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Errors in Photogrammetry

It is important that one understand the errors, their sources, characteristics, and relative magnitudes in order to apply photogrammetric materials effectively.

ABSTRACT: *To teach and work effectively with photogrammetry, one should have a basic understanding of the sources and relative magnitude of errors inherent in aerial photographs. Exact calculus approaches are often so complicated that they cause one to want to forget about errors entirely and pretend they do not exist. The approach described herein equates all source error effects to a percentage, and as long as the mathematical manipulations are multiplication and division, the same percentages can be applied to the final answer to ascertain its probable error. The method described provides estimates of errors identical to those obtained using calculus, but the described method is much easier. The method provides students and users with a ready and quick method for analyzing errors and of obtaining a feeling for the relative magnitudes of errors in photogrammetry work.*

INTRODUCTION

TO TEACH OR WORK effectively with photogrammetric or remote sensing images, a basic understanding is necessary of sources of photogrammetric errors and their relative and approximate magnitudes. Often the subject of errors is covered in such mathematical detail that it leaves the user so confused that he simply overlooks these errors entirely. What is really needed is a simple approach for analyzing errors and understanding their effects for (1) applications where aerial photos are used as map substitutes, (2) where photos are used in conjunction with stereoscopes and parallax bars, or (3) where photos are used with stereo plotters or analytical photogrammetry. This paper presents a simple, comprehensive, and practical method of understanding these errors and ascertaining their approximate magnitudes. The techniques presented here have been developed by the author in six years of teaching photogrammetry and have proven very effective in analyzing and understanding errors and dealing with them in various aspects of photogrammetry and remote sensing work.

BASIC RELATIONSHIPS IN ANALYSIS

In dealing with the subject of errors of any kind, it is important to realize that there are different rules governing the propagation of errors depending on whether the process used is addition and subtraction, or is multiplication and division.

SIGNIFICANT FIGURES IN ADDITION AND SUBTRACTION

If one adds the number 1.2345 (five significant figures) to the number 123.4 (four significant figures), the answer is 124.6, accurate only one place to the right of the decimal point because the second number added is no more accurate than that.

This same approach is used if one subtracts rather than adds.

SIGNIFICANT FIGURES IN MULTIPLICATION AND DIVISION

With multiplication, if one takes the number 1.234 (four significant figures) and multiplys it by the number 1.23 (three significant figures), the answer 1.51782 is rounded off to 1.52 (three significant figures) because the product can have no more significant figures than the least in the number being multiplied. The same general approach also holds for division. The error analysis for photogrammetric work herein described is derived from the rules governing propagation of errors for multiplication and division. This is important to remember even though almost all photogrammetric equations to be analyzed involve multiplication or division in their solution.

Photogrammetric Error Propagation—Simple Example

Photogrammetric equations are based on the fact that all light rays pass through the front and rear nodal points of the camera lens with direction unchanged. If one neglects the thickness of the lens, the geometry becomes that of the pinhole camera. (See Figure 1).

In Figure 1, by similar triangles $Z/X = f/I$ or $Z = (f/I)X$. Let us assume that the following values and probable errors in their measurement are known (*in* = inch, *ft* = foot):

f = 10.000 in ± 0.001 in
X = 1000 ft ± 1 ft
I = 5.00 in. ± 0.01 in.

The problem presented here is that Z is to be computed along with the error in Z due to the errors in f, X, and I. Two approaches will be presented, the first is an exact calculus approach. The second is herein called the *Governing Percentage Method* which is used in the rest of the paper for analyzing more complex error situations.

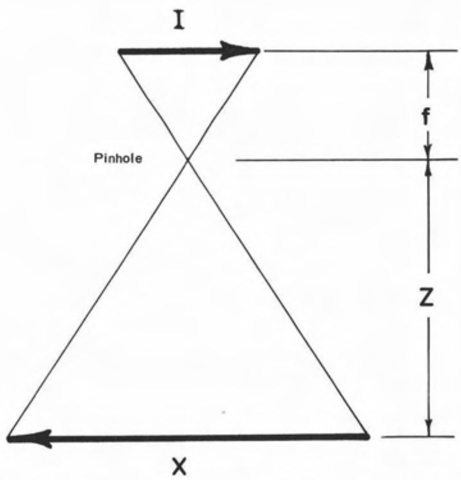

FIG. 1. Simple photogrammetric relationship; the pinhole camera geometry. $Z/X = f/I$ or $Z = fX/I$. Legend: f, focal length; I, image; X, object; Z, flying height above terrain.

Calculus Method of Determining Error Propagation

From Figure 1 and the given values of f, x, and I,

$$Z = (f/I)X = \frac{10.000\ in \times 1000\ ft}{5.00\ in}$$

$$Z = 2000\ ft \pm some\ error.$$

One can calculate the total probable value of this error by taking derivatives of Z with respect to f, I, and X and combining the effects.

The error in Z due to f by derivatives is:

$$dZ = (X/I)\ df = (1000\ ft/5\ in) \times (0.001\ in)$$
$$dZ_{(f)} = \pm\ 0.2\ ft$$

The error in Z due to X is:

$$dZ = (f/I)\ dX = (10\ in/5\ in) \times 1\ ft$$
$$dZ_{(X)} = \pm\ 2\ ft$$

The error in Z due to I is derived from:

$$Z = (f/I)X = I^{-1} fX \text{ and } (dZ/dI) = (-fX/I^2)$$

or:

$$dZ = (-fX/I^2)\ dI = (-10\ in \times 1000\ ft/5\ in \times 5\ in) \times .01\ inch$$

$$dZ_{(I)} = \pm\ 4\ ft$$

According to the theory of probability, the combination of these various errors results in:

$$Total\ Probable\ Error = \sqrt{[(0.2)^2 + (2)^2 + (-4)^2]}$$
$$= 4.4\ ft \text{ or approximately } 4\ ft.$$

As any error can be either positive or negative, accumulative or compensating, one can say that the total resulting error in Z due to f, X, and I may be as high as 0.2 + 2 + 4 = 6.2 ft. Assuming by some coincidence that some of the errors in f, X, and I really approach zero, then the total error on Z will perhaps lie between 0 to 6 ft; perhaps about 4 feet.

Determining Error Propagation by Governing Percentage Method

Rather than using calculus, a simpler approach using percentages of errors will give the same results. This method is herein called the *Governing Percentage Method,* and is extensively elaborated on later in this paper.

It is possible to express the errors in f, X, and I in terms of percentages of the numbers themselves:

$f = 10.000$ in.; error in $f = 0.001$; percent error in

$f = 0.001$ in/*10.000 in* $= 0.01\%$

$X = 1000$ ft; error in $X = 1$ ft; percent error in

X = 1 ft/1000 ft = *0.1%*

$I = 5.00$ in; error in $I = 0.01$ in; percent error in

I = *0.01 in/5.00 in* = 0.2%.

If we sum the percentages of .01%, 0.1% and 0.2% caused by f, X, and I, we have .31%. This same percent error will carry through to the calculated value of Z. Assuming accumulating error, the error in Z is calculated as follows:

Error in Z = .31% of 2000 ft = 6.2 ft

which is the same as the error obtained by calculus. By analyzing the percentages of errors on the input figures we can see that the governing or largest percentage is 0.2% and the final propagated error it causes is approximately 0.2% × Z, or 4 ft. This same governing percentage approach can be applied to various and complex photogrammetric calculations, involving multiplication and division.

Errors in Photogrammetric Work

Whether or not a particular error is of concern depends on the photogrammetric technique being used. Many aerial photos are used simply as map substitutes. Where this is done, certain errors should be understood.

AERIAL PHOTOS USED AS MAP SUBSTITUTES

In Figure 1, we assume that the object X is on flat ground and that there is no relief displacement, i.e., difference in scale over the photo caused by difference in distance between the ground and the photo. We assume that there is no relief-displacement error.

Also, in Figure 1, we assume that the film is parallel to the flat ground, or that I is parallel to X. We assume that there are no tilt errors.

We also assume that the light rays pass straight through the lens—direction unaltered. We assume, therefore, no lens distortion errors.

We assume that the distance I measured on the photo is the true distance projected by the camera. We, therefore, assume no error due to the film or paper-print shrinkage. In some instances, we can enlarge images by projectors which take care of film shrinkage. We then assume that the shrinkage is uniform across the photo. We assume that there is no differential film shrinkage.

Likewise, we assume that the plane upon which the image was projected was truly flat, that it had no bumps on it caused by dust behind the film or caused by uneven thickness of the film emulsion. We, therefore, assume no focal-plane flatness errors.

One other important error in any photogrammetric work is the error caused in measurement. For analysis of Figure 1, we already stated that the error in measurement of I was ± .01 in, so the measurement error has already been accounted for in this example. Other errors such as atmospheric refraction may be present, but are usually insignificant compared to the errors already listed.

Generally speaking, if a photo is used as a map substitute, we assume that the following errors are zero:

- Relief displacement.
- Tilt.
- Paper or film shrinkage.
- Differential shrinkage.
- Lens distortion.
- Focal-plane flatness.

PHOTO PRINTS USED IN CONJUNCTION WITH STEREOSCOPES AND PARALLAX BARS

If one views two overlapping photos side by side and measures parallax with a parallax bar, all of the errors listed for the single photo exist and may be doubled except the error of relief displacement which is really the parallax being measured*. Several very significant additional errors must also be considered in this situation. In Figure 2 by similar triangles $Z_A/B = f/P_a$ where Z_A, f, and B are as shown in Figure 2, and P_a is the parallax of point a. This is the equation that is commonly used to calculate the difference in elevation between the aircraft and any point on the ground. If we take the parallax of the top and bottom of the tree in Figure 2 we have:

$$P_a = fB/Z_A \text{ and } P_c = fB/Z_c$$

The difference in parallax between the top and bottom of the tree is:

$$dp = P_a - P_c = [fB(Z_c - Z_A)]/Z_cZ_A.$$

so

$$Z_c - Z_A = dp\ Z_cZ_A/fB.$$

$Z_c - Z_A$ also equals dh and from

$$P_a = fB/Z_A,\ Z_A = fB/P_a$$

*According to the Theory of Probability if we use 2 photos the resulting error in the combination can be expected most probably to have a magnitude of $\sqrt{2} = 1.414$ times the errors in a single photo. It is also conceivable to have a maximum error 2 times the magnitude of the errors of a single photo.

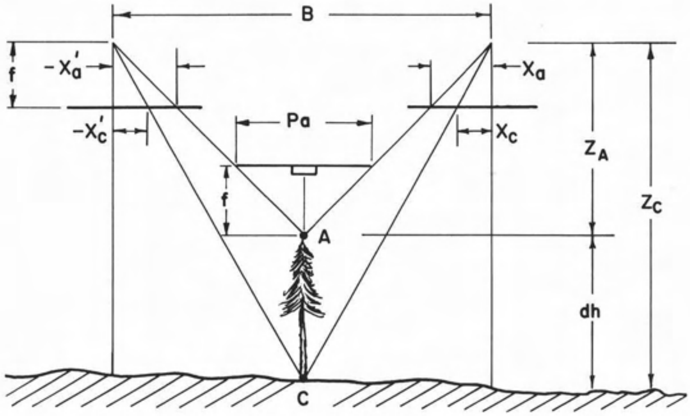

FIG. 2. Geometry for using overlapping photos and a parallax bar.

Therefore,

$$dh = (dpZ_c/fB)\,(fB/P_a) = dpZ_c/P_a.$$

If dh is small compared to Z_c, P_a approaches b, the photo base, and

$$dh = dpZ_c/b.$$

This is the equation that is often used to relate difference in parallax to difference in elevation.

In these equations we assume a common flying height for both photos, but there may be up to 100 feet difference in flying height. Also, significant measuring errors are usually caused by transferring principal points and calculating b or B. In summary, for work with parallax bars, we have all of the errors associated with a photograph used as a map substitute except the relief displacement error; this is reflected in the parallax which is measured. The remaining error effects are all increased and, possibly, doubled because two photographs are used. Additional errors are due to unequal flying heights and errors in measuring and transferring principal points.

ERRORS IN STEREOPLOTTING AND ANALYTICAL PHOTOGRAMMETRY

With stereoplotting we measure the relief displacement as with the parallax bar. However, each projector is also adjusted to take out the tilt effects and, therefore, the errors due to tilt. The projectors are adjusted to take out any errors caused by unequal flying heights. If we use the proper projection lens or projection techniques, we can handle the lens-distortion errors. We adjust the projectors until the projected image matches the plotted ground control distances so that uniform film or plate shrinkage is of no concern. The errors that still exist with the stereoplotter is of no concern. The errors that still exist with the stereoplotter are errors caused by differential film and plate shrinkage, errors of focal-plane flatness and any stereoplotter measuring errors.

The same error analysis applies for analytical photogrammetry as well as for stereoplotting. With special cameras, it is possible with *reseau* grids etched on the focal plane to handle the errors due to differential film shrinkage. Very special cameras using glass plates rather than film can almost eliminate both the effects of differential film shrinkage and focal-plane flatness. Such cameras are very special indeed and are not normally used for operational photogrammetry. For operational photogrammetry, be it stereoplotting or analytical work, the limiting errors will normally be (1) errors due to differential film shrinkage, (2) errors due to focal-plane flatness, and (3) errors due to measurements.

Any other less-precise photogrammetric operation will be limited by some combination of the errors previously listed. Following is a detailed investigation of each error source and an attempt to ascertain the magnitude of each.

ERRORS DUE TO UNEVEN TERRAIN

In Figure 1, the scale of the photograph is I/X which is also equal to f/Z or *(focal length)/(flying height)*. Of course, as Z changes, so does the scale. As a point is moved up or down in elevation, its image is displaced on the photograph.

In Figure 3 the general equation that expresses this displacement is:

$$dr/r = dz/z^*.$$

*If this relationship is not readily apparent from Figure 3, any good photogrammetric text will show its derivation.

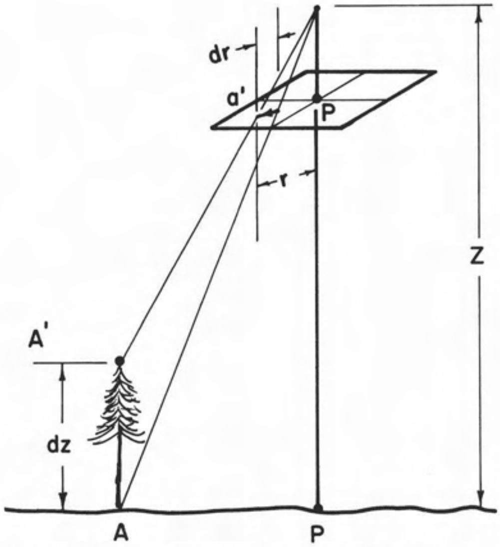

FIG. 3. Displacements on an aerial photo due to differences in elevation.

The distance r is really a photo representation of the ground distance AP. As point A is moved upward by dz, a' is moved on the photo by the distance dr. The resulting error in scale of the line r is dr; the percentage error in line r is dr/r, which is equal to dz/z. Therefore, the percent error due to relief is equal to dz/z. The absolute magnitude of this error varies depending on the ruggedness of the terrain and flying height. If z is 1000 ft and dz is 100 ft, the percent error due to uneven terrain is $100/1000 = 10\%$.

Errors Due to Tilt

The most significant error in a single photo used as a map substitute is the error caused by uneven terrain. The next most significant error is the error caused by tilt. For the photos of Figures 1 and 3, the assumption is that the photographs are vertical. Generally speaking, due to air turbulence, etc., such photos are likely to have up to 3° tilt. Following is an analysis to arrive at the magnitudes of computational error which might be caused by 3° tilt.

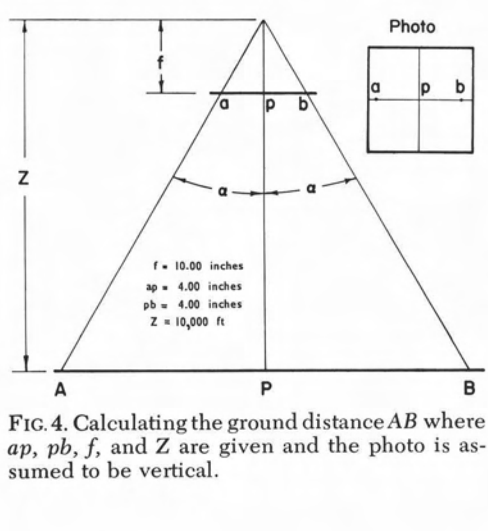

FIG. 4. Calculating the ground distance AB where ap, pb, f, and Z are given and the photo is assumed to be vertical.

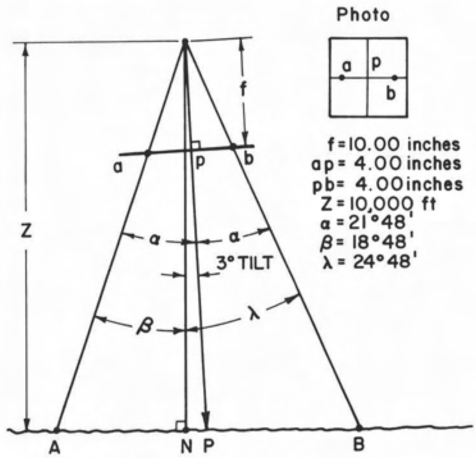

FIG. 5. Calculating the ground distance AB where ap, pb, f, and Z are given and the photo is tilted 3°.

Let us assume in Figure 4 that the images on the photo are used to calculate the ground distance AB, first we assume a vertical photo. From Figure 4,

$$\tan \alpha = 4.00/10.00;\ \alpha = 21°48' \text{ and}$$
$$AP = PB = \tan \alpha \times 10{,}000\ ft = 4000\ ft$$
$$AB = AP + PB = 8000\ ft.$$

Now let us assume that there was really 3° tilt in the photo at the time of exposure as shown in Figure 5. From Figure 5,

$\alpha = 21°8'$ (as in Figure 4) and
$B = \alpha - 3° = 18°48'$
$\lambda = \alpha + 3° = 24°48'$
$AN = 10{,}000 \times \tan B = 3404$ ft
$NB = 10{,}000 \times \tan \lambda = 4620$ ft
$AB = AN + NB = 8024$ ft.

If we assume a vertical photo and it was, in fact, tilted 3° as shown, there is an error of 8024-8000 = 24 ft in the calculated length of AB due to the 3° tilt. The relative error due to the 3° tilt in this instance is *24 ft/8000 ft* or 0.3%. Other methods of analyzing the effects of the 3° tilt produce errors of the same general magnitude. As a general rule, the errors due to normal tilt can be expected to be between 0 and 0.3%.

Errors Due to Shrinkage

To calculate the errors due to paper print shrinkage, one has to measure the distance

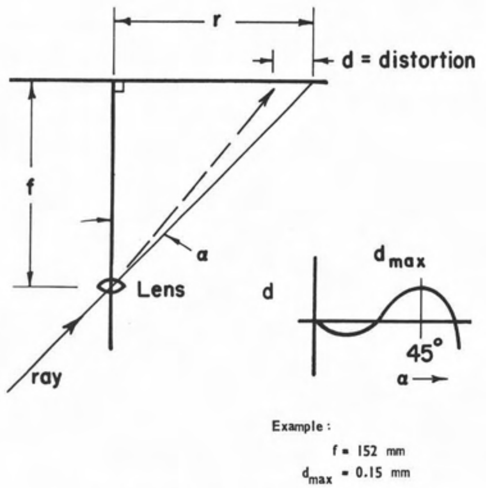

FIG. 6. Effects of lens distortion.

between fiducial marks on a finished print and compare that to the distance on the negative. The resulting difference over the average distance is the relative error. Good-quality papers will produce shrinkage errors from 0 to 0.2%, whereas poorer-quality papers will produce shrinkage errors as high as 0.5%.

To obtain the *film shrinkage,* one compares the negative to the camera opening. The shrinkage errors of most aerial films can be expected to be less than 0.1%. *Differential film shrinkage* or non-uniformity of this shrinkage in any one direction will perhaps be about 1/10 to 1/100 of this or about 0.005%.

Errors Due to Lens Distortion

Figure 6 shows a sketch of lens distortion and a typical distortion curve for an older camera lens. In this example, if $\alpha = 45°$, the distance r is equal to $f = 152\ mm$. The lens distortion at this angle is $d_{max} = 0.15\ mm$. The relative error in the distance r due to the lens error is $d_{max}/r = 0.15\ mm/152\ mm$ = 0.1%. As a general rule, the errors on a photo due to lens distortion will be less than 0.1% and considerably less on higher-quality cameras.

Errors of Focal-Plane Flatness

In photogrammetric calculations, we assume a flat focal plane. Aerial cameras have vacuum systems to flatten out the film for this purpose. However, pieces of dust may catch between the film and the vacuum platen, or the thickness of the film itself may vary. Figure 7 shows a sketch of such errors.

In Figure 7, the ray striking a truly flat focal plane would be imaged at a. However, because of the deviation from the flat focal plane due to distance t, the ray is really imaged at b. The error on the flattened image is the distance d. The relative error in distance r is d/r. Typical values for t are about 10 μm or 0.01 mm. For $\alpha = 45°, d = t$. The value of r for a normal camera will be about 150 mm. The relative error then is $0.01\ mm/150\ mm$ or less than 0.01%.

The magnitude of errors caused by lack of focal plane flatness will be in the magnitude of less than 0.01%.

Measuring Errors

Measuring errors are always present and depend entirely on the technique used. The percentage of this error is, of course, calculated by forming a ratio of the probable error Δd and the distance measured d,

$$\begin{matrix}\textit{Percent error}\\ \textit{due to}\\ \textit{measurement}\end{matrix} = \frac{\begin{matrix}\textit{Probable error}\\ \textit{in measurement}\end{matrix}}{\begin{matrix}\textit{measured}\\ \textit{length}\end{matrix}} = \frac{\Delta d}{d}.$$

Combined Effects of Errors

From the foregoing analysis, the magnitudes of the different errors can be summarized as follows:

USING A SINGLE PHOTO AS A MAP SUBSTITUTE

Table 1 summarizes the errors that exist if a single photo is used as a map substitute. Assuming flat terrain one can see from Table 1 that the expected error will still be up to about 0.5% which corresponds to a precision of $0.5/100 = 1/200$.

It is clear that unless one corrects for paper shrinkage and tilt effects, there is no use worrying about lens distortion, film shrinkage, or focal-plane flatness in this example.

Assuming terrain differences of 400 ft and a flying height above average terrain of 4,000

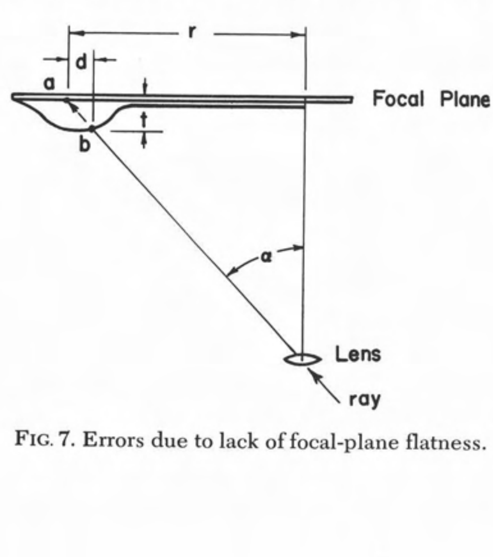

FIG. 7. Errors due to lack of focal-plane flatness.

TABLE 1. ERRORS WHICH EXIST IF ONE USES A SINGLE PHOTO AS A MAP SUBSTITUTE.

Uneven terrain ($\Delta z/z$)	(varies, depends on terrain)
Tilt	0 to 0.3%
Paper Shrinkage	0 to 0.5%
Film Shrinkage	0 to 0.1%
Differential Film Shrinkage	0 to 0.005%
Lens Distortion	0 to 0.1% %
Lack of Focal Plane Flatness	0 to 0.01%
Measuring ($\Delta d/d$)	(varies with technique)

ft, the terrain error becomes *400/4000 = 10%,* and terrain is clearly the governing error in this application and will limit the expected precision.

In this example, if one scales distances directly from the photo used as a map, the errors are about 10% which gives a precision of *10/100 = 1/10.* Calculated ground distances from such a photo can have errors as large as 10% of the calculated length.

Terrain effects are almost always governing in such instances and there is little point in worrying about tilt, lens distortion, or film shrinkage if there are significant differences in terrain.

PHOTO PRINTS USED WITH STEREOSCOPES AND PARALLAX BARS

If two overlapping photos are fastened down side by side and used with a parallax bar to obtain elevations, the uneven terrain error of Table 1 drops out because this is what is being measured.

All the other errors still exist and are increased and may be doubled because of the two photographs. Also, significant errors are introduced due to transfer of principal points, due to measuring, and due to unequal flying heights, since the basic equations for such work assume common flying heights.

In any event, assuming perfect transfer of principal points and perfect measurements and common flying heights, the resulting errors may still be as high as is shown in Table 2.

If one uses the equation $(Z_a/B) = (f/P_a)$ or $Z_a = Bf/P_a$ for the sketch in Figure 2 to get absolute elevation of the terrain at Point *A*, there may be up to 2% error in Z_a. This is a precision of 2/100 = 1/50. If Z is about 5,000 ft, this will give a 100 ft error. This acounts for the experience of many people that parallax bars are almost worthless for determining *absolute elevations* for normal work; 2% of 5000 ft is an error of 100 ft.

TABLE 2. ERRORS WHICH MAY EXIST IF ONE USES A STEREOSCOPE AND PARALLAX BAR

Tilt	2 × 0.3% = 0.6%
Paper Shrinkage	2 × 0.5% = 1.0%
Film Shrinkage	2 × 0.1% = 0.2%
Lens Distortion	2 × 0.1% = .2%
	Total 2%

On the other hand if one wishes only to measure the height of say a tree, as in Figure 2, the difference in parallax between the top and bottom of the tree can be measured directly as *dp*. The equation $dh = dp \times b/Z_c$ can be used and a 2% error is applied to the calculated value of *dh*. In this instance, if the calculated value *dh*, or the height of the tree, is 100 ft, the error will be 2% × 100 ft or ± 2 ft which is indeed satisfactory for most forestry work. This accounts for the fact that stereoscopes and parallax bars or parallax wedges are indeed useful tools for measuring differential heights such as tree heights above ground. If the above equation is used, the 2% error is applied against the tree height and not against the large distance between the plane and the tree. An error of 2% of a tree height of 100 ft results in an error of 2 ft which is acceptable to most foresters.

STEREOPLOTTERS AND ANALYTICAL PHOTOGRAMMETRY

With stereoplotters and analytical photogrammetry, the terrain heights are what is measured, whence terrain effects are no longer an error. Paper prints are not used, so the paper shrinkage error disappears. Images are appropriately enlarged to match-plotted ground control so film shrinkage is no problem. Differential film shrinkage still does exist. Lens distortion errors are normally taken out by proper projection lenses or correction plates so this error drops out. Errors due to lack of focal-plane flatness still exist. Measuring errors are a factor and are incorporated into the *C*-Factor for the particular plotter used. Table 3 shows the errors that still exist with stereoplotters.

TABLE 3. ERRORS THAT EXIST WITH STEREOPLOTTERS AND ANALYTICAL PHOTOGRAMMETRY

Differential Film Shrinkage	.005%
Lack of Focal-Plane Flatness	.01%
Measuring (varies with machine)	

As it is shown, one can expect up to 0.01% error with a stereoplotter which converts to a precision of 0.01/100 = 1/10,000. A common

C-factor for stereoplotters is 1000, which is equal to:

C-Factor = Z/(*Contour Interval*) *or*
(*Contour Interval*/Z) = 1/1000.

Here the errors are still less than one-tenth of the Contour Interval. The operator's measuring ability is also a very important factor in limiting the accuracy of stereoplotters or transferring points in analytical photogrammetry.

One can easily see that it is the error due to lack of focal-plane flatness (as well as measuring) that controls the accuracy of stereoplotter work.

Analytical photogrammetry really consists of doing with a computer what is done graphically on the stereoplotter. Therefore, all the principles that apply to errors on the stereoplotters also apply to analytical photogrammetry. In other words, the errors due to focal-plane flatness are governing in analytical photogrammetry to about 0.01% or a precision of 1/10,000. This will allow for a probable error in calculated points to be as low as 1/10,000 of Z (the flying height).

It is interesting to note that the standard errors in operational analytical photogrammetry are about 1/5000 of the flying height and with the most careful research work, they approach 1/10,000 of the flying height.

Conclusions

To be an effective user or teacher of photogrammetry, one must have a basic workable understanding of errors, their sources and their magnitudes. The simplified procedure herein described as the *Governing Percentage Method* reduces all source errors to a percentage. The source error percentages reflect the percent error in the calculated answer as long as the calculations are basically multiplication and division.

For using a single photo as a map substitute, the governing error source is uneven terrain which produces an error of $\Delta z/z$ = (terrain elevation difference)/(flying height above terrain), which in operational work can commonly be as high as 10%. Less significant error sources in this case are tilt and paper shrinkage both approaching a possible 0.5%.

With stereoscopes and parallax bars, the governing source errors are measurements, tilt, and shrinkage and will result in errors as high as 2% which effect either the calculated distance between the plane and the point in question or the height of an object such as a tree, depending on the equations used.

With stereoplotters and analytical photogrammetry, the governing error source is measuring limitations and lack of focal plane flatness, which can cause errors of up to 0.01% or a precision of 1/10,000. This corresponds to the operational limits of accuracy of both stereoplotters and analytical photogrammetry.

The *Governing Percentage* technique of error analysis has proven understandable, practical, and invaluable for teaching of errors in photogrammetry education.

Acknowledgement

The author wishes to express appreciation to Prof. Paul Wolf, Prof. James Clapp, Ed Hughes and John Haverberg for their help in constructively reviewing the paper, and to all others who helped in the review and preparation.

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