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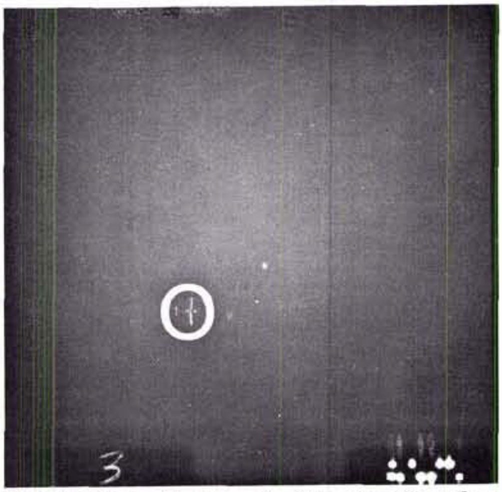
Tests of an Airborne Tilt-Indicator

The system will permit a significant increase in the measurement accuracy of tree variables from large-scale photographs



Frontispiece. Photograph of the aircraft's reflection in the water and coded ADL-1 output on lower right-hand corner.

(Abstract on next page)

Introduction

Determination of tree height, one of the most important variables in the computation of forest inventory statistics, has been the subject of many research studies. So far, the main concern has been the effect of isolated factors, such as the photographic scale, the focal length of the camera, the instrument used for the evaluation, etc., upon the measurement of the differential parallax (Nielsen, 1971). Interactions of these factors have not received the attention they deserve. Also tilt, recognized by Johnson (1962), Schut and van Wijk (1965), Brun (1972), and Aldred (1973) as one of the main sources of errors in the photogrammetric determination of tree height, has been largely neglected by the user of aerial photographs. The tilt problem has become increasingly important with the use of large-scale and particularly, small-format (70-mm) aerial photographs. At the Forest Management Institute (FMI), the tilt problem has been approached in two ways. One, by using a gyro-stabilized camera mount which keeps the camera within ±0.5 degrees of vertical 90 percent of the time (Hall[1]). The other approach is the use of a gyro-stabilized recording system such as the Airborne Tilt-Indicator designated ADL-1 (Airborne Data Logger) which measures the angles of longitudinal and lateral tilt and displays the output into the secondary optics of the camera. These data, recorded on the picture frame, permit analytical corrections to be made to eliminate the effect of tilt.

The aim of this paper is to summarize the effect of longitudinal tilt upon the determination of tree heights, and to present test results of the ADL-1.

[1] Hall, J. "Operational trial of A-28 stabilized vertical camera mount." Unpublished manuscript.

Effect of Tilt in the Determination of Tree Heights

Schut and van Wijk (1965) analyzed the distortion of parallax caused by tilt, and demonstrated that the error depends on the formulas that were used to compute tree height. Among other values, they used the following:

H	(flying height above ground)	= 150 m
B	(distance between exposure stations)	= 42 m
f	(focal length of camera)	= 100 m
dp	(differential parallax)	= 3.818 m
Pt	(absolute parallax of top of tree)	= 31.818 m
Pb	(absolute parallax of base of tree)	= 28 mm.

The formulas analyzed were:

$$h = \frac{H\,dp}{Pt} \tag{1}$$

$$h = \frac{Bf\,dp}{Pt Pb} \tag{2}$$

$$h = \frac{H\,dp}{(Bf/H) + dp} \qquad (3)$$

where h is the height of a tree. For the values given above, all three formulas give a tree height of 18 m.

If the right-hand photograph is assumed tilted one degree towards the left-hand photograph (Figure 1), the maximum errors computed were:

Formula 1: +1.0 m or +5.6%
Formula 2: +2.3 m or +12.8%
Formula 3: −0.2 m or −1.1%.

These errors consider parallax only. However, camera tilt also affects the determination of flying height and photographic base. Although the distortion of flying height is negligible (especially if measured with the radar altimeter), the distortion of the photographic base is more serious. The distance B between stations used in the formulas above, cannot be measured (unless the technique of fixed-base photography is used). B is either measured on the ground or calculated from measurements on the photographs (for example, using FMI's stereotope digitizer system, on the left-hand photograph). In both instances, this distance is distorted (Figure 1). For the values given, the distortion is

$\tan 1° \times H = 0.0175 \times 150 = 2.625$ m.

Thus B becomes

$42 - 2.625 = 39.375$ m or, at the scale given.

b' (photo base) = 39.375:1500 = 26.25 mm.

Introducing this value into the formulas, all three give approximately the same result, i.e. Formula 1 will not be affected, in Formula 2 there is a certain amount of compensation and thus the error decreases and for Formula 3 the error increases.

For the following more typical set of values— H = 300 m, f = 150 mm, b = 22.4 mm, h = 20 m, and 70-mm nominal format— the error for one degree of longitudinal tilt on the right-hand photograph is about 12 percent, computed with any of the formulas and for any position of the tree within the model.

These examples demonstrate that tilt cannot be neglected. The stabilized camera mount is a significant improvement, but for the two examples given, ±0.5 degrees of longitudinal tilt still means a possibility of about ±3 percent and ±6 percent error respectively in the determination of tree height.

Airborne Tilt-Indicator

The ADL-1 was described at the Aerospace Electronics Symposium, Saskatoon, 1973.[2] Briefly, the system consists of a gyroscope, two opto-electronic measuring units, the logic and the display. The opto-electronic measuring units, one for longitudinal tilt and one for lateral tilt, consist of a series of light-emitting diodes each, which are placed exactly opposite to an equal number of photo transistors. These units are mounted one on the main chassis and one on the longitudinal axis gimbals of the gyroscope and are, therefore, related to the (moving) longitudinal and lateral tilt axes of the aircraft (or camera). A gap between light-emitting diodes and photo-transistors permits free movement of two coded masks which are mounted one on the longitudinal and one in the lateral axes gimbals of the gyroscope and thus, are related to the axes of the vertical gyroscope. Any change in the relative position between the stable gyroscope axes and those of the moving aircraft (or camera) promote a change in the code. The electronics provide holding circuits to ensure that the code changes only if the sensor is exactly in the center of a code line. The output is fed to another set of light-emitting diodes which are photographed through the secondary optics of the camera. A holding circuit freezes the reading just before the shutter reaches the secondary optics, avoiding a code change during exposure.

The system has a range of 7.5 degrees in

ABSTRACT: *Distortions of parallax and photographic base caused by tilt introduce significant errors in the photogrammetric determination of tree height. An airborne system to measure and record tilt at the instant of photographic exposure was tested. Longitudinal tilt was measured to an accuracy of ±0.3 degree 95 percent of the time. It is concluded that, for photogrammetric values typically used in large-scale aerial photography applied in forest inventories, this accuracy is satisfactory.*

[2] Nielsen, U. 1973. "A new system for recording aircraft attitude". *Canadian Aeronautics and Space Journal*, 19(10): 525-527.

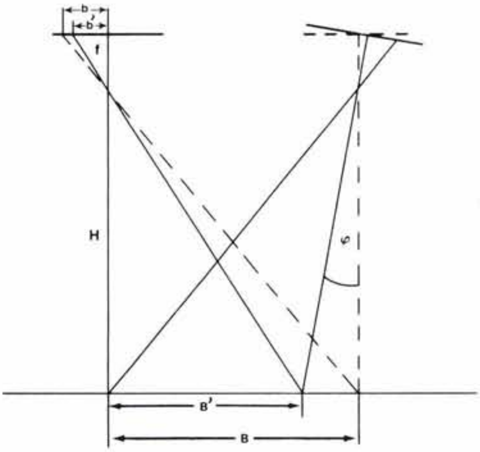


FIG. 1. Base distortion caused by tilt of the right-hand photograph.

any direction. Resolution is 6 minutes of arc within the range of ±6 degrees, and 30 minutes of arc for the rest of the range. These limits are specific to the system tested.

In Figure 2, the system is installed in an aircraft. The gyroscope with the built-in measuring units is mounted on a platform firmly attached to the camera mount. The camera and the gyroscope chassis thus form one rigid unit. The logic unit is not shown.

The Frontispiece is an example of the coded readout of the system and the image of the aircraft's reflection. A table is used to convert the binary code to lateral and longitudinal tilt readings.

This system can be obtained from Presentey Engineering Products Limited in Ottawa, at a cost of approximately $15,000 to $20,000 depending on specifications.

Test Procedure

In order to test the system, a simple technique to determine longitudinal and lateral tilt of photographs was used. For this purpose, the system was flown and operated over smooth water and the reflection of the aircraft was photographed. The water surface acts as a horizontal mirror, i.e., the reflection of the camera in the water always coincides with the nadir (Sayn-Wittgenstein and Aldred, 1967). If the optical axis if vertical at the time of exposure, the reflection will also coincide with the principal point of the photograph. Deviations from this position permit easy calculation of longitudinal and lateral tilt.

The following two factors introduced systematic errors in the measurements:

- The exact location of the principal point was not known (measurements were related to approximately located fiducial marks);

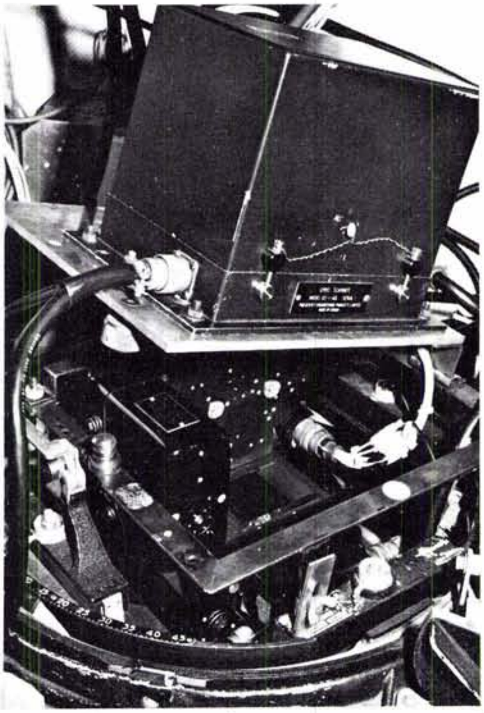


FIG. 2. The ADL-1 and camera system mounted in an aircraft. Note that the platform holding the gyroscope is lifted.

- The accuracy of alignment of the lens axis and the vertical gyroscope axis was not known (they were assumed to be parallel).

Several lines were flown, one with the aircraft in normal attitude, a second with induced longitudinal and a third with induced lateral tilt angles within the range of the system, and a fourth with both tilt angles changed simultaneously. Photographs were obtained at the rate of one frame per second.

Results

The average differences between the computed tilt and the ADL-1 readings were assumed to be the combined bias caused by residual misalignment of fiducial marks and of optical and gyro axes. For the longitudinal tilt, this bias was 1.24 degrees; for the lateral tilt, it was 0.81 degree. These systematic errors were then subtracted from all differences. Results are listed in Table 1. The maximum errors are found under the columns labeled *Extremes*. The standard deviations were computed for the differences between calibrated and ADL-1 measured tilts.

The standard deviations for all differences are:

Longitudinal tilt 0.17 degree
Lateral tilt 0.20 degree.

TABLE 1. ERRORS OF ADL-1 IN DEGREES FOR FOUR DIFFERENT FLIGHTS

No. of photos	Range of tilts in degrees		Errors ADL-1 in degrees			
			Longitudinal		Lateral	
	Long.	Lat.	Extremes*	s.d.	Extremes†	s.d.
16	0.5	1.5	0.2/0.4	0.17	0.6/0.2	0.21
30	3.5	1.6	0.3/0.3	0.16	0.5/0.4	0.23
30	1.5	9.2	0.3/0.3	0.15	0.2/0.3	0.14
13	3.6	11.2	0.5/0.4	0.26	0.5/0.3	0.21

*First value for nose down, second for nose up.
†First value for left wing down, second for left wing up.

DISCUSSION OF RESULTS AND CONCLUSIONS

Tilt is one of the major sources of errors in the photogrammetric determination of tree height. The use of a gyro-stabilized camera mount is an improvement but, for very large-scale and small-format aerial photographs, still insufficient. A further disadvantage of the mount is its size and weight, which complicate installation in small aircraft.

The ADL-1 is easy to install and results are more accurate. The theoretical error of this system is slightly less than 0.1 degree. Because measurement is taken at the gyroscope's axes gimbals and no analog-to-digital conversion takes place, accuracy depends only on the accuracy of the gyroscope itself. Aircraft accelerations should not cause significant errors. Sudden changes in aircraft attitude are normally very fast, i.e., one or two seconds. If, during this time, the mercury in the switches of the erector circuits is displaced due to the acceleration force, the gyroscope is brought off the vertical position. But, as this change in position is slow (about 0.1 degree per second), the error introduced is not significant.

The test provided the following results:

- ★ For longitudinal tilt, a systematic error of 1.24 degrees (nose up) and a standard deviation of the differences between calibrated and ADL-1 measurements of 0.17 degree;
- ★ For lateral tilt, a systematic error of 0.81 degree (left wing up) and a standard deviation of 0.20 degree.

If the systematic errors are removed, the readings of the ADL-1 systems are within ±0.34 and ±0.40 degree for longitudinal and lateral tilt respectively 95 percent of the time. With the given data, this translates into a maximum tree-height error of about ±2 feet or ±3 percent, which is satisfactory for forest-inventory purposes. The systematic errors are attributed to misalignments of fiducial marks (reference of calibration measurements) and of vertical axis of gyro-scope in relation to optical axis.

A factor which may contribute to the random errors obtained is the limited accuracy of the calibration measurements. The film obtained for the test was badly underexposed, making the image of the reflection of the plane in the water sometimes barely discernible. This caused some uncertainty and consequently an error in the determination of the position of the camera on the image. It is estimated that the measurement accuracy of the distance from the assumed principal point to the image of the camera reflection was about ±0.3 mm, which translates into slightly over ±0.1 degree of tilt. Otherwise, however, the *reflection technique* used to determine the actual longitudinal and lateral tilts present at time of exposure, is extremely simple and effective. The measurements of the ADL-1 system can be related directly to both tilt angles of the aircraft if the camera is aligned so that the edge of the film is parallel to the longitudinal axis of the aircraft (no correction for drift).

It is concluded that the ADL-1 system will permit a significant increase in the measurement accuracy of tree variables from large-scale photographs. Furthermore, the system will also find application in other fields where photogrammetric techniques are applied, and where the ground control required for the orientation of the photographs is insufficient or does not exist.

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