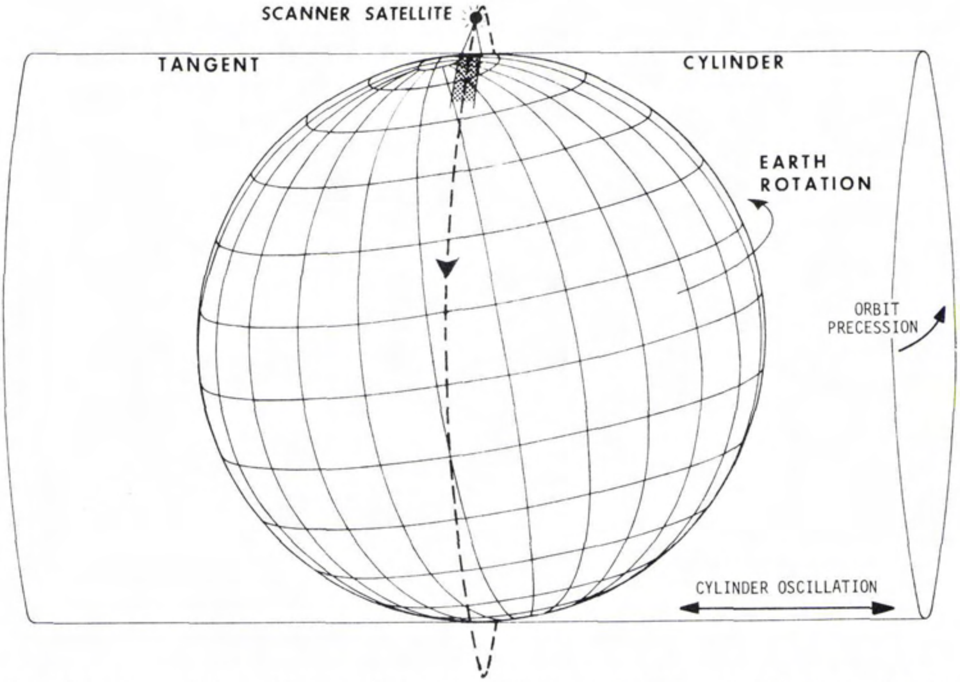

FIG. 1. The Space Oblique Mercator Projection. ERTS-1 satellite images the Earth from N 82° to S 82° every 18 days. The motions that produce effects on the imagery include: the scanner sweep, satellite orbit, Earth rotation, and orbit precession.

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Space Oblique Mercator*

A new map projection of the Earth lends itself well to the utilization of ERTS imagery.

(Abstract on next page)

INTRODUCTION

HISTORICALLY MAP projections have been based on static conditions. The figure of the Earth, perspective center (if there is one), and projection surface are all fixed with respect to one another. As long as the Earth is imaged by a framing camera that records a scene instantaneously, the static condition holds, and film returned from aircraft or spacecraft generally fits into this category. Moreover video systems that transmit recorded scenes, such as those on the Lunar Orbiters and the Return Beam Vidicons (RBV's) on the Earth Resources Technology Satellite (ERTS-1), also represent the static mode, in which the image is considered to have been obtained instantaneously.

In all such applications a perspective image of the Earth is recorded in a unique but definable form, and the image can then be fitted or transformed to one of the conventional map projections. However, we now have orbiting spacecraft equipped with scanning devices that are imaging the Earth scene continuously. Such satellites have mapping capabilities that open the door to an entirely new concept of map projections in which relative motion, and therefore time,

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*It has been suggested that this be called *Colvo's Projection* to facilitate reference to the subject.—*Editor*.

ABSTRACT: *The Earth Resources Technology Satellite (ERTS) Multispectral Scanner (MSS) is producing imagery of rather amazing geometric fidelity. The positional errors (rms) of points on a properly controlled image are less than the 80-meter instantaneous field of view (picture element) of the scanner. Such accuracy is attributed to the stability of the scanner and spacecraft and to the corrections that are being made by NASA before each image is printed. The image is in fact formed on a cylindrical surface in space which can be defined as a specific map projection and results in the mapping of the world (between the 82° parallels) every 18 days. Moreover the projection is mathematically definable and thus has the potential of developing into an automated mapping system in which the picture element (pixel) can be discretely related to its position on the figure of the Earth.*

becomes a mapping parameter. Thus the conventional static concept of map making is replaced by a dynamic concept. The basic conditions for this dynamic concept of mapping are found in the polar-orbiting weather satellites, such as Nimbus, ITOS, and DAPP (*D*ata *A*cquisition and *P*rocessing *P*rogram, which utilizes U. S. Air Force weather satellites). Imagery from these satellites has so far not been defined with respect to the figure of the Earth with the precision expected of a map projection. The multispectral scanner (MSS) of the ERTS-1 satellite, however, creates an image of sufficient resolution and geometric fidelity to warrant definition as a true map projection.

DEVELOPMENT

The ERTS-1 MSS has an instantaneous spot size (pixel:picture element) on the Earth of 79 m and therefore is considered a relatively low-resolution system for Earth sensing. The effective resolution (in optical terms) of ERTS is no better than 200 m. Nevertheless, ERTS imagery has high geometric fidelity, which results in cartographic products that have spatial errors on the order of 50 to 150 m (*rms*). This is the basic imagery, referred to by NASA as bulk or system-corrected, which has in turn been related to the Earth's figure through ground control points[1]. With such control and the application of appropriate procedures, geographics (lat/long) or a plane coordinate grid such as the Universal Transverse Mercator (UTM) can be fitted to the ERTS imagery. The UTM grid unit is not a true square if it is so fitted, but the deviations are so small that grid anomalies are not detectable if measurements are made with reference to the nearest lines of a nominal grid square.

The procedure for gridding ERTS imagery was developed by the U. S. Geological Survey under NASA sponsorship, with the Ohio State University providing the initial computer programs.[2] The small size of the spatial anomalies observed if a geodetic grid is applied to the MSS imagery is the first indication that scanner imagery as corrected and printed by NASA is in fact on a continuous map projection of definable form. Further information relative to the procedures used in grid fitting are available.[4] The NASA ERTS Data Users Handbook[1], pages G-17 and G-18, describes the 14 basic geometric corrections applied to ERTS MSS data *before* printout as bulk imagery. (Unfortunately these corrections have not, as yet, been applied to the digital computer-compatible tapes produced by NASA.)

If all 14 corrections are applied, a pseudocylindrical map projection of rather curious characteristics results. Maximum distortions of the Earth figure due to the projection are on the order of only 1:1,000, which makes it acceptable for mapping purposes. The corrections were designed to give MSS imagery geometric characteristics similar to those of the ERTS RBV's, which are frame cameras and thus have perspective geometry. The MSS does have geometric fidelity comparable to that of the RBV's and thus warrants its own optimized map projection, which would have maximum distortions of only about 1:10,000. A further discussion of projections is contained in reference 5, in which a *Space Oblique Mercator* projection is described and recommended for ERTS-type imagery.

The projection could have any one of several characteristics, but precise map makers generally consider the characteristic of conformality as dominant. Conformality retains equal scale locally in all directions and preserves angular relationships. The conformal cylindrical projection was conceived by *Mercator* and in this application is defined in *Space;* it is *Oblique* to the polar axis. Definition of the projection is given in nonrigorous but nevertheless geodetic terms in the following section.

GEODETIC CONSIDERATIONS

Conceptually one can start with a spherical Earth and then develop the elliptical modifications which, unfortunately, cannot be ignored. Figure 1 illustrates a cylinder defined by a truly circular ERTS orbit with the projection surface tangent to the Earth's spherical figure. Although four motions (scanner sweep, satellite orbit, Earth rotation, and orbit precession) are involved, the imagery can be recorded on the simple cylindrical surface which, if developed into a continuous plane, is in fact a map projection.

To keep Earth rotation from distorting the image, the cylindrical surface oscillates along its axis at a compensating rate which varies with latitude. Motion is otherwise uniform and symmetrical with respect to the orbit, and thus every orbit exactly repeats its path on the projection plane even though the Earth scene changes nearly 26° in longitude with each orbit. Thus the projection coordinate values are repeated each orbit even though a different portion of the Earth's surface is mapped on each successive orbital pass. This means that the Earth figure coordinates λ,ϕ are related to the projection coordinates x,y as a function f of time t. That is, $\lambda,\phi = f(x,y,t)$; t must expand through the 18 day system and then revert to zero for the next cycle. Figure 2 (exaggerated) shows how the image (if continuous) is cast on the developed projection surface.

The fact that the orbit has precessed in space by a few minutes of arc with each revolution does not affect the projection surface, which is defined by the orbit. Orbit precession, which retains the orbit's angular relationship to the Sun, does slightly modify the effects of Earth rotation.

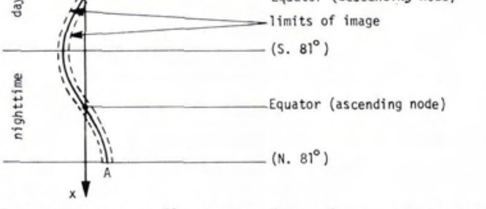

FIG. 2. Diagram illustrating how the image is cast on the developed projection surface.

Inasmuch as the Earth's figure is an ellipsoid instead of a sphere, several modifications must be considered. First, the Earth's polar radius is over 20 km less than the equatorial radius. An orbit which is at a fixed height above the Earth's figure and thus always images the Earth at the same scale would in fact have to have an elliptical orbit with two perigees which remain at the 81° points of maximum inclination, and this is contrary to the laws of physics. In practice, a truly circular or prescribed elliptical orbit is impossible to maintain, but NASA must (and does) consider the Earth's ellipticity as well as orbital ellipticity in computing satellite altitude.

A second consideration is that the scanner, controlled by horizon sensors, is referenced to the local geometric vertical rather than the direction to the center of mass of the Earth, which is the computational center for the orbit as well as the Earth's figure. The maximum difference between these directions approaches 12 minutes of arc. As the orbit is only 9° off the pole, the angular difference is principally along track and thus slightly affects the time relationship of the satellite to the Earth's figure. The slight cross-track angle (3.6 min. of arc maximum) between the local geometric vertical and the vector to the Earth's center does in effect deform the projection surface. The deformation constitutes a deviation from the concept of a uniform map projection and also disturbs the precise conditions of conformality in the projection. These considerations are probably academic and will never be found by the map user, but for the mathematician who defines the map projection in rigorous terms they are important. At 81° latitude the 3.6 minutes of arc subtend nearly 1 km on the Earth's surface. As the orbit approaches the Equator, the cross-track deviation steadily decreases from 1 km to 0.

The actual path of the satellite on the Earth's figure as defined by the local geometric vertical is also of interest. To see this, we should first forget the Earth's rotation and merely consider the figure generated by the local geometric vertical from the orbit as it intersects a fixed figure of the Earth. This is not the true ellipse (great circle) that would result from passing a plane obliquely through the center of an ellipsoid, and it is not a geodesic, which is the shortest distance between two points on the elliptical Earth surface. Regardless of what the actual figure is, it must be defined in mathematical terms because it creates the locus of image centers. NASA probably could define ERTS imagery with respect to the direction to the center of mass of the Earth and thus simplify the computational problem. However, this solution would create a slightly tilted image with respect to the Earth's figure that is probably undesirable for any analog portrayal. Once a comprehensive mathematical analysis has been made, the various conditions stemming from the Earth's ellipticity can be fully evaluated. Only then can the decisions be made as to which conditions and terms must be considered or ignored.

SPECIFIC PARAMETERS

Certain parameters, assumptions, and nomenclature relative to the ERTS system must be defined before a mathematical model and transformation equations can be rigorously defined. Recommendations with pertinent values provided by NASA/Goddard are as follows:

Earth figure (Figure 1).
- a, semimajor axis, 6,378,165 m.
- f, flattening of the ellipsoid, 1/298.3.
- R, nominal radius of curvature in the cross-track direction, 6,388,000 m.

Orbit, nominal:
- Circular radius, 7,294,690 m.
- Altitude, computational, 918,592 m.
- Inclination, 99.092°. [This is the angle of the ascending node with respect to due East. The maximum latitude of the orbit is 80.968°. Imagery is taken on the descending (daytime) portion of the Sun-synchronous orbit.]
- Period, 103.267 minutes.
- Time of descending node (equatorial crossing), 9.42 a.m. local Sun time.
- Coverage cycle duration, 18 days (251 revolutions)
- Distance between adjacent ground tracks at Equator, 159,380 m.

Imaging System, Multispectral Scanner (MSS) (See Figure 3).
- β, viewing angle of scanner with respect to nadir has a maximum value of 0.100749 rad, about 5.76°. The plane of the scanner motion is now defined as perpendicular to the plane of the orbit.
- γ, angle of Earth curvature (max≈0.83°).
- f, effective focal length of scanner. Based on mirror size and f number, this is 730 mm; however, this dimension is immaterial with respect to the projection.
- N, nadir point, based on local geometric vertical.
- P, point on Earth imaged by MSS sensor.
- C, cylindrical image surface, develops into image plane.
- T, cylindrical projection surface, develops into projection plane.

Let X be the distance in instantaneous orbital direction on Earth figure along orbital path*. (This orbital path would be a great circle or orthodrome on a spherical nonrotating Earth.)

Y is the distance normal to instantaneous orbital direction on Earth figure from nadir $Y = \gamma R$*.

x is distance on projection plane in instantaneous orbital direction.

y = distance on projection plane normal to x (computed; not projected).

l = actual orbital path as imaged.

τ = skew angle (varies with latitude).

Then the Space Oblique Mercator projection basic formulas (scale factor = 1) are:

$x = X$

and based on origin at nadir,

$y = R\int sec\gamma \, d\gamma = R \, log_e (sec \gamma + tan \gamma) = R \, log_e \, tan (\gamma/2 + \pi/4).$

The term y will generally contain another term to take care of the cylinder oscillation due to Earth rotation. A false value of perhaps 1,600,000 m should be given to the x axis as shown in Figure 2 in order to eliminate negative values of y.

APPLICATION

To make ERTS MSS-type imagery fully suitable for mapping, several steps must be taken, as follows:

1. Parameters for the system must be set and adhered to within stated limits.
2. The projection must be carefully defined, and system corrections must be applied with results comparable to or better than those now being achieved with ERTS-1.
3. The mathematical relationship between the projection (model) and the figure of the Earth must be rigorously developed.
4. Image-identifiable control must be cataloged and used for system calibration. The

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*If one disregards the small error introduced by Earth rotation during the scan sweep (the maximum displacement in the x direction is only about 200 m for the 185-km scan length), the y direction on the image is that of the scan lines (as now configured). However the orbital path is skewed on the projection by as much as 4° with respect to the instantaneous orbital direction, again due to Earth rotation. (See Figure 2)

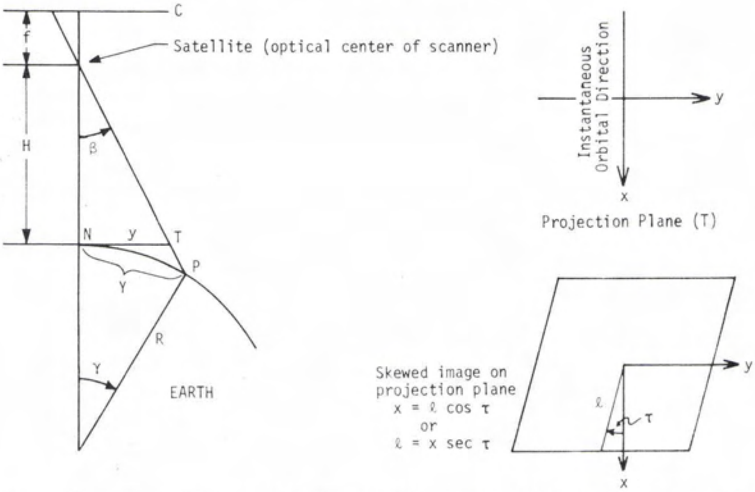

FIG. 3. Geometry of ERTS Multi Spectral Scanner (orbital plane is perpendicular to the plan on the left).

density or form of the control is not known at present, but there are indications that spacing may be in the hundreds or even thousands of kilometers, with a few test sites of denser control (20 to 50 km spacing) for detailed system analysis.

5. System corrections to be applicable to tapes as well as imagery.

6. Precision (scene-corrected) processing to be of two forms:

   a. Precise application of geodetic indicators (lat/long or UTM coordinates) to the system-corrected imagery (and perhaps tapes) without altering the system-corrected structure or projection.

   b. Transformation of imagery (and perhaps tapes) to a conventional map projection, such as the UTM or polar stereographic, and addition of appropriate geodetic indicators. Transformation should be required for only a small percentage of the recorded imagery.

Steps 1 and 2 can be based on ERTS-1 performance as it is assumed that the performance of ERTS-1 can be equalled or exceeded on future ERTS-type spaceflights.

The mathematical problem (Step 3) is of paramount concern. I suggest that NASA, with technical input from USGS (and others), take the lead. Here is a real challenge to the cartographic community. As geodesists and photogrammetrists, we must carefully examine the problems and the various solutions possible. Then, as mapmakers representing the map users of the world, we should spell out exactly what we need. A considerable and dedicated effort will still be needed to develop the mathematical model and associated computer programs. Because the programs thus developed could be applied to Earth imaging systems other than ERTS, the programs should have appropriate flexibility and precision.

Once the model and programs are developed, they should be tested against a variety of ground control arrays, and thus the requirements for control (step 4) can be defined. Steps 5 and 6 require the provision of appropriate processing at some centralized point. With these 6 steps made effective, we believe that ERTS-type images and tape in cartographic form and with geodetic precision can be introduced in a matter of days after acquisition—particularly if the continuous and uniform Space Oblique Mercator projection is employed. Perhaps the era of automated mapping, based on Earth-sensing space systems, is not far off.

REFERENCE

1. NASA, *Earth Resources Technology Satellite Data Users Handbook*. Prepared and maintained by General Electric Corporation, 1971 to date.
2. Bender, Lee U., *An Algorithm for Gridding Satellite Photographs*. Prepared by the Ohio State University (Dept. of Geodetic Science) for U. S. Geological Survey, April 1970.
3. Colvocoresses, A. P., and McEwen, R. B., "EROS Cartographic Progress," *Photogrammetric Engineering*, 39:12, pp. 1303-1309, 1973.
4. Colvocoresses, A. P., "Status of Positional Referencing of ERTS Imagery (EC-17-ERTS)." Memorandum submitted to NASA in part of an ERTS experiment, July 1973.
5. Colvocoresses, A. P., "Towards an Operational ERTS." To be published by NASA as part of the Third ERTS Symposium of Dec. 10-13, 1973.

Reference 5 contains the following additional references pertinent to this paper:

a. Konecny, G., "Geometric Aspects of Remote Sensing." Invited paper Commission IV International Congress of Photogrammetry, Ottawa, 1972.

b. Kratky, V., "Cartographic Accuracy of ERTS Images," *Photogrammetric Engineering,* 40:2, pp. 203-212, 1974.

c. Forrest, R. B., Mapping from Space Images, *Bendix Technical Journal,* Summer/Autumn 1970.

d. Colvocoresses, A. P., "ERTS-A Satellite Imagery," *Photogrammetric Engineering,* 36:6, pp. 555-560, 1970.

e. Thomas, Paul D., *Conformal Projections in Geodesy and Cartography,* Coast and Geodetic Survey Sp. Pub. No. 251, Government Printing Office, Washington, D.C., 1964.

Opportunities for Research in Lunar Science

THE National Aeronautics and Space Administration (NASA) is continuing to encourage and support research on all aspects of lunar science. NASA's goal is to improve the scientific understanding of the origin, evolution, structure, and composition of the moon, and of its relationships to the earth and the solar system. NASA's broadly based lunar programs include the following: (a) experimental and theoretical research on lunar materials (samples and Surveyor parts); (b) lunar data analysis and synthesis, using published or other generally available data; and (c) supporting research and technology designed to support the general goals of the other lunar programs, such as theoretical studies, laboratory simulations, meteorite research, advanced experiment concepts etc. NASA especially encourages well-qualified scientists not now in the lunar programs to participate if they have new ideas, techniques, or research capabilities. An announcement giving details on where and how to propose may be obtained from Dr. Noel W. Hinners, Director, Lunar Programs Office, Code SM, NASA, Washington, D.C. 20546.

Analytical Methods Developed for Application to Lunar Samples Analysis, Special Technical Pub. 539, American Society for Testing and Materials, 1916 Race St., Philadelphia, Pa. 19103. 156 pages, hard cover, $15.

THIS SYMPOSIUM was organized to provide an opportunity for those outside the National Aeronautics and Space Administration lunar sample program to learn of some of the advances in methods and instrumentation resulting directly from the program.

The papers describe the present status of advanced testing methods used in lunar sample analysis. Particular emphasis is placed on the description and evaluation of the various experimental techniques as opposed to other lunar science conferences which have emphasized interpretation of the results. Since contamination control is such a vital consideration in lunar sample work, several papers are presented dealing with that aspect.