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# Plotter Orientation from Aerotriangulation Output

## Formulas are listed for the Zeiss C-8 Stereoplanigraph, Wild B-8 Aviograph, and the Santoni Stereosimplex II-C.

Abstract: *If a stereoplotter is equipped with dials and scales to indicate the attitudes of the projectors, then the relative and absolute orientation of a stereomodel may be accomplished in the plotter by setting known values into the dials and scales for the projectors. These setting values can be determined from the exterior orientation parameters of the photographs of the stereomodel that are computed in analytical aerotriangulation programs.*

### Introduction

The elements of exterior orientation for the photos of a stereopair may be analytically determined in several ways. A computer program for space resection will compute the exterior orientation parameters for a single photo from the photo coordinates and the ground coordinates of three or more points on the photo.[1] The points may be artificial points and the ground coordinates may be aerotriangulated coordinates. In one method of analytical aerotriangulation, referred to as the simultaneous adjustment of photos, the exterior orientation parameters of all the photos in a strip or block are determined as an integral part of the adjustment. It is the purpose of this paper to detail how the values of the exterior orientation parameters may be used to aid in the relative and absolute orientation of models in those stereoplotting instruments that have dials and scales to indicate the attitudes of the projectors.

The exterior orientation parameters that are output from the aerotriangulation computer program are the camera station coordinates and the rotations associated with all the photos in the strip or the block that was adjusted. The coordinates, designated as $X^C$, $Y^C$, $Z^C$, are in the ground coordinate system. The rotations generally used in analytical photogrammetry are the sequential rotations $\omega$, $\phi$, $\kappa$ which, if applied to the axes of the ground coordinate system, make them parallel to the photo coordinate system. Further explanation of these rotations may be found in Reference 2.

The camera station coordinates and the rotations for the photos numbered 1 and 2 are designated as

$$X_1^C,\ Y_1^C,\ Z_1^C,\ \omega_1,\ \phi_1,\ \kappa_1$$
$$X_2^C,\ Y_2^C,\ Z_2^C,\ \omega_2,\ \phi_2,\ \kappa_2.$$

For the purpose of this paper, there are two types of plotting instruments. One type, such as the Zeiss C-8 Stereoplanigraph, can accommodate a liberal $bz$ motion in the right projector. The other type, such as the Wild B-8 Aviograph and the Santoni Stereosimplex II-C, are equipped with a common *phi* motion instead of a $bz$ motion. The motions associated with both types of instruments are the $x$-tilt, $y$-tilt and swing ($\kappa$) for each projector.

The entire procedure for the Zeiss C-8 is presented first. The procedures for the other instruments are modifications of this procedure.

### Zeiss C-8 Procedure

1. Photo 1 is placed in the left projector, and Photo 2 in the right projector. The unit of rotation is grads. The instrument base scales are in millimeters.
2. Compute the air base components:

$$BX = X_2^C - X_1^C$$
$$BY = Y_2^C - Y_1^C$$
$$BZ = Z_2^C - Z_1^C.$$

3. The instrument base is

$$bx = m\,(BX^2 + BY^2)^{1/2}$$

where $m$ is the scale of the model times a factor to convert the ground unit to millimeters. If the model scale is 1/1500 and the ground unit is feet,

$$m = (1/1500)\,(305)\ mm/ft = 0.203.$$

4. The $bz$ setting for the right projector is

$$bz = m\,(BZ).$$

5. The angles symbolized by $\omega$, $\phi$, $\kappa$ are sequential rotations with $\kappa$ tertiary. In the plotting instruments, $\kappa$ is applied first. Let $\omega^*$, $\phi^*$ symbolize the rotations after $\kappa$ has already been applied. Then

$$\omega_1^* = \omega_1 \cos \kappa_1 + \phi_1 \sin \kappa_1$$
$$\phi_1^* = \phi_1 \cos \kappa_1 - \omega_1 \sin \kappa_1$$
$$\omega_2^* = \omega_2 \cos \kappa_2 + \phi_2 \sin \kappa_2$$
$$\phi_2^* = \phi_2 \cos \kappa_2 - \omega_2 \sin \kappa_2$$

6. The left and right $x$-tilt settings are

$$c_L = 100 + \omega^*_1$$
$$c_R = 100 + \omega^*_2$$

7. The left and right $y$-tilt settings are

$$d_L = 100 + \phi^*_1$$
$$d_R = 100 + \phi^*_2.$$

8. The swing is cleared by optical observation of the model.

### Wild B-8 Procedure

Modify the Zeiss C-8 procedure as follows:

4. Compute common-*phi* as,

$$\Phi = \frac{63.7\,(BZ)}{(BX^2 + BY^2)^{1/2}}$$

where 63.7 is the number of grads per radian. The instrument setting is

$$a = 100 + \Phi.$$

7. The left and right $y$-tilt settings are

$$d_L = 100 + \Phi + \phi_1^*$$
$$d_R = 100 + \Phi + \phi_2^*.$$

### Santoni II-C Procedure

Modify the Zeiss C-8 procedure as follows:

4. The common-*phi* setting is applied equally to both the right and the left scales and is:

$$a = \frac{1000\,(BZ)}{(BX^2 + BY^2)^{1/2}}.$$

The common-*phi* angle is

$$\Phi = a\ (63.7\ /\ 1000).$$

(6) The left and right $x$-tilt settings are

$$c_L = 200 - \omega^*_1$$
$$c_R = 200 - \omega^*_2.$$

(7) The left and right $y$-tilt settings are

$$d_L = 100 + \Phi + \phi_1^*$$
$$d_R = 100 + \Phi + \phi_2^*.$$

These procedures will produce a model that is *very nearly* cleared, scaled, and leveled. There are several reasons why the procedures do not produce the model perfectly: (1) the aerotriangulation values are not exact; (2) film distortion, lens distortion and atmospheric refraction are analytically corrected in the aerotriangulation process, but are not corrected in the analog process; (3) operator bias; (4) instrument calibration.

### Sample Calculation

The computations for an actual model set in a Santoni Stereosimplex II-C are presented below. The photographs were taken with a 6-inch mapping camera at a flight height of 900 feet. The photo scale is 1:1800 and the model scale is 1:900. The exterior orientation parameters for the two photos of the model, Photos 5 and 6, are shown in Table 1.

Table 1. Exterior Orientation Parameters for a Pair of Photographs

| | *Photo 5* | *Photo 6* |
|---|---|---|
| $X^C$ | 1,719,149.8 | 1,719,570.6 |
| $Y^C$ | 218,838.2 | 218,064.7 |
| $Z^C$ | 904.1 | 899.2 |
| $\omega$ | 0.24 | −0.37 |
| $\phi$ | 0.15 | −1.06 |
| $\kappa$ | 30.14 | 31.80 |

The dial and scale settings are determined as follows:

1. First, photo 5 is placed in the left projector and photo 6 is placed in the right projector.

2. $BX = 1{,}719{,}570.6 - 1{,}719{,}149.8 = 420.8$
   $BY = 218{,}064.7 - 218{,}838.2 = 226.5$
   $BZ = 899.2 - 904.1 = -4.9$

3. $m = (1 / 900)\,(305) = 0.339$
   $bx = (0.339)\,(420.8^2 + 226.5^2)^{1/2}$
   $= (0.339)\,(477.9) = 162.0$

4. $a = 1000\,(-4.9) / (477.9) = -10.25$
   $\Phi = (-10.25)\,(63.7 / 1000) = -0.653$

5. $\omega_5^* = (0.24)\cos(30.1^g) + (0.15)\sin(30.1^g)$
   $= (0.24)\,(0.8903) + (0.15)\,(0.4554)$
   $= 0.282$
   $\phi_5^* = (0.15)\,(0.8903) - (0.24)\,(0.4554)$
   $= 0.024$
   $\omega_6^* = (-0.37)\,(0.8778) + (-1.06)\,(0.4790)$
   $= -0.840$
   $\phi_6^* = -1.06)\,(0.8778) - (-0.37)\,(0.4790)$
   $= -0.753$

6. $c_L = 200 - 0.282 = 199.72$
   $c_R = 200 - (-0.840) = 200.84$

7. $d_L = 100 + (-0.653) + 0.024 = 99.37$
   $d_R = 100 + (-0.653) + (-0.753)$
   $= 98.59.$

8. After the swing was cleared, the model was clear to within *one dot* of parallax.

After the model had been completely cleared, leveled and scaled by optical observation, the dial and scale readings were $b_x = 162.0$, $a = -10.25$, $c_L = 199.71$, $c_R = 200.74$ $d_L = 99.33$, $d_R = 98.55$.

### References


1. Keller, M. and Tewinkel, G. C., "Space Resection in Photogrammetry," *ESSA Technical Report* C&GS 32, September 1966.
2. American Society of Photogrammetry, *Manual of Photogrammetry*, 3rd Edition, pages 46-53, 1966.


## Space Observations for Management of World Resources

Annual Meeting of the American Association for the Advancement of Science, Americana Hotel, New York City, Jan. 27-31, 1975.

Arranged by J. J. Doyle and R. B. McEwen of the American Society of Photogrammetry for 9:00 a.m., Friday, Jan. 31.

*Significant Results from ERTS and Skylab*
J. M. Denoyer and W. A. Fischer, EROS Program, USGS.
*Observation System Capabilities*
R. J. Ondrejka, ITEK Corp.
*Ground Processing of Space Observations*
W. Murray Strome, Canadian Centre for Remote Sensing.
*International Information Exchange*
H. G. S. Murthy, United Nations
*Politics, Security and International Law*
Prof. Samuel D. Estep, Univ. of Michigan.