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Analytic Block Adjustment

Final summary of ISP Commission III Working Group reports 1968-1972.

Abstract: *The Commission III Working Group consisted of participants from Canada, Finland, Germany, Japan and the United States with a total of 12 organizations being involved. During the period 1968-72, each participant of the Group performed independent adjustments of a 5 × 20 block of simulated near-vertical photography, having an approximate scale of 1:66,000. Methods used were: (a) simultaneous adjustment of photographs (bundle adjustment); (b) simultaneous adjustment of independent models; and (c) sequential polynomial adjustments. A single block size and uniform control arrays were specified allowing realistic comparisons. The bundle adjustments produced the most accurate results at a higher cost than simultaneous adjustment of independent models and the polynomial adjustments.*

Introduction

A Commission III Working Group report on analytical adjustment of a block of simulated aerial photographs was presented at the 1968 11th Congress of the International Society of Photogrammetry in Lausanne. A broad spectrum of block sizes and test cases was studied in that report and many interesting conclusions drawn from the results. However, the diversity of block dimensions, control arrays, and experiments in that set of tests prevented meaningful comparison of the various methods. Consequently, one of the charges to Commission III at the 11th Congress was to continue investigations of analytical block adjustment using simulated photography.

Suggestions were solicited (via a questionnaire) from potential participants with respect to: (a) test block dimensions; (b) desired sidelap and overlap; (c) points/photographs; (d) control point configurations; and (e) desired flight arrangements. On the basis of responses to these questionnaires, guidelines were established for tests with a uniform block size and specified control arrays.

The general objectives of the study were to: (1) evaluate and compare different methods of analytical aerotriangulation and adjustment of blocks using uniform block size and control configurations; (2) evaluate and compare several ground control configurations; and (3) assess effects of residual systematic perturbations remaining in observed plate coordinates after coordinate refinement.

As ultimately formed, the Working Group included 12 organizations from Canada, Finland, Germany, Japan, and the United States. Results from these 12 participants have been received and analyzed. This report constitutes a final summary and evaluation of the significant contributions of the Working Group participant's individual reports.

Simulated Test Block

A 5 × 20 block of simulated, near-vertical

* "Summary of Working Group Reports" presented at the XIIth Congress of the International Society of Photogrammetry at Ottawa, Canada, July-August 1972. Final Report presented at the Annual Convention of the American Society of Photogrammetry in Washington D.C., March 1972.

aerial photographs was provided for each of the participants.* This simulated block is composed of fictitious photographs taken from approximately 11,000 meters above terrain containing up to 1,000 meters of relief. The camera focal length is 152.00 mm resulting in an approximate photo scale of 1:66,000. Theoretically perfect or unperturbed plate coordinates are given in micrometers for an approximately rectangular array of 25 images per photograph. Two sets of perturbed plate coordinates are provided in which perturbations consist of: (1) random normal deviates having a standard deviation of 6 micrometers; and (2) random normal deviates (standard deviations of 6 micrometers) plus residual systematic deviations designed to simulate systematic errors resulting from faulty camera calibration and incomplete film distortion compensation.

* The basic data for the simulated block were generated by the United States Army Topographic Command (4) with subsequent transformations by E. H. Ramey at (NOS) NOAA.

These simulated systematic deviations were based on: (a) an analysis by Professor Egon Dorrer, University of New Brunswick, of a set of measurements of photographic film distortion made at the National Research Council of Canada; and (b) results of camera calibration studies provided by Mr. Lawrence W. Fritz of the National Ocean Survey.

Conditions for Experiment

Each participant was requested to run tests using:

1. Five strips (strips 1, 3, 5, 7, and 9) of 20 photographs each, having 20 to 25 percent sidelap.
2. Block arrays and control configurations *A*, *B*, and *C* as illustrated in Figure 1.
3. Plate coordinates for nine points per photograph, arranged as indicated in Figure 1 (Test Case B), and perturbed with:
   (a) random normal deviates *only* (Test cases 1A, 1B, 1C).

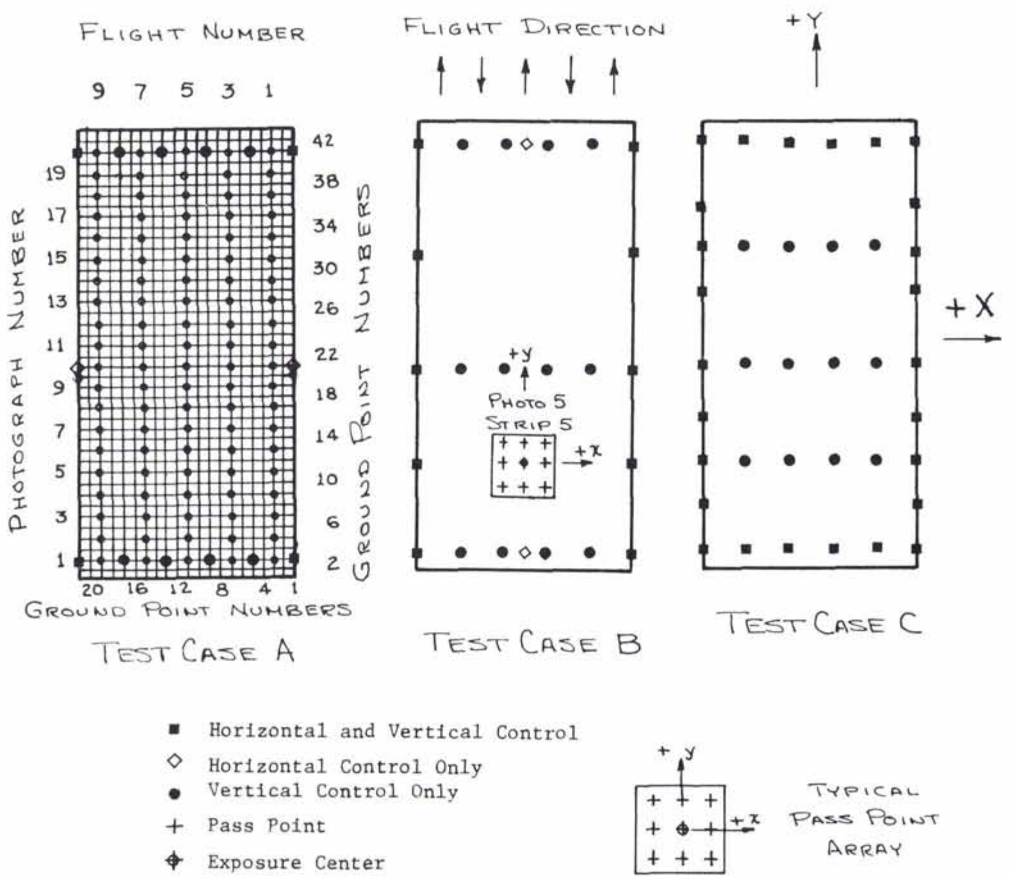

FIG. 1. Control configurations.

(b) random normal deviates *plus* residual systematic deviations (Test cases 2A, 2B, 2C).

Analytic Triangulation Procedures Tested

The procedures tested are divided into three general groups: (1) sequential adjustments; (2) simultaneous adjustment of independent models; and (3) simultaneous or bundle adjustment of photographs. Participants classified according to these groups are:

Group 1. Sequential Adjustments

1. National Research Council of Canada-Ottawa, Canada, Mr. G. H. Schut
2. University of Wisconsin, Madison, Wisconsin, U.S.A., Dr. Paul Wolf & Mr. Steven Johnson
3. Pacific Aero Survey Co., Ltd., Tokyo, Japan, Mr. Hiroshi Morito and Mr. Hitoshi Tamura
4. Asia Air Survey Company, Tokyo, Japan
5. Kokusai Aerial Survey Co., Ltd., Tokyo, Japan, Mr. Sohachi Kurihara
6. Toyo Aerial Survey Co., Ltd., Tokyo, Japan, Mr. Isamu Yamamoto
7. The Ohio State University, Columbus, Ohio, U.S.A., Dr. Sanjib K. Ghosh

Group 2. Simultaneous Adjustment of Independent Models

8. Institute Fur Angewandte Geodasie, Frankurt, West Germany, Prof. Dr. R. Forstner and Universitat Stuttgart, Stuttgart, West Germany, Prof. Dr. Ing. F. Ackermann

Group 3. Simultaneous or Bundle Adjustments

9. Helsinki University of Technology, Otaniemi, Finland, Prof. R. S. Halonen
10. United States Army Topographic Command, Washington, D.C., U.S.A., Mr. Richard L. Penrod
11. D.B.A. Systems, Inc., Melbourne, Florida, U.S.A., Mr. John A. Strahle
12. National Ocean Survey (NOS) NOAA,

Table 1. Sequential Adjustments Summary of Procedures

Participant	*Equation*	*Basic Unit*	*Unit Assembly By*	*Adjustment Procedure Remarks*
1	Coplanarity	2 photo	Concurrent with Relative Orientation	Iterative block adjustment of strips. Sequential XY and Z using specified degree polynomial
2	Collinearity	2 photo	Concurrent with Relative Orientation	Iterative block adjustment of strips. Sequential XY and Z using specified degree polynomial
3	Y-parallax	2 photo	Successive rotation and scaling	Linear transformation followed by polynomial adjustment of a specified degree
4	Y-parallax	2 photo	Concurrent with Relative Orientation	Strip 3 used as base strip. Other strips transformed into this system using 2nd degree equations.
5	Coplanarity	2 photo	Successive rotation and scaling	Strips oriented absolutely using a linear transformation. Adjust planimetry and heights separately using 1st and 2nd order conformal transformations
6	Y-parallax	2 photo	Concurrent with Relative Orientation	Method I—Planimetry and elevations adjusted separately using specified degree equation; Method II-3-D Linear Transformation
7	Collinearity	4-15 photo sub-blocks 4-10 photo sub-blocks	Linear 3-Dimensional Transformation	Planimetry and elevations adjusted simultaneously, linear in Y and Z and with the potential of using a 3rd degree term in X.

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Procedural characteristics and results obtained by individual participants classified and numbered as above are tabulated and summarized in subsequent sections. First consider a brief resume of details of the respective major groups of procedures.

SEQUENTIAL ADJUSTMENTS

Approaches for sequential procedures are categorized according to the type of condition utilized in relative orientation, size of basic unit, method of unit assembly, degree of equations employed, and method of basic unit assembly. These characteristics are summarized for Sequential Adjustments in Table 1.

SIMULTANEOUS ADJUSTMENT OF INDEPENDENT MODELS

Independent models are formed analytically in arbitrary space using the Y-parallax equations. All independent models so formed are then assembled and adjusted to ground control using a similarity transformation performed simultaneously for all models with alternating plan-height iterations.[1] A large number of unknowns are involved, resulting in banded normal equations which are solved using a recursive partitioning algorithm. Participant 8 was the only organization to develop and use this procedure.

SIMULTANEOUS ADJUSTMENTS

This group includes procedures in which the desired parameters are adjusted using a direct simultaneous least squares adjustment of the block. Estimates are required for exposure station positions and orientations plus estimated coordinates for all object points.

Procedural characteristics for simultaneous methods are listed in Table 2. All participants in this group used the collinearity condition equation for the adjustment.

WEIGHTS

Choice of weights can influence the results of the adjustment. In the sequential procedures weights (as given by those participants reporting use of weights) were assigned to ground control points relative to a weight of one for tie points between strips. Weights assigned to ground control points, as reported by participants 1 and 2 are summarized in Table 3 along with degree of equation utilized for the adjustment and number of iterations required for convergence. Weight as defined in Table 3 is the value by which the contribution of a point to the normal equations is multiplied.

Weights incorporated into simultaneous solutions are generally taken as being inversely proportional to the estimated variances of the observed values. Factors used by Participants 9, 10, 11, and 12 for weighting their respective simultaneous solutions are tabulated in Table 4.

TEST BLOCK CONTROL CONFIGURATIONS

Control configurations were specified and are illustrated in Figure 1. Note that arrays *A*, *B*, and *C* represent near minimum, mod-

TABLE 2. SIMULTANEOUS ADJUSTMENT SUMMARY OF MATHEMATIC PROCEDURES

Participant	*Parameters Adjusted in Simultaneous Solution*	*System of Normals Solved By*	*Estimates Required For*	*Manners of Acquiring Estimates*
9	$(X, Y, Z, \omega, \Phi, \kappa)_o$ $i = 1, 2, \ldots, m$ $(XYZ)_j$ $j = 1, 2, \ldots, n$	Iterative Method	Exp. Sta. Parameters Ground Points	Analytic Sequential Triangulation
10	$(X, Y, Z, \omega, \Phi, \kappa)_o$ $i = 1, 2, \ldots, m$ $(XYZ)_j$ $j = 1, 2, \ldots, n$	AUTORAY Algorithm	$(X, Y, Z, \omega, \Phi, \kappa)_o$ $(XYZ)_j$	Perturbed Exposure Sta. and Orientations and Ground Coordinates
11	$(X, Y, Z, \omega, \Phi, \kappa)_o$ $(XYZ)_j$	Recursive Partitioning	$(X, Y, Z, \omega, \Phi, \kappa)_{oi}$ $(XYZ)_j$	$(\omega, \Phi, \kappa)_o = (0.0, 0.0, 0.0)$, $(X, Y, Z)_o$ for 1st and last photo in each strip scaled from base map. Use approx. least squares algorithm to calculate $(XYZ)_j$ for $j = 1, 2, \ldots, n$
12	$(X, Y, Z, \omega, \Phi, \kappa)_o$ $(XYZ)_j$	Gauss-Cholesky Elimination	$(X, Y, Z, \omega, \Phi, \kappa)_{oi}$ $(XYZ)_j$	Preliminary solution using 3 photo strip adjustment program

TABLE 3. WEIGHTS USED IN SEQUENTIAL PROCEDURES

Participant	*Test*	*Wt. Applied* *Planimetric*	*Elevation*	*Deg.* *XY*	*Deg.* *Z*	*Number of* *Iterations*
1	1A	20	10	2	2	6
1	1A	20	10	3	2	15
1	1B	10	5	3	3	6
1	1C	5	2.5	3	3	4
2	1A	15	10	2	2	14
2	2A	15	10	2	2	15
2	1B	15	10	3	3	18
2	2B	15	10	3	3	20
2	1C	15	10	3	3	12
2	2C	15	10	3	3	20

Note: Weights for tie points between strips equal to unity.

TABLE 4. FACTORS USED FOR WEIGHTING SIMULTANEOUS ADJUSTMENT OF PHOTOGRAPHS

Participants	*Plate Coord. (Micrometers)*	*Weights* *Ex. Sta. Parameters*	*Ground Positions*
9	$\sigma_X = \sigma_Y = 6$ for all images, all tests	°	°
10	$\sigma_X = \sigma_Y = 6$ all images for random sample $\sigma_X = 18$, $\sigma_Y = 8.5$ random + sys.	$\sigma_{X_o} = \sigma_{Y_o} = \pm 100$ meters $\sigma_{Z_o} = \pm 50$ meters $\sigma_W = \sigma\varphi = \pm 0°30'0''0$ $\sigma_U = \pm 1°00'0''0$	$\sigma_X = \pm 0.3$ m $\sigma_Y = \pm 0.3$ m $\sigma_Z = \pm 0.3$ m
11	$\sigma_X = 6$ $\sigma_Y = 6$ all images, all tests	$(X, Y, Z, \omega, \Phi, \kappa)_{oi}$ adjusted as free parameters	(X Y Z) Ground Control Points Assumed Errorless, (X Y Z) Pass Points adjusted as free parameters
12	Weighting is empirical. Ground control parameters have a factor of 5 while the corresponding observation equations in vx and vy are given a weight factor of 3. Camera parameters are not weighted. Image weighting is a function of point location relative to photo center and image identifiability.		

TABLE 5. NUMBER OF CONTROL POINTS AND CHECK POINTS

Array	*Planimetric Pts.* *Control*	*Check*	*Vertical Pts.* *Control*	*Check*
A	6	214	12	208
B	12	208	22	198
C	26	194	38	182

erate, and dense amounts of control, respectively. The number of planimetric and height control and check points in these arrays are shown in Table 5.

In general, participants restricted their tests to the specified arrays *A*, *B*, and *C*. However in one instance, Participant 1 modified array *A* by adding two elevations to the planimetric control points located at the mid-points of the sides of the block.

TEST RESULTS

Tests using specified control arrays are labeled as shown in Table 6.

RESULTS WITH SPECIFIED TESTS

Test results for all participants and all tests are tabulated in Table 7. Displayed in

TABLE 6. TYPES OF CONTROL ARRAYS

Control Point Array	*Test Case Label*	*Plate Coordinate Perturbations*
A	1A	Random Normal
B	1B	Deviates only
C	1C	($\sigma = 6$ micrometers)
		Random Normal
A	2A	Deviates
B	2B	($\sigma = 6$ micrometers)
C	2C	plus residual systematic deviations

TABLE 7. RMSE (IN METERS) DISCREPANCIES IN PLANIMETRY AND ELEVATIONS OF CHECK POINTS, ALL TESTS

		1A *Random Only*		*2A* *Random + Sys.*		*1B* *Random Only*		*2B* *Random + Sys.*		*1C* *Random Only*		*2C* *Random + Sys.*	
	Participant	m_{XY}	m_Z	m_{XY}	m_Z	m_{XY}	m_Z	m_{XY}	m_Z	m_{XY}	m_Z	m_{XY}	m_Z
Sequential	1	{3.1** {2.5**	3.3** —	{3.5** {2.6**	3.5** —	1.4	2.1	1.5	2.4	{1.3 {1.1	{1.7 {1.2	1.3	1.4
	2	4.14	5.33	6.11	6.10	1.74	2.24	2.54	2.53	1.66	1.71	2.21	1.89
	3	5.80	9.73	10.77	16.90	{4.67 {1.94	{2.99 {2.99	{10.02 { 2.24	{8.66 {8.66*	{1.80 {1.87	{2.76 {2.75	{2.10 {3.26	{7.92 {2.81
	4	2.71	25.09	—	—	2.67	2.89	—	—	2.58	2.33	—	—
	5	7.67	9.84	11.98	15.80	3.04	3.52	2.71	6.93	1.52	2.18	2.66	3.24
	6	3.95	7.08	3.97	7.33	2.57	3.33	2.90	3.38	{2.23 {1.28*	{3.07 {1.32*	{2.55 {1.87*	{3.13 {1.38*
	7	18.38	15.56			6.82	13.51			5.12	8.80		
	8†	1.34	4.73	2.27	5.20	1.09	1.82	1.92	2.45	0.95	1.18	1.33	1.66
Simultaneous	9	1.08	2.03	2.34	2.39	0.67	1.66	1.31	2.03	0.51	0.88	0.69	1.32
	10	1.14	5.92	2.22	9.65	0.68	1.54	—	—	0.49	0.83	—	—
	11	1.14	2.70	2.21	3.18	0.74	1.66	2.12	2.25	0.51	0.78	0.68	0.89
	12			1.45	3.19			1.41	1.86			0.70	0.90

* 3 sections per strip
** using modified control Array A
† simultaneous adjustment of independent models

TABLE 7(a). NUMBER OF CHECK POINTS USED

Participant	Number of Check Points Used 1A & 2A XY	1A & 2A Z	1B & 2B XY	1B & 2B Z	1C & 2C XY	1C & 2C Z
1†	214	206	208	198	194	182
1†	214	206	208	196	194	182
2	214	208	208	196	194	182
3	214	208	208	196	194	182
4	214	208	208	196	194	182
5	214	208	208	196	194	182
6	214	208	208	196	194	182
7	214	208	208	196	194	182
8	182	182	182	182	182	182
9	206	206	196	196	194	182
10	208	198	208	198	194	182
11	214	208	208	198	194	182

† Performed with Test Array A Modified Using 21-1 and 21-21 as Horizontal and Vertical Control

this table are: root-mean-square errors (RMSE)* of discrepancies in position (m_{XY}) and elevation (m_Z) for Test Cases 1A, 2A, 1B, 2B, 1C, 2C. The number of check points used by each participant to calculate respective RMSE's are given in Table 7(a). Note that all participants did not use the same number of check points but utilized the minimum number in Test Cases 1C, 2C. The maximum differences in number of points are not large (182 *vs.* 208) but because the deleted points are on the block perimeter (weaker points) a significant difference in the RMSE could occur.

The RSME's in position and elevation for exposure stations are listed in Table 8 for all test cases.

Additional items of interest output from direct simultaneous procedures are the estimated standard deviations of unit weight, m_o, in plate coordinate residuals for the respective adjustments, listed in Table 9. Variance ratios are given in Table 10. Note that no significant difference exists between calculated and tabulated values of F for Test Cases 1A, 1B and 1C indicating that a valid distribution of random normal deviates were applied to the plate coordinates. On the other hand, comparison of calculated with tabular values of F for Test Cases 2A, 2B, and 2C, indicates the presence of a significant amount of systematic error in the plate coordinates at the 90 percent confidence interval.

* RMSE (root-mean-square error) $= (\Sigma v^2/n)^{1/2}$ where v = calculated value minus true value and n = number of check points.

TABLE 8. RMSE (IN METERS) FOR DISCREPANCIES IN EXPOSURE STATIONS FOR ALL TEST CASES

Group	Participant	1A Random Only m_{XY}	1A m_Z	2A Random + Sys. m_{XY}	2A m_Z	1B Random Only m_{XY}	1B m_Z	2B Random + Sys. m_{XY}	2B m_Z	1C Random Only m_{XY}	1C m_Z	2C Random + Sys. m_{XY}	2C m_Z
Sequential	2	5.37	7.07	8.56	10.09	3.59	3.60	7.06	5.33	3.29	2.70	5.58	3.31
Sequential	6	7.11	6.66	—	—	5.38	2.65	—	—	5.13	2.54	—	—
	8	8.56	4.35	9.55	4.44	9.02	1.30	9.86	1.61	8.46	0.72	8.95	1.09
Simultaneous	9	2.80	1.74	6.06	1.82	3.58	1.46	5.91	1.47	1.67	0.54	3.00	0.92
Simultaneous	10	3.80	5.67	9.26	8.96	1.96	1.32	—	—	1.29	0.44	—	—
Simultaneous	11	2.96	2.40	6.20	2.72	2.22	1.49	5.70	1.87	2.23	0.40	2.55	0.43
Simultaneous	12			5.21	2.89			5.23	1.58			2.57	0.41

COMPARISONS

A comparison of the average RMSE's in the lowest discrepancies of Participants 1, 2, 3, and 6 (Sequential Polynomial Adjustments) with Participants 9, 10, 11, and 12 (Simultaneous or Bundle Adjustments) is possible by examining Table 11. Also given in this table are the percentage changes in average RMSE's.

On the average, Participants 1 (Sequential) and 9 (Simultaneous), which were

TABLE 9. ESTIMATED STANDARD DEVIATIONS OF UNIT WT. FOR SIMULTANEOUS ADJUSTMENTS

Participant	*1A*	*2A*	*1B*	*2B*	*1C*	*2C*
9	1.05	1.67	1.05	1.65	1.05	1.72
10	1.03	0.92	1.00	–	0.96	–
11	1.05	1.67	1.05	1.65	1.05	1.74

Note: Assuming plate coordinates are equally weighted inversely proportional to the estimated variances, then the estimated unit variance σ_0^2 for the ideal case would be unity.

among those performing all tests, achieved the lowest discrepancies in their respective groups. A comparison of RMSE's in discrepancies for these two participants is given in Table 12. The percent change through use of the direct simultaneous solution (Participant 9) is from −13 to −56 in position and from −17 to −50 in elevation.

The simultaneous linear transformation of independent models (Participant 8) is a relatively new development. A comparison between simultaneous, independent models (Participant 8) and a sequential polynomial adjustment (Participant 1) is given in Table 13.

COMPUTER STORAGE AND TIME REQUIREMENTS

Comparisons among procedures with respect to time are not too meaningful due to the variable characteristics of different computer systems. Unfortunately, no single participant performed both a sequential and simultaneous adjustment on the same system. Consequently, a valid comparison of times for these two basic groups of procedures is not feasible with the data available. Central processor time for sequential procedures varied from 1 to 8 seconds per photograph while simultaneous solutions require from 2 to 10 seconds per photograph. Thus, sequential procedures still require less time than do simultaneous methods, but the gap is closing. Increased efficiency of solving the normal equations by iterative methods and recursive partitioning is most probably the reason for this narrowing gap.

CONCLUSIONS

Formal reports including conclusions were not solicited from working group participants. The conclusions that follow represent those drawn by the authors.

- Using uniform block size and control arrays the simultaneous or bundle adjustment of photographs produced average RMSE's in planimetry and heights 16 to 71 and 44 to 53 percent smaller, respectively, than were achieved by computing with sequential procedures (refer to Table 11).
- Using the near-minimum control array

TABLE 10. VARIANCE RATIOS, SIMULTANEOUS ADJUSTMENTS

Test Case	f_1	f_2	m_0^2/σ_0^2 9	10	11	*Tabular F*	*Remarks*
1A	528	∞	1.11	1.06	1.10	1.1	
1B	572	∞	1.10	1.00	1.10	1.1	No Significant Difference
1C	660	∞	1.11	0.92	1.10	1.1	
2A	528	∞	2.81	0.85	2.78	1.1	
2B	572	∞	2.73	–	2.72	1.1	Significant Difference
2C	660	∞	2.95	–	3.03	1.1	

TABLE 11. COMPARISON AVERAGE RMSE DISCREPANCIES SEQUENTIAL VS. SIMULTANEOUS

	Average RMSE in Discrepancies (meters) for Participants					
Test Case	*1*,2,3,6 Sequential* m_{XY}	m_Z	*9,11,12 Simultaneous or Bundle Adj.* m_{XY}	m_Z	*Percent Change Through Use of Simultaneous Solution* m_{XY}	m_Z
1A	3.89	6.36	1.12	3.55	−71	−44
2A	5.39	8.45	2.06	4.60	−62	−46
1B	2.46	2.73	0.70	1.62	−72	−41
2B	1.91	4.24	1.61	2.05	−16	−51
1C	1.46	1.75	0.50	0.83	−66	−53
2C	1.81	1.87	0.69	1.04	−63	−44

* Using modified control array for Tests 1A, 2A.

TABLE 12. COMPARISON RMSE IN DISCREPANCIES PARTICIPANTS 1 (SEQUENTIAL) AND 9 (SIMULTANEOUS)

	Average RMSE in Discrepancies (meters) for Participants					
Test Case	*1 Sequential* m_{XY}	m_Z	*9 Simultaneous or Bundle Adj.* m_{XY}	m_Z	*Percent Change Through Use of Simultaneous or Bundle Adjustment* m_{XY}	m_Z
1A	2.5*	3.3*	1.08	2.03	−56	−40
2A	2.6*	3.5*	2.34	2.39	−12	−31
1B	1.4	2.1	0.67	1.66	−50	−19
2B	1.5	2.4	1.31	2.03	−13	−17
1C	1.1	1.2	0.51	0.88	−55	−25
2C	1.3	1.4	0.69	1.32	−46	−50

* Using modified control Array *A* for Tests 1A, 2A.

TABLE 13. COMPARISON RMSE IN DISCREPANCIES PARTICIPANTS 1 (SEQUENTIAL) (SIMULTANEOUS INDEPENDENT MODELS)

	Average RMSE in Discrepancies (meters) for Participants					
Test Case	*1 Sequential* m_{XY}	m_Z	*8 Simultaneous Indep. Models* m_{XY}	m_Z	*Percent Change Through Use of Simultaneous Indep. Models* m_{XY}	m_Z
1A	2.5*	3.3*	1.34	4.73	−48	+34
2A	2.6*	3.5*	2.27	5.20	−15	+49
1B	1.4	2.1	1.09	1.82	−21	−14
2B	1.5	2.4	1.92	2.45	−28	+5
1C	1.1	1.2	0.95	1.18	−10	—
2C	1.3	1.4	1.33	1.66	—	+14

* Using modified control array for Tests 1A, 2A.

TABLE 14. COMPARISON RMSE IN DISCREPANCIES PARTICIPANTS 8 (SIMULTANEOUS, INDEP. MODELS) AND 9 (SIMULTANEOUS, OR BUNDLE ADJ.)

	Average RMSE in Discrepancies (meters) for Participants					
Test Case	*8 Simultaneous Indep. Models* m_{XY}	m_Z	*9 Bundle Adj.* m_{XY}	m_Z	*Percent Change in RMSE Through Use of Simultaneous or Bundle Adjustment* m_{XY}	m_Z
1A	1.34	4.73	1.08	2.03	−19	−57
2A	2.27	5.20	2.34	2.39	+3	−54
1B	1.09	1.82	0.67	1.66	−37	−9
2B	1.92	2.45	1.31	2.03	−16	−17
1C	0.95	1.18	0.51	0.88	−46	−25
2C	1.33	1.66	0.69	1.32	−48	−20

* Using modified control Array *A* for Tests 1A, 2A.

A (Figure 1) as a base unit, it is possible to state in approximate terms that: (a) *if random perturbations only* are present, doubling the control results in reduction of planimetric and vertical discrepancies of 40 and 55 percent, respectively; and (b) quadrupling the control yields decreases in discrepancies of ~50 and ~65 percent in planimetry and elevation, respectively. Similarly if *random + systematic perturbations* are present, doubling control results in decreases in planimetric and vertical discrepancies of 55 percent, whereas quadrupling control yields a decrease of about 70 percent.

- Systematic perturbations applied to image plate coordinates produced significant systematic errors as indicated by statistical tests of the standard error of unit weight from the simultaneous adjustments, with one exception. Participant 9 (Tables 4, 9, 10) selected weights for the Y-plate coordinates based on the random and systematic errors in the sample, and chose the weights for X-plate coordinates so as to produce a standard deviation of unit weight for the image coordinate residuals close to unity. Information of this type is usually not available and selection of weights would be considerably more approximate. Hence, it is felt that the simulated systematic perturbations are significantly large.

- In sequential procedures where polynomials are used for block adjustment, 3rd-degree equations are necessary to correct for systematic errors. Division of strips into sections (3 sections/strip) for block adjustment produced a substantial decrease in discrepancies (Table 7).
- Use of 25 points per photograph resulted in less than a 10 percent decrease in the RMSE in discrepancies. (Participant 1 results not tabulated). Use of 25 points per photograph and 60 percent sidelap resulted in decreases in RMSE's of discrepancies of about 30 and 60 percent, respectively, in planimetry and elevation. (Participant 7, results not tabulated).
- The simultaneous linear adjustment of independent models (Participant 8) if compared with the sequential polynomial adjustment of Participant 1 showed changes in the RMSE for position and elevation of from zero to −48 percent and −14 to +49 percent, respectively (Table 13). A comparison of the bundle adjustment of Participant 9 with Participant 8 revealed changes in RMSE for position and elevation of +3 to −48 and −9 to −57 percent, respectively (Table 14).
- Tests performed using UTM versus secant-plane coordinates revealed no significant differences in the RMSE's in discrepancies. These tests were run by Participant 1 and are not tabulated in the report.

The simulated data block continues to be a powerful tool for experimental studies in block adjustment. Further efforts should be made to determine the proper parameters for generating simulated residual systematic perturbations which best duplicate those found in practical applications. Subsequent efforts with simulated blocks should be directed toward experimental studies for establishing: (a) criteria for weighting which reflect the true worth of observed values; (b) realistic specifications for the observed quantities utilized in the triangulation adjustment; and (c) optimum ground control point arrays.

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