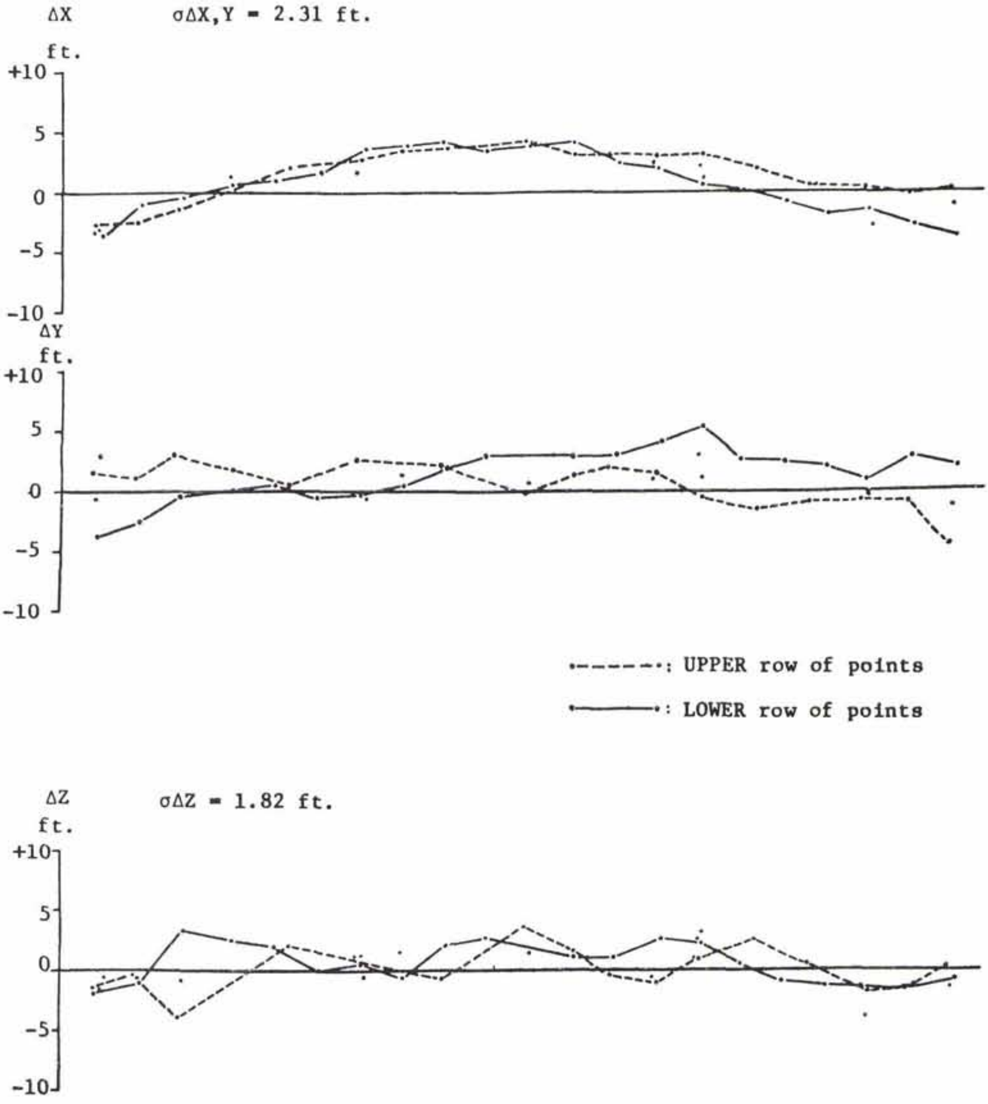

FIG. 9. Strip Deformations.

described for the semi-analytical method. The absolute orientation permitted the comparison between the given and computed ground coordinates of points.

This comparison is given in graph form in Figure 9. It can be seen from this figure that the strip deformation in planimetry is about the same as presented for the semi-analytical experiment. The deformation in the elevation shows a marked improvement. The standard residual error in Z coordinate is ± 1.82 ft. as compared to ± 5.88 ft. and 43.04 ft. for the mathematical and physical semi-analytical methods.

The results of the strip adjustment performed by polynomials are given in Table 4.

It can be seen from the Table that there is no appreciable decrease in the amount of residual errors beyond the second order polynomial. Consequently, a lower order polynomial is needed if the systematic errors are corrected.

TABLE 4. RESIDUAL STANDARD ERRORS AFTER STRIP ADJUSTMENT IN ANALYTICAL AERO-TRIANGULATION (IN. FT.)

	Adjustment (XY)			*Adjustment (XY)*		
	Order of Polynomial			*Order of Polynomial*		
Method	*1st*	*2nd*	*3rd*	*1st*	*2nd*	*3rd*
Analytical	2.33	0.93	0.90	1.82	1.01	1.06

CONCLUSIONS

The performance of the instrument according to the accuracy achieved is very good, considering that no electronic readout was used in these experiments. The model coordinates were obtained from the mechanical coordinatograph of the instrument, having a least reading of 50 micrometers and by estimation 25 micrometers. It is believed that the instrument can provide considerably better results than this coordinatograph permits; thus, the use of electronic readout is strongly recommended for a least reading of 10 micrometers or smaller.

Semi-analytical and analytical aero-triangulations, as performed with the instrument, are equally acceptable in practice. In semi-analytical aero-triangulation, the accumulation of errors is more rapid than in analytical triangulation due to the uncorrected systematic rather moderate, the adjustment of the strip instrumental errors. Since in analytical trirequires a lower order polynomial than the angulation the accumulation of errors is semi-analytical method. This means that considerably fewer control points are required for analytical than for semi-analytical triangulation, which may favorably balance the extra time requirement in the observation of photo-coordinates.

In semi-analytical aero triangulation, the distribution of ground control points is important. These points should cover the entire strip evenly. In analytical triangulation the control point distribution has less influence.

The instrument can be calibrated considerably better than experienced in this case. It must be pointed out, however, that this improved calibration can be made possible only if electronic readout is available to the instrument. The calibration should be carried out projector by projector, only after satisfactory results, and checking of the calibrations should be done by stereoscopic observation of the grid plates. It was computed that the e_3 eccentricity error was 12 micrometers, for the right projector and 5 micrometers for the left projector. It is advisable that these eccentricity errors do not exceed 5 mircometers if the instrument is to be used for semianalytical aero triangulation.

Finally, it must be pointed out that in these experiments the Santoni Stereosimplex IIc was used outside its intended field. The potential inherent in its design was exploited in an area where it had not been used before to the author's knowledge. The analytical solution provides excellent results if the instrument is handled in the same manner as the aerial camera, i.e., if its systematic error is corrected. The instrument is rather stable, maintaining its calibration values for two years.

The most important point one may infer from these experiments is that the Santoni Stereosimplex IIc is capable of aero-triangulation. Because of its design, it can provide the needed data for analytical triangulation. This makes the instrument one of the most flexible in this category of plotters.

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