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Local Variation of Photogrammetric Refraction

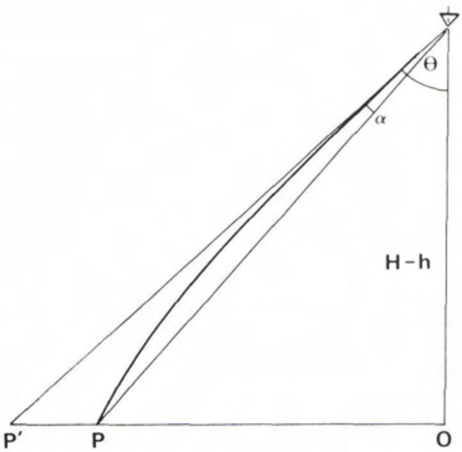

FIG. 1. Photogrammetric refraction α.

Errors caused by the assumption of a standard atmosphere are relatively small under normal photographic conditions.

(Abstract on page 296)

Introduction

In a recent report on image geometry, Ziemann[1] includes photogrammetric refraction as one of the sources of error which causes image distortion. Complying with current usage, the term *photogrammetric refraction* is confined in the report to the bending of light rays in an undisturbed atmosphere. Of other air refraction phenomena causing image distortion, internal refraction within the aircraft itself, although not strictly belonging to the photogrammetric refraction proper, has been included in the present discussion because of the important role it may have in the final outcome of the total refraction effect.

Referring to Figure 1, the photogrammetric refraction is customarily defined as the angular separation α of the true and apparent positions P and P' of a terrain point viewed from an aerial camera. It is a function of nadir distance θ:

$$\alpha = R \tan \theta \qquad (1)$$

where R, in turn, is a function of the vertical density structure of the atmosphere, flying height H, and terrain height h. For a light ray making an angle of 45° with the vertical, $\alpha = R$. We shall use R, expressed in radians, henceforth as a convenient general measure for photogrammetric refraction.

Neglecting, in this connection, the curvature of the earth, the apparent radial shift of a terrain point is $\overline{PP'} = R \tan\theta \sec^2\theta \, (H - h)$, from the geometry of the figure. The resulting photographic image of a horizontal terrain plane is consequently enlarged by scale factors m_1 and m_2 in the radial and tangential directions, respectively;

$$m_1 = 1 + R + 3R(r/f)^2$$
$$m_2 = 1 \infty R + R(r/f)^2,$$

f being the focal length of the camera, and r the distance from the principal point. Away from the center of the photograph, where $m_1 \neq m_2$, the enlargement is not even conformal but involves distortion of small detail as well.

As one might expect with good reason, at least some of the refraction error is eliminated in the photogrammetric process, a stereomodel was set up from 15 simulated ground control points in a pattern shown in Figure 2. On each photograph, assumed to be taken from 6000 meters, the points were shifted radially so as to simulate the effect of standard refraction. The residuals which remained after relative and absolute orientations, and linear transformation to ground control are given in Table 1. In this particular example of a single stereomodel, the residuals would apparently be lost among other errors from different sources. Similar results were obtained in a test involving points 1 to 6 only.

However, the errors due to refraction are systematic and, on a strip of photographs, they may tend to accumulate from model to model. For this reason, the application of ra-

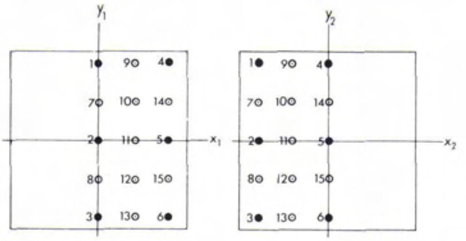

FIG. 2. Test pattern.

dial corrections for refraction is justified in analytical photogrammetry.

REFRACTION IN STANDARD ATMOSPHERE

The summation method used by Bertram[2] and Schut[3] for the computation of photogrammetric refraction in a standard atmosphere has a simple analytical interpretation which deserves a closer examination. Referring to Figure 3, one finds immediately that in Schut's formula, which may be written

$$R = \frac{0.000226}{H - h} \sum_{\rho = \rho_H}^{\rho = \rho_h} (z - h)\, d\rho$$

$$= \frac{1}{H - h} \sum_{n = n_H}^{n = n_h} (z - h)\, dn,$$

the photogrammetric refraction is computed dividing the triangular area $\bar{A}\bar{B}\bar{C}$ which falls under the refractive index curve (or the curve for density ρ) by triangle height $\bar{A}\bar{B}$. If we now plot a second curve showing the average refractive index $\bar{n}$ (or the average density $\bar{\rho}$) from ground level up to any flying height, the areas of the two shaded triangles in the figure will be equal, giving the important relationship,

ABSTRACT: *Photogrammetric refraction causes a non-homogeneous increase in the scale of an aerial photograph, the enlargement factor growing with radial distance from the nadir point. Scaling of the photograph during the evaluation process will, on the whole, reduce the refraction error, but it can be eliminated only by radial corrections based upon the refractive properties of the atmosphere. Investigation of local variations of refraction caused by changes in atmospheric pressure, temperature and humidity shows that, up to flying heights of 2000 to 3000 meters, the largest variations are associated with changes in the vertical gradient of temperature, whereas for flying heights over 5000 meters the amount of refraction largely depends on the absolute temperatures rather than the temperature gradients. With the exception of winter photography in the middle and high latitudes, local variations of refraction should not constitute a serious source of error in aerial photogrammetry, and corrections for refraction computed from standard atmosphere may be considered adequate if the photographs have been taken in unconditioned camera environment.*

TABLE 1. RESIDUALS FROM UNCORRECTED REFRACTION
($\bar{H}$ = 6000 m; R = 59 μrad; f = 152.4 mm)

Point	*dx*, (*μm*)	*dy*, (*μm*)	*dz*, (*μm*)	*Point*	*dx*, (*μm*)	*dy*, (*μm*)	*dz*, (*μm*)
1	0	0	+1	9	0	0	+2
2	0	0	−2	10	0	+1	−2
3	0	0	+1	11	0	0	−2
4	0	0	+1	12	0	−1	−2
5	0	+1	−2	13	0	0	+2
6	0	0	+1	14	0	0	0
7	−1	0	0	15	0	0	0
8	−1	0	0				

$$R = \bar{n} - n_H = 0.000226(\bar{\rho} - \rho_H), \quad (2)$$

i.e., *photogrammetric refraction R is equal to the refractive index at the camera level subtracted from the average refractive index in the air column between the ground and camera levels.* This interpretation of the photogrammetric refraction is valid regardless of the atmosphere considered.

In most standard atmospheres, refractive index and density can be expressed mathematically as integrable functions of height; those functions, in Equation 2, yield formulas for the photogrammetric refraction in terms of flying height and ground height. For example, taking the standard distribution of pressures and temperatures given by Brunt[4] for the I.C.A.N. Atmosphere, we have in this atmosphere the density distribution

$$(z \leq 11\ km) : \rho = 1.2256\ (1 - 0.02257\ z)^{4.256}$$

$$(z \geq 11\ km) : \rho = 0.3638\ e^{-0.1578\ (z-11)}$$

where density ρ is in kg/m^3, height above sea level, z is in kilometers, and e stands for the base of natural logarithms. Integration of these density functions gives the following expressions for the photogrammetric refraction in the I.C.A.N. Atmosphere:

Flying heights of up to 11000 meters–

$$10^6 R = \frac{2335}{H-h}\left[(1 - 0.02257\ h)^{5.256} - (1 - 0.02257\ H)^{5.256}\right] - 277.0\ (1 - 0.02257\ H)^{4.256} \quad (3a)$$

Flying height over 11000 meters–

$$10^6 R = \frac{2335}{H-h}\ (1 - 0.02257\ h)^{5.256} - 0.8540^{H-11}\left(82.2 + \frac{521}{H-h}\right) \quad (3b)$$

where camera height H and ground height h are both in kilometers above the sea level. Comparison of Formulas 3a and 3b with the refraction tables by Schut[3], the latter based on the U. S. Standard Atmosphere, 1962, shows trivial differences not exceeding 0.2 μrad.

Much simpler, though formally less accurate, formulas for the computation of the photogrammetric refraction at moderate flying heights can be obtained approximating refractive index n as a second-order function of height:

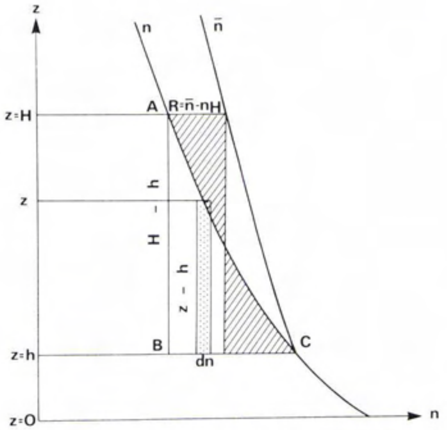

FIG. 3. Photogrammetric refraction as a function of refractive index.

$$n = n_0 + c_1 z + c_2 z^2 . \quad (4)$$

This gives

$$\bar{n} = \frac{1}{H-h}\int_h^H n\ dz$$

$$= n_0 + \tfrac{1}{2}\ c_1(H+h) + \tfrac{1}{3}\ c_2(H^2 + Hh + h^2)$$

$$n_H = n_0 + c_1 H + c_2 H^2$$

and

$$R = \bar{n} - n_H =$$

$$-\tfrac{1}{2}\ c_1\ (H-h)\left[1 + \frac{2c_2}{3c_1}\ (2H+h)\right]. \quad (5)$$

Coefficients c_1 and c_2 are evaluated for any atmospheric model desired so as to obtain the best possible fit for the vertical distribution of the refractive index in Equation 4.

In the U. S. Standard Atmosphere, 1962, one obtains the approximate formula:

Flying heights of up to 9000 meters—

$$10^6 R = 13(H - h)\ [\ 1 - 0.02(2H + h)] \quad (6)$$

where again, camera height H and ground height h are both in kilometers above the sea level. This formula gives the standard values of photogrammetric refraction correctly to within $\pm$ 0.5 μrad.

In climates vastly different from standard conditions it may be advisable to determine coefficients c_1 and c_2 on the basis of regional monthly aerological data[5].

Local Variations

The actual atmosphere in which photographs are flown is a dynamic medium where the vertical distribution of density and, therefore, the photogrammetric refraction constantly departs more or less from that in a standard atmosphere. Whether or not such departures can be large enough to have an adverse effect on the photogrammetric accuracy shall be investigated in view of the observed variations in those meteorological elements that govern the vertical density structure: pressure and temperature at the sea level, vertical gradient of temperature, and water vapor content of the air.

A formula which expresses the photogrammetric refraction in terms of meteorological elements was recently given by the author[6]:

$$10^6 R = 2.316 \left(\frac{p_0 - p_H}{H} - 34.11\, p_H / T_H \right). \quad (7)$$

We assume here, for simplicity, that the ground is at sea level, where the symbols have the following meanings: p_0 and p_H are the barometric pressures in millibars at the sea level and at the camera level, respectively, H is the flying height in kilometers above the sea level, and T_H is the absolute temperature in kelvins at the camera level. Denoting the vertical gradient of temperature by β (°C/km), we have also

$$T_H = T_0 + \beta H$$

and, as was shown in the reference,

$$p_H / p_0 = (T_H / T_0)^{-34.11/\beta} = \left(1 + \frac{\beta H}{T_0}\right)^{-34.11/\beta}$$

t_0 being the absolute temperature at the sea level. If these values are substituted into Equation 7, and the result expanded into a binomial series, one obtains the following relationship between photogrammetric refraction R and the basic meteorological elements:

$$10^6 R = 39.6\,(34.1 + \beta)\,(p_0 / T_0^2)\, H \times$$
$$\left[1 + c_1 (H/T_0) + c_2 (H/T_0)^2 + \cdots \right] \quad (8)$$

where the coefficients are

$$c_1 = -\tfrac{2}{3}\,(34.1 + 2\beta)$$

$$c_2 = \tfrac{1}{4}\,(34.1 + 2\beta)(34.1 + 3\beta)$$

$$\cdots\cdots$$

$$c_n = \frac{2(-1)^n}{n!(n+2)}\,(34.1 + 2\beta)(34.1 + 3\beta)$$
$$\ldots \left[34.1 + (n+1)\,\beta\right]$$

$$\cdots\cdots$$

The magnitude of local variations in refraction can be investigated with the aid of Equation 8.

Sea-Level Pressure

Barometric pressure indicates the weight of the overlying atmospheric column. Consequently pressure changes at the ground level, which necessarily require physical transport of air towards regions of increasing pressure, are much smaller than pressure changes observed in the free atmosphere at any higher altitude where they are caused by vertical movement of air within the column itself.

The highest sea-level pressures are observed in winter over the continents (1075 mb in Siberia, December 1877), the lowest (below 900 mb) in tropical storms. In weather conditions favorable for photographic flights, however, the sea-level pressure should rarely depart more than ±30 mb from the standard value of 1013 mb. According to Equation 8 this will cause a proportional variation of up to ±3 percent in the standard photogrammetric refraction.

Sea-Level Temperature

The daily average sea-level temperatures vary in different parts of the world, generally within a range of about ±6 percent of the standard temperature 288 K, with the exception of the middle and high latitudes where winter temperatures often fall much below this range. The effect of the temperature variation on the photogrammetric refraction depends on the flying height.

At low flying heights, the last, bracketed factor in Equation 8 can be disregarded; consequently the photogrammetric refraction at lower flying heights, other things being the same, is inversely proportional to the square of sea-level temperature T_0. A decrease of temperature by 6 percent will thus increase the photogrammetric refraction by 12 percent, and a temperature rise will similarly decrease it.

With increasing flying height, the last factor in Equation 8 becomes more and more prominent, gradually decreasing the effect of the sea-level temperature on the refraction. At the tropopause, i.e., for a flying height of 11000 meters, a change of 6 percent in the sea-level temperature corresponds to a change of approximately 8 percent in the photogrammetric refraction. Finally, at altitudes high enough for pressure p_H to be negligible, the sea-level temperature has no effect whatever on the photogrammetric refraction, as is evident from Equation 7.

VERTICAL GRADIENT OF TEMPERATURE.

Because the atmosphere is an effective absorbent of long-wave radiation from the earth's surface, but practically transparent to direct sunlight, the diurnal variation of temperature is in the free atmosphere much smaller (except over arid deserts seldom more than 2 or 3°C at an altitude of 2000 meters) than the daily range of surface temperatures, which may exceed 20°C between a minimum occurring shortly before sunrise and a maximum during midafternoon. Consequently the vertical gradient of temperature, and therefore also the photogrammetric refraction, is extremely variable in the lower 2000 or 3000 meters of the atmosphere; at higher altitudes over 5000 meters the variations are small, and do not have any significant effect on refraction.

The decrease of pressure with elevation cannot maintain a decrease in air density if the vertical temperature gradient becomes sufficiently steep, the limit for constant density being $\beta = -34°C/km$, approximately. A steeper gradient will produce a state of instability, in which the density of air increases with height and the refraction of light rays becomes inverted giving rise to certain types of mirage phenomena (such as the familiar road mirage).

On a calm day, such a gradient may obtain in the first few meters above strongly heated dry ground, such as desert rock or sand, often producing violent dust whirls. On low photographic flights over such areas one may expect abnormally steep temperature gradients, say −20°C/km or more, instead of the standard gradient −6.5°C/km. Under these conditions the photogrammetric refraction will be reduced, according to Equation 8, by more than 50 percent.

At the other end of the scale, extremes of the temperature gradient are found in ground inversions in which the temperature, instead of decreasing, increases with height. Ground inversions are produced by contact cooling of the air at the surface. With the exception of warm air flowing over a cold surface, they are essentially due to the radiational cooling of the ground during the nights, and are often referred to as radiation inversions. The strongest ground inversions, which may persist both day and night, occur in the high latitudes in winter when the long nights provide excessive periods of radiational cooling.

In the middle latitudes during the summer months, nocturnal inversions are commonly observed over low-lying land when the sky is clear and the wind is light. They are generally confined within the first kilometer of the atmosphere, and have a temperature gradient of, say, +10°C/km, at the most. Under such conditions the photogrammetric refraction at lower flying heights will be increased, according to Equation 8, by as much as 60 percent of the standard value.

The atmosphere is usually more or less stratified, and inversion layers which affect refraction are not restricted to ground inversions alone. Mechanical turbulence of air by surface winds, for example, tends to induce an inversion of temperature at the top of the mixed layer. In such inversions, the rise in temperature may be only a few degrees, but if the layer happens to be a short distance below the aerial camera, a significant effect on the photogrammetric refraction may result. Actual limits for such effects are, however, difficult to establish; a few test computations would suggest a maximum increase of standard refraction by 15 to 20 percent.

For flying heights over 6000 meters, observed variations from the standard temperature gradient do not change the photogrammetric refraction by more than 1 percent. But even in this instance, the photogrammetric refraction cannot be determined from observations made at the ground level alone, because equivalent sea-level temperature T_0, which must be referred to the standard temperature gradient extending from top to bottom through the troposphere, is indeterminate without knowledge of the actual temperature gradients in the lower levels.

VAPOR PRESSURE

The ordinary standard atmosphere is considered absolutely dry, but in the actual atmosphere some water vapor is always present. Under given pressure and temperature, moist air is lighter in density than dry air by a factor of $1 - 0.378\, e/p$, where e/p denotes the ratio of vapor pressure to the total pressure. On a very hot and humid day, air density in the lowest levels may thus be reduced by as much as 2 percent which, in buoyancy, has the same effect as would have a rise in temperature by roughly 6°C, corresponding to a reduction in the photogrammetric refraction by 4 percent.

Furthermore, because vapor pressure normally decreases with elevation much more rapidly than the total pressure, this equivalent rise will diminish with height, producing the same effect as a steepening temperature gradient, that is, a further reduction in the photogrammetric refraction. Assuming that vapor pressure decreases with height according to a formula proposed by the author[6], the equivalent rise in temperature owing to humidity would drop from 6°C at sea level to about 3¾°C at a height of 1000 meters and to less than 1°C at 6000 meters, corresponding to a reduction in the photogrammetric refraction by 7 and 1 percent, respectively.

The above calculations suggest that the photogrammetric refraction at flying heights below 3000 meters may conceivably be reduced by up to 10 to 15 percent due to the presence of water vapor in the air.

TOTAL EFFECT OF METEOROLOGICAL ELEMENTS

High barometric pressure, low temperature, strong inverted temperature gradient, and negligible humidity are features which contribute towards *strong* refraction and, peculiarly, all of them are typical of one extreme of climate and weather. Similarly, the other extreme which is characterized by low barometric pressure, high temperature, steep negative temperature gradient, and high humidity, contains all of the features tending towards *weak* refraction.

The results from the foregoing discussion have been summed up in Figure 4, in which the solid curves indicate the estimated approximate limits for the local variations of refraction due to meteorological elements. The dashed curve shows the actual average refraction in Verhojansk, January 1970, computed on the basis of radiosonde data[5].

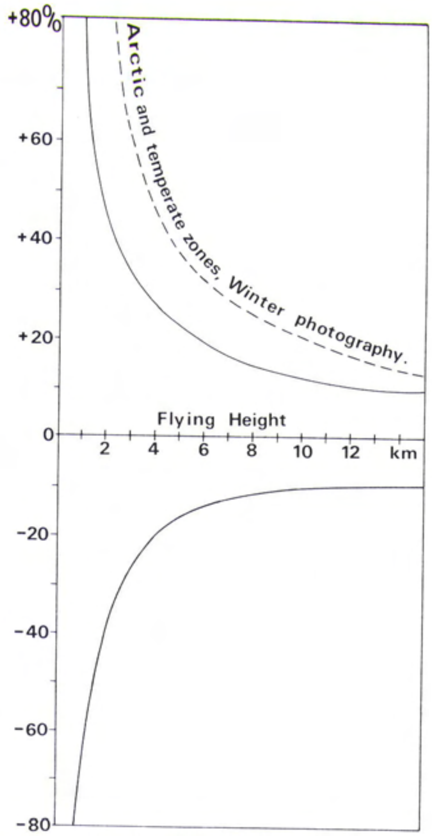

FIG. 4. Limits for the variation of photogrammetric refraction in percent of standard refraction.

CAMERA ENVIRONMENT

In routine computation of the photogrammetric refraction from an atmospheric model or from meteorological observations, the assumption is made that the density of air inside and immediately below the aerial camera equals the air density of free atmosphere at the same level. This may not always be true, and certainly not if the aerial photographs are taken through a port-glass from a heated, pressurized cabin of a modern aircraft. In such a situation, light rays will be refracted at the port-glass, as they do at any surface which separates air of different density.

For exactly vertical photographs taken through a precisely horizontal, plane-parallel glass plate, the effect of the camera environment on the photogrammetric refraction is obtained at once from Equation 2,

$$\Delta R = n_H - n_A \tag{9}$$

i.e., the difference between refractive index n_H of the actual atmosphere at the camera level and refractive index n_A of the air inside the camera carrier must be added algebraically to photogrammetric refraction R. (Change in $\bar{n}$ due to the short light path from port-glass to camera can be omitted.)

As an example, consider aerial photographs from an altitude of H = 6000 meters in standard atmosphere (p_H = 472 mb; T_H = 249 K; R = 59 μrad). Let the air pressure in the cabin be equalized with outside pressure at 2500 meters (p_A = 747 mb) and the temperature kept constant at 5°C (T_A = 278 K). We have then[6],

$$(n_H - 1)\, 10^6 = 79.0\, (p_H/T_H) = 150$$

$$(n_A - 1)\, 10^6 = 79.0\, (p_A/T_A) = 212$$

$$\Delta R = -62\ \mu rad.$$

The total photogrammetric refraction will be practically nil, $R + \Delta R = -3\ \mu$rad.

Internal refraction ΔR, if appreciable, makes it impossible to determine standard-atmospheric refraction corrections without making observations of pressure and temperature (or refractive index) both inside and outside the aircraft. But if measurements are to be made, it will be better to resort to Equation 7, which now becomes

$$10^6\,(R + \Delta R) = 2.316 \left(\frac{p_h - p_H}{H - h} - 34.11\, p_A/T_A \right) \qquad (10)$$

and gives the total photogrammetric refraction under the actual conditions from equally simple measurements.

In two articles published in Geodesy and Aerophotography, Kushtin[7,8] derives corrections to image coordinates due to internal refraction effects on photographs taken through a tilted glass plate, or through a spherical window.

Conclusion

Interpretation of the estimated meteorological limits for refraction shown by the solid curves in Figure 4, in terms of image coordinates at the corners of the format (θ = 45°), suggests a maximum uncertainty of $\pm 3\mu$m, quite independent of flying height, in radial corrections determined on the basis of standard atmosphere. Because the greater part of this error becomes eliminated in the photogrammetric scaling process, corrections for refraction computed from standard atmosphere can be considered adequate for most practical purposes.

The practice of taking aerial photographs through a port-glass from conditioned camera environment may have a deteriorating influence on photogrammetric accuracy, at least as long as the port-glass is considered as a fixture of the aircraft rather than a part of the camera optical system.

Acknowledgement

I wish to thank my colleague Z. Jaksic for his suggestions and advice in arranging the test computations described in the introduction to this research.

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