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Analysis of Panoramic Photographs

Because of their high resolution and satisfactory results, pan photos are considered to be advantageous for various problems.

Abstract: *It is necessary in panoramic imagery to consider the time-differential terms due to the motion of the aircraft during the period of the exposure. An analytic procedure is developed for taking these time-differentials into proper account.*

Orientation Definition

The panoramic photograph is obtained by means of a scanning procedure. On the average, the scanning velocity is about one radian per second, and the time required for producing a photograph is in the order of 1000 times longer than the exposure time of a usual photograph. In evaluating an ordinary photograph, a basic assumption is accepted which regards the entire frame as recorded simultaneously, and that each region on the photograph has one and the same orientation in space with regard to an assumed reference system. This assumption does not necessarily hold for a panoramic photograph.

Because of the duration of the scanning, it stands to reason that the camera does not maintain a constant angular orientation within the time interval needed for taking a photograph, and that its path does not lie in a horizontal plane. In addition, it may be assumed that the movement of the camera is not compensated entirely by the IMC. The logical deduction from these assumptions is that the time element has to be incorporated in the description of the camera orientation, and that the orientation elements should be regarded as variables depending on time. In accordance with the above statements, the mathematical model of describing the orientation of the photograph is expanded by introducing six additional elements, namely, three linear and three angular velocities of the camera. The orientation of the photograph is defined then by the following elements:

$$
\begin{aligned}
X_0 &= X_0^c + t\,\dot{X}_0 \\
Y_0 &= Y_0^c + t\,\dot{Y}_0 \\
Z_0 &= Z_0^c + t\,\dot{Z}_0 \\
\omega &= \omega^c + t\,\dot{\omega} \\
\varphi &= \varphi^c + t\,\dot{\varphi} \\
\kappa &= \kappa^c + t\,\dot{\kappa} \;.
\end{aligned}
\qquad (1)
$$

The quantities denoted by the superscript c are the constant components of the orientation elements. They relate to an initial instant t_0 from which the time is counted, and are equivalent to the usual definition of an orientation. The quantities with the dots are the respective velocities.

The panoramic photograph is in effect an assembly of exposed slits. From Equations 1 it follows that each slit has its own particular set of orientation elements. To use Expressions 1 for computation purposes, a means to relate the time must be established. Time is usually measured by a physical process occurring at a uniform rate. Such a process is available in the camera; it is the movement of the film. Regarding the sweep velocity of the film as constant, the time can be expressed as:

$$t = C\,y' \qquad (2)$$

where y' is measured along the photograph.

For the sake of simplicity, the proportionality factor C in Expression 2 is made equal to *1*. Then the time will be expressed in terms of the ordinate y', i.e., $t = y'$.

The instant t_0 is accepted to be that at which the slit defined by the equation $y' = 0$ is recorded. All time instants which precede t_0 are negative; they correspond to negative y'-values. Similarly, time instants which follow t_0 are associated with positive y'-values.

Coordinate Axes on the Photograph

Measuring coordinates requires the definition of a reference system. This may present certain difficulties because there are usually no fiducial marks on the camera. But even in the absence of fiducial marks a reference system can still be introduced on the photograph. The y'-axis is defined as a line parallel to the long edge of the frame, and the x'-axis is assumed to be perpendicular to it, i.e., parallel to the axis of the slit. The x'-axis may be located on the photograph arbitrarily. It is convenient to place it approximately in the middle of the photograph. The origin of the reference system coincides with the center point of the x'-axis (see Figure 1).

In the figure x,y represent the axes of the measuring device, x',y' represent the axes of the photograph, *1* is an arbitrary *fiducial mark* with coordinates x_1,y_1, and *2* is a *fiducial mark* with coordinates x_2,y_1.

It is mandatory that $y' \| y$ which requires appropriate positioning of the photograph on the measuring device.

Hence, the coordinates of the origin O are given by

$$x_0' = (x_1 + x_2)/2 \qquad y_0' = y_1 \tag{3}$$

and the coordinates of a point in the photograph system are:

$$x' = x - x_0' \qquad y' = y - y_0' \tag{4}$$

The choice of the location of the x'-axis defines the initial time instant t_0 and affects the components of the orientation elements.

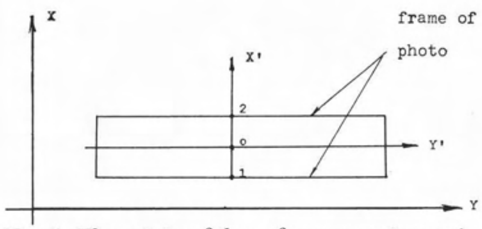

FIG. 1. The origin of the reference system coincides with the center point of the x'-axis.

Orientation of a Single Photograph

The projective relations between image coordinates (x',y',f) of a point lying in a slit recorded at the instant $t = y'$ and their counterparts (X,Y,Z) in the ground system are expressed as follows:

$$\begin{aligned} X &= X_0^c + t\,\dot{X}_0 + (Z - Z_0^c - t\,\dot{Z}_0)\,u/w \\ Y &= Y_0^c + t\,\dot{Y}_0 + (Z - Z_0^c - t\,\dot{Z}_0)\,v/w\ . \end{aligned} \tag{5}$$

The components u,v,w are obtained from the transformation

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \mathbf{A} \begin{bmatrix} x \\ y \\ -f \end{bmatrix} \tag{6}$$

The matrix **A** is derived from the orientation angles ω,φ,κ each one being a sum of two terms as shown by Formulas 1. The coordinates x,y,f entering into Expression 6 are deduced from the panoramic photograph in accordance with the known transformation

$$\begin{aligned} \theta &= y'/f \\ x &= x'/\cos\theta \\ y &= f\tan\theta\ . \end{aligned} \tag{7}$$

Expressions 5 can be used to solve the orientation elements of the photograph. For this purpose the expressions have to be linearized, which requires the knowledge of an initial set of approximate elements. These are deduced easily by assuming that the six velocities are equal to zero and by solving the remaining six elements by a usual resection in space. Expanding the Formulas 5 into a series with respect to the initial solution and neglecting all non-linear terms, yields:

$$\begin{aligned} X &= \bar{X} + dX_0^c + t\,d\dot{X}_0 - \frac{\bar{u}}{\bar{w}}\,dZ_0^c - t\,\frac{\bar{u}}{\bar{w}}\,d\dot{Z}_0 + \\ &\quad (Z - \bar{Z}_0^c - t\bar{\dot{Z}}_0)\frac{du}{\bar{w}} - (Z - \bar{Z}_0^c - t\bar{\dot{Z}}_0)\frac{\bar{u}}{\bar{w}}\frac{dw}{\bar{w}} \\ Y &= \bar{Y} + dY_0^c + t\,dY_0 - \frac{\bar{v}}{\bar{w}}\,dZ_0^c - t\,\frac{\bar{v}}{\bar{w}}\,dZ_0 + \\ &\quad (Z - \bar{Z}_0^c - t\bar{\dot{Z}}_0)\frac{dv}{\bar{w}} - (Z - \bar{Z}_0^c - t\bar{\dot{Z}}_0)\frac{\bar{v}}{\bar{w}}\frac{dw}{\bar{w}} \end{aligned} \tag{8}$$

All quantities derived from the approximate orientation elements are denoted with a bar.

The differentials du, dv, dw are obtained from differentiating Equation 6 with respect to both components of each orientation angle:

$$\begin{bmatrix} du \\ dv \\ dw \end{bmatrix} = \begin{bmatrix} \partial u/\partial\kappa & \partial u/\partial\varphi & \partial u/\partial\omega \\ \partial v/\partial\kappa & \partial v/\partial\varphi & \partial v/\partial\omega \\ \partial w/\partial\kappa & \partial w/\partial\varphi & \partial w/\partial\omega \end{bmatrix} \begin{bmatrix} d\kappa \\ d\varphi \\ d\omega \end{bmatrix} + t \begin{bmatrix} \partial u/\partial\kappa & \partial u/\partial\varphi & \partial u/\partial\omega \\ \partial v/\partial\kappa & \partial v/\partial\varphi & \partial v/\partial\omega \\ \partial w/\partial\kappa & \partial w/\partial\varphi & \partial w/\partial\omega \end{bmatrix} \begin{bmatrix} d\dot{\kappa} \\ d\dot{\varphi} \\ d\dot{\omega} \end{bmatrix}. \quad (9)$$

Inserting the *du, dv, dw*, from Equation 9 into 8 provides a system of observation equations which includes 12 unknowns. Hence, six control points must be available on the photograph. If the number of control points exceeds six, the unknowns are solved from an adjustment procedure. The solved corrections are added to the previously existing orientation elements and a new approximate orientation is formed. The orientation process is terminated and the orientation is regarded as completed when a predefined criterion has been satisfied.

It should be stressed that for each iteration step, the expansion of Equation 5 is made around the last set of orientation elements, which implies that all coefficients in Equation 8 have to be computed for each step anew. Secondly, the matrix **A** for computing the coordinates u, v, w, as well as its derivatives for the determination of *du, dv, dw*, vary with each control point, and have to be computed for each point separately.

A Pair of Photographs

Due to the unbalanced format of the panoramic photograph, it is not advisable to base the solution of the orientation only on points lying in the overlapping area of the two consecutive photographs. The system of equations from which the elements have to be solved may be ill-conditioned. Therefore control points located at the edges of the photographs, outside the overlapping zone have to be considered.

The following scheme for forming a model is recommended. A limited number of points well distributed over the overlapping area is selected and incorporated into the orientation process (in the following they are referred to as new points). The process is iterative and consists of a repetition of two basic steps, namely, resections for orienting each photograph of the pair and intersections for determining coordinates of new points. The procedure starts with an approximate orientation of both photographs which is obtained from performing two resections as described in the preceding section using the given control points. If the number of points available is insufficient, the additional six elements for each photograph are disregarded. With these orientation elements a series of intersections is computed and the coordinates of the new points are determined.

The intersection itself is performed as follows. The relation between the corresponding rays in the photographs and their counterparts in the ground system can be presented as

$$\vec{r}_1 \times \vec{R}_1 = \vec{v}_1 \qquad (10)$$
$$\vec{r}_2 \times \vec{R}_2 = \vec{v}_2$$

The right hand side of Equations 10 are non-zero vectors. Because of the incomplete orientation it is assumed *a priori* that the vectors $\vec{r}$ in the oriented photograph system, and $\vec{R}$ related to the ground system, are not collinear. Each one of the Expressions 10 provides three observation equations as follows:

$$\begin{bmatrix} 0 & -w & v \\ w & 0 & -u \\ -v & u & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} - \begin{bmatrix} 0 & -w & v \\ w & 0 & -u \\ -v & u & 0 \end{bmatrix} \begin{bmatrix} X_0^c + t\,\dot{X}_0^c \\ Y_0^c + t\,\dot{Y}_0^c \\ Z_0^c + t\,\dot{Z}_0^c \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}. \quad (11)$$

The components u, v, w of the vector $\vec{r}$ are found from the transformation presented by Formula 6. The two Equations 10 provide a system of equations for the determination of the coordinates X, Y, Z of the intersected point. The quantities entering into the observation equations have not been directly observed. Nevertheless they can be regarded as quasi-observations and the required coordinates may be found from an adjustment procedure which disregards the existing correlations. The application of an adjustment which considers correlated quantities does not seem justified here. The assumption inherent in Formula 10 implies that the two corresponding rays are skew to each other. It can be shown that the determination of the coordinates X, Y, Z of the point by minimizing the sum of squares of the components of r_1 and r_2, locates the intersected point in the middle of the shortest distance between the two skew rays[1].

The coordinates of the new points obtained after performing all intersections enter into the resection procedures, and the orientation of the two photographs is determined anew. Here a question arises of how to weight the coordinates of the various points. It is self-evident that the given control points have to be assigned weights which are considerably larger than those assigned to the intersected points. Consequently, any assumed relation between weights which considers this basic fact would be acceptable.

This successive chain of resections and intersections continues until neither the orientation elements nor the coordinates of the new points alter significantly. Choosing the above iteration process we avoid the necessity of handling a large system of equations, which is inevitable if one solves the orientation and the coordinates simultaneously. The process converges rapidly; experience shows that three or four iteration steps are sufficient. Moreover, the solution towards which the process converges is equivalent to that which is obtained from a simultaneous determination of the orientation and the coordinates[1].

A few additional facts have to be stressed.

- Starting the second iteration step the angular and linear velocities have to be regarded by the resection routines.
- The Transformation 6 is performed with orientation angles which include components resulting from the angular velocities, hence each point on the photograph is associated with a different orientation matrix.
- After the first series of intersections, the approximate coordinates of each intersected point are known. The intersection routine for the following iteration steps can then be modified by utilizing these approximate values.
- The vector R in Expression 10 is substituted by $R = \overline{R} + \Delta R$, $\overline{R}$ being defined by the approximate coordinates of the point. Assuming this substitution, Expression 11 is rewritten as:

$$\begin{bmatrix} 0 & -w & v \\ w & 0 & -u \\ -v & u & 0 \end{bmatrix} \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} + \begin{bmatrix} 0 & -w & v \\ w & 0 & -u \\ -v & u & 0 \end{bmatrix} \begin{bmatrix} \bar{X} - (X_0^c + t\,\dot{X}_0^c) \\ \bar{Y} - (Y_0^c + t\,\dot{Y}_0^c) \\ \bar{Z} - (Z_0^c + t\,\dot{Z}_0^c) \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} \tag{12}$$

The intersection based on Formula 12 would be advantageous from the point of view of computation techniques.

Summary

Geometric concepts of evaluating a single panoramic photograph and a stereopair have been discussed. Only the basic approach to the evaluation of the photographs is outlined, the detailed derivation of all working formulas is omitted here. The derivation presents no difficulties and may be done easily by the reader himself. Practical tests conducted with actual photographs have proved the applicability of the described procedures.

In view of the satisfactory results of the evaluation of the photographs and the high resolution of the panoramic camera, we believe that the panoramic photograph can be used to advantage to solve various photogrammetric problems.

References

1. A. Perlmuter, Block triangulation, D.Sc. Thesis, Haifa 1971.
2. Case, James B., "The Analytical Reduction of Panoramic and Strip Photography", *Photogrammetria,* pp. 127-141, 1967.