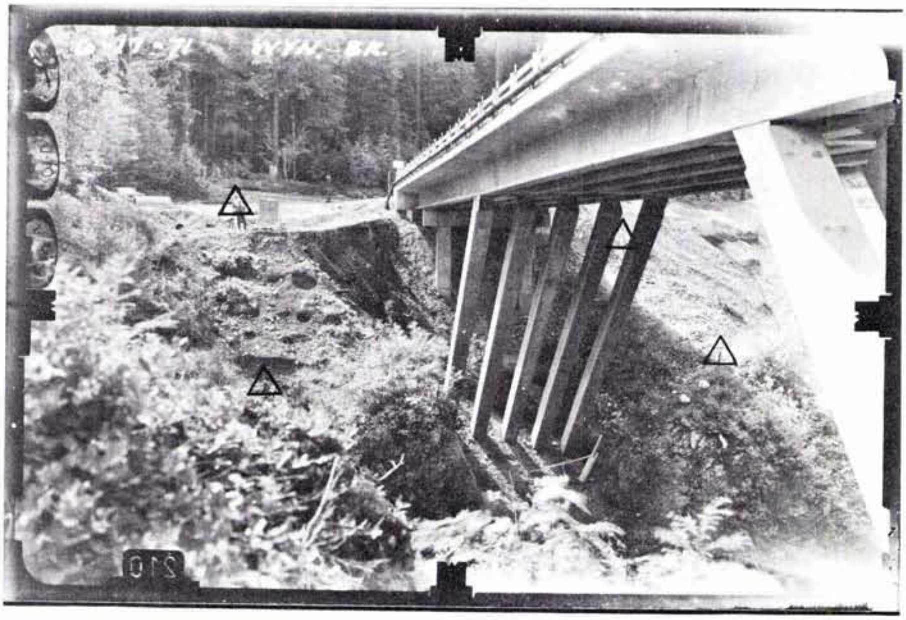

FIG. 1. Example of a phototheodolite survey in which the location of four control points results in a weak solution.

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Contemporary Problems in Terrestrial Photogrammetry

Orientation elements can be obtained through the establishment of ground control points or by directly measuring the phototheodolite.

(Abstract on next page)

INTRODUCTION

TERRESTRIAL PHOTOGRAMMETRY in this paper is regarded as a photogrammetric application of a photograph taken of an object more than 200 feet from the camera. Any photogrammetric application of photographs having less than 200 feet photographic distance is regarded to be in the category of close-range photogrammetry, which is outside the scope of this paper. The development of terrestrial photogrammetry during the last few years has concentrated mainly on close-range photogrammetry. In spite of the elaborate work of Zeller[9], terrestrial photogrammetry has been used relatively little in photogrammetric practices. Although Zeller's approach was a cartographic type of photogrammetry, there have been several reports recently which differ substantially in concept as well as in application from the classical method.

These reports indicate the use of analytical terrestrial photogrammetry. Erez[2] used analytical photogrammetry for measurement of seasonal motions of the "Barrage de la

Loutre" dam. Cotovanu[1] used terrestrial photogrammetry for monitoring the motion of bridges under load. Gutu[4] determined the size of cracks on the walls of rock salt mines. Planicka[6] reported excellent accuracy determining the deformation of dams.

Beginning in 1968 the Washington State Highway Department, in cooperation with the U.S. Bureau of Public Roads, sponsored a research project[7,8] to determine the motion and deflection of retaining walls by photogrammetric means. In 1970 the U.S. Army Corps of Engineers, Seattle District, realizing the practical and economical potential of analytical photogrammetry, sponsored studies and undertook projects to monitor structures and potential slide areas.[3] During the last three years several substantially different types of projects were measured and/or monitored, providing the opportunity to evaluate methods and instrumentation on semi-routine bases. These projects were different in nature, ranging from bridge and dam surveys to monitoring structures and potential slide areas, with varied accuracy requirements. Problems encountered during the execution of these projects, both in methodology and instrumentation, will be discussed here.

ABSTRACT: *The conventional phototheodolite can be used for structural surveying if a classical analytical photogrammetric method is employed. It may not be possible to overcome the difficulty of proper control point location under certain topographic conditions. This problem can be solved by methodology, namely, to measure the orientation elements of the camera. If such measurements are to be made, a more elaborate camera calibration is required. Camera calibration and accuracy standards are given.*

Classical Approach

The first experimentations were done by the implementation of the research conducted for the Washington State Highway Department.[7,8] This method is a classical approach in that it utilizes analytical space resection and intersection common in aerial photogrammetry.

The mathematical concept of space resection and intersection is based upon the well-known collinearity equation, i.e.,

$$x = f\,\frac{M_1X}{M_3X}, \quad y = f\,\frac{M_2X}{M_3X} \tag{1}$$

where

$$M = \begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix}, \text{ rotational matrix}$$

$$\text{and } X = \begin{bmatrix} X - X_o \\ Z - Z_o \\ Y - Y_o \end{bmatrix},$$

X, Y, and Z are the coordinates of ground control point or points to be determined, and X_o, Y_o and Z_o are the coordinates of the camera station. The observation equations formed from the above two functions f_1 and f_2 are:

$$v_{x_i} = \frac{\partial f_1}{\partial \omega}\Delta\omega + \frac{\partial f_1}{\partial \phi}\Delta\phi + \frac{\partial f_1}{\partial \kappa}\Delta\kappa - \frac{\partial f_1}{\partial x_o}\Delta x_o - \frac{\partial f_1}{\partial y_o}\Delta y_o - \frac{\partial f_1}{\partial z_o}\Delta z_o + \frac{\partial f_1}{\partial x}\Delta x + \frac{\partial f_1}{\partial y}\Delta y + \frac{\partial f_1}{\partial z}\Delta z - x_i + f_1(x_o, y_o, z_o)$$

and

$$v_{y_i} = \frac{\partial f_2}{\partial \omega}\Delta\omega + \frac{\partial f_2}{\partial \phi}\Delta\phi + \frac{\partial f_2}{\partial \kappa}\Delta\kappa - \frac{\partial f_2}{\partial x_o}\Delta x_o - \frac{\partial f_2}{\partial y_o}\Delta y_o - \frac{\partial f_2}{\partial z_o}\Delta z_o + \frac{\partial f_2}{\partial x}\Delta x + \frac{\partial f_2}{\partial y}\Delta y + \frac{\partial f_2}{\partial z}\Delta z - y_i + f_2(x_o, y_o, z_o) \tag{2}$$

These equations are used to perform space resection-intersection. It has been shown[4] by deriving error propagation formulas that residual deformations are introduced at the computed coordinates by errors existing in the variables. Further, such deformations can be compensated for to a large extent if one performs a tri-scale or affine transformation, i.e.,

$$\begin{bmatrix} X \\ Z \\ Y \end{bmatrix} = A \begin{bmatrix} \lambda_x \hat{X} \\ \lambda_z \hat{Z} \\ \lambda_y \hat{Y} \end{bmatrix} + \begin{bmatrix} dX \\ dZ \\ dY \end{bmatrix} \tag{3}$$

The A_{ij} individual terms in the A matrix are the direction cosines between the two coordinate systems (X,Y,Z) and $(\hat{X},\hat{Y},\hat{Z})$, dX, dY and dZ are the translational elements, $\hat{X}$, $\hat{Y}$ and $\hat{Z}$ are the computed elements and X, Y and Z are the given control coordinates.

This method provides very good results. In general, the residual errors in coordinates were approximately 1/40,000 of the photographic distance. The basic criterion of the method is to have at least four control points in the object space. The determination of these control points can be done with a phototheodolite at the time of photography.

The basic problem of the method is the location of control points. There are structures such as bridges which are linear in nature, and under certain conditions it is very difficult to place four control points in such a location that they provide a strong geometry for the resections. If the control points are located too closely to each other, the determination of rotational elements ω, ϕ, κ becomes uncertain. In terrestrial photogrammetry the ω rotation around the X axis, which is parallel to the base line, is the most critical. This is due to the fact that the lower portion of the photograph is occupied by the technically useless foreground, whereas the upper portion is the sky. This problem can be seen from the standard error of the ω, which according to Reference 5 after some simplification is:

$$\sigma_\omega = \sigma_o \sqrt{\frac{7f^2}{18a^4}} \tag{4}$$

where f is the focal length of the camera, σ_o is the standard error of unit weight, and a is the distance of the control point from the X axis of the photograph. It can be seen from this equation that if a becomes too small, the weight coefficient approaches infinity.

It was found by our experimentation that if $2a < \frac{1}{3}$ of the format size, the determination of the ω rotation is not sufficiently accurate for a precise structural survey. An example for this is shown by Figure 1 where the Wynoochee Bridge (Washington) under survey with a Wild P-30 phototheodolite is exhibited. It can be seen that $2a \approx \frac{1}{3}$ of the format size. The points on the photograph were determined photogrammetrically as well as by conventional ground survey. The residual standard errors were computed and the average found to be $\sigma_x = \pm 0.06$ and $\sigma_z = 0.11$ inch. It can be seen that the usually strong Z vertical coordinate deteriorated to the point where its standard error was nearly two times as large as that of the X coordinate.

The above described problem shows the limitation of the method concerning the applicability to various structures. The use of terrestrial photogrammetry in structural surveys can be extended further if the exterior orientation elements are measured rather than the control points. This makes the orientation of the camera independent from ground control points and thus from the given geography and structure.

Measurement of Orientation Elements

Using an analytical photogrammetric method, whereby the exterior orientation elements are measured along with the coordinates of the camera station (frontal nodal point of the objective), has more flexibility in connection with structural survey than that of the previously described approach. However, this raises the following question: How accurate must the measurement of the orientation elements be in order to meet the required specifications? This is an important question because the deformation in the ground coordinate will not be partially compensated by three-dimensional affine transformation as was done in the previous method.

The deformations in the ground coordinates caused by the interior and exterior orientation elements have been derived[7] for terrestrial analytical photogrammetry. Here, therefore, the problem will only be demonstrated for the deformation dX in the X direction. The formula is:

$$\begin{aligned}
dX &= \frac{X}{B}\, dB \\
&+ \left\{\frac{X^2}{YB}\left[\left(\frac{B}{X} - 1\right) \sin \phi_1 - \sin \phi_2\right]\right. \\
&+ \left.\frac{X}{B}\left[\left(\frac{B}{X} - 1\right) \cos \phi_1 + \cos \phi_2\right]\right\} dx \\
&+ \left\{\frac{X}{f} + \frac{X^2}{YB}\left[\left(1 - \frac{B}{X}\right) \cos \phi_1 - \cos \phi_2\right]\right. \\
&+ \left.\frac{X}{B}\left[\left(\frac{B}{X} - 1\right) \sin \phi_1 + \sin \phi_2\right]\right\} df \\
&+ \frac{XZ}{YB}(B - X)\, d\omega_1 + \frac{X^2Z}{YB}\, d\omega_2 \\
&+ \frac{XY}{B}\left[\left(\frac{B}{X} + \frac{XB}{Y^2} + \frac{X^2}{Y^2}\right) d\phi_1 + \left(\frac{X^2}{Y^2} + 1\right) d\phi_2\right] \\
&+ \frac{XZ}{Y}\left[\left(1 - \frac{X}{B}\right) \sin \phi_1 + \left(1 + \frac{Y}{X} \cos \phi 1\right] d\kappa_1 \\
&+ \frac{XZ}{YB}(X \sin \phi_2 - Y \cos \phi_2)\, d\kappa_2
\end{aligned} \tag{5}$$

where the dx and df are the errors of interior orientation elements, $d\omega_1$, $d\omega_2$, $d\phi_1$, $d\phi_2$, $d\kappa_1$

and $d\kappa_2$ are the errors of exterior orientation elements for the left and right camera stations, respectively, and dB is the error of the baseline. The X, Y and Z are the coordinates of a discrete point on the structure. The coordinate system was chosen in such a way that X is parallel to the baseline and the Z is in the vertical above the optical axis.

In order to demonstrate the effect of errors of the various orientation elements, excluding the interior orientation elements, let us assume the following numerical example which closely resembles the values of an actual structural deformation monitoring project currently underway at the U.S. Army Corps of Engineers, Seattle District. Therefore, let the baseline be B = 700 ft. and the convergency angles $\phi_1 = \phi_2 = 35°$. Further, the coordinates of a point of interest are X = 350 ft, Y = 600 ft, and Z = 50 ft.

Practical experimentation has shown that the standard error of the baseline measurements is ± 0.006 ft, using a Hewlett-Packard Distance Meter. It may be noted that this error is included for the reason of completeness; but in practice it creates only a scale error which can be disregarded because the same base is reoccupied at the subsequent measurements. The rotational angular errors can be 2, 5, 10 and 30 seconds of arc, respectively. Using these examples, numerical values can be computed for various errors as summarized in Table 1.

TABLE 1. ERRORS IN FEET CAUSED BY ERRORS IN ORIENTATION

caused by	2 sec	5 sec	10 sec	30 sec
Errors in X coordinate				
dB	0.0030	0.0030	0.0030	0.0030
$d\omega$	0.0003	0.0007	0.0013	0.0041
$d\phi$	0.0013	0.0031	0.0063	0.0190
$d\kappa$	0.0005	0.0013	0.0025	0.0076
Errors in Z coordinate				
$d\kappa$	0.0069	0.0165	0.0031	0.1005

The values in the table are given in feet. It can be seen from the table that the X coordinate is affected most by an error in the ϕ rotational angle as it is a natural consequence of the geometry. The most influential factor in the Z direction is the error in the κ angle. An outline of this influence is given in the second part of the table.

Practical experiments were conducted with a Wild P-30 phototheodolite. The values obtained from these experiments closely resemble the values used for the computation of the table. The average residual coordinate error as compared to classical field measurements in X was 0.031 feet; in Y, 0.006 feet; and in Z, 0.192 feet. These data are in remarkably close agreement with the theoretical data shown in the table. Considering that the sensitivity of the plate level of the theodolite is ± 20 sec of arc, it is therefore possible to have ± 30 sec of κ error in the leveling of the instrument, which gives:

$$dX_{\text{computed}} = 0.023 \text{ ft}, \; dX_{\text{actual}} = 0.031 \text{ ft}$$

$$dZ_{\text{comp. } (\kappa \text{ only})} = 0.110 \text{ ft}, \; dZ_{\text{actual}} = 0.192 \text{ ft}.$$

The same experiments were repeated with a BC-4 ballastic camera converted to phototheodolite and the results indicated the same agreement. For example, dX actual = 0.011 ft compared to the computed 0.008 ft, realizing that in this instance a striding level was used to set the κ angle whose sensitivity is 6 sec of arc.

In view of the large error caused by the κ rotation, one must realize that this rotation takes place at the base of the phototheodolite and not at the center of the photograph. This further aggravates the question, thus explaining the larger discrepancies between the computed and actual errors in the Z direction.

It can be concluded that the conventional phototheodolite cannot be used for such a precise structural survey beyond the photographic distances of approximately 300 feet, unless they are equipped with striding level, so that the $d\kappa$ is approximately ± 5 sec. However, it must be pointed out that if the accuracy standards are relaxed, as for slide areas where the yearly motion may exceed 0.5 ft, the phototheodolite can be used to monitor the motion up to 600 ft or more of photographic distance.

CALIBRATION REQUIREMENTS

. As was mentioned earlier, it is required to determine the coordinates of the camera station, that of frontal nodal point of the objective, in order to obtain independence from the ground control points. The camera in a phototheodolite is suspended through its gravity or rotational center, which balances its weight. This kind of suspension leads to a new geometrical quantity which will be called Eccentricity Constant and is not calibrated by the manufacturer. Figures 2 and 3 present this concept, in which O is the perspective center of the objective, and R is the rotational center of the camera. The Eccentricity Constant, shortened to EC, represents the distance between R and O. During the ground survey, the coordinate of R is obtained instead of the camera station O. Consequently, this disparity must be accounted for by determining the EC.

This problem does not occur with the method described in the previous section because the coordinates of O are determined by the space resection.

Realizing the geometry exhibited in Figures 2 and 3, the coordinates of the camera station are:

$$X_{O_1} = X_{R_1} + EC \cos \psi_1 \cos \omega_1$$

$$Y_{O_1} = Y_{R_1} - EC \sin \psi_1 \cos \omega_1$$

$$Z_{O_1} = Z_{R_1} + EC \sin \omega_1$$

$$X_{O_2} = X_{R_2} + EC \cos \psi_2 \cos \omega_2$$

$$Y_{O_2} = Y_{R_2} - EC \sin \psi_2 \cos \omega_2$$

$$Z_{O_2} = Z_{R_2} + EC \sin \omega_2 \qquad (6)$$

These equations are correct only for those phototheodolites whose vertical and horizontal rotational axes intersect. This is not true for the Wild P-30 phototheodolite in which the camera's vertical rotational axis coincides with the vertical rotational axis of the theodolite and the horizontal rotational axis of the camera is eccentric to that. Consequently, this instrument has a vertical Eccentricity Constant EC_v and a horizontal Eccentricity Constant EC_h. These constants modify the above equations to:

$$X_{O_1} = X_{R_1} + EC_h \cos \psi_1 \cos \omega_1$$

$$Y_{O_1} = Y_{R_1} - EC_h \sin \psi_1 \cos \omega_1$$

$$Z_{O_1} = Z_{R_1} + EC_v \sin \omega_1. \qquad (7)$$

They are similarly modified also for the right camera station. This means two Eccentricity Constants must be determined for this instrument.

There are several methods to determine the Eccentricity Constant. For example, photographing and measuring a subtense bar from various distances from the camera[2] and from the photographic and theodolite measurement, the EC is found by taking the difference between the two. This method has been found to lead to too large residual errors for cameras with long focal lengths, such as the Wild BC-4 camera (f = 305 mm, where the shortest photographic distance is about 200 ft). A direct measurement of the distance between the rotational center and the photographic frame proved to be more accurate. The measurements were performed repeatedly with a precise micrometer and the residual standard error of the EC was found to be ± 0.040 mm. This is satisfactory for structural surveying.

The photogrammetric structural survey often consists of surveillance of the structure. This type of monitoring activity means that the photographic stations must be occupied at predetermined time intervals, and the structure measured and compared to the first or previous measurements. Most of the phototheodolites presently on the market have no optical plummet which would permit the precise reoccupation of the stations. This difficulty has been overcome by a special casing. The design is illustrated by Figure 4, which consists of two parts. An 8-inch diameter steel pipe is located underground deep enough to provide a complete stability (*1* in the figure). The upper end of the casing

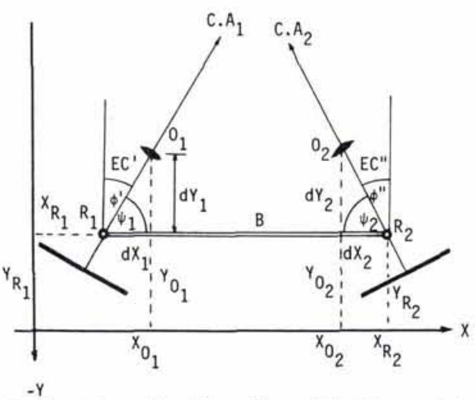

FIG. 2. Example of the effect of the Eccentricity Constant.

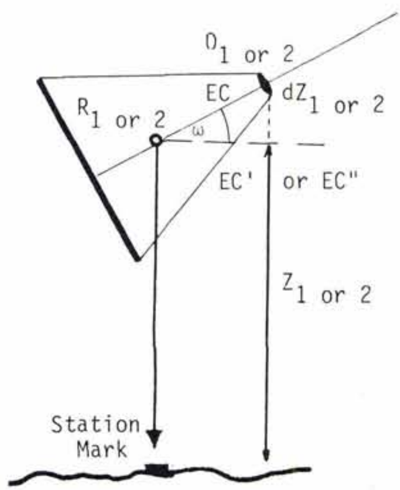

FIG. 3. Depiction of the Eccentricity Constant.

is threaded, permitting the second part, the theodolite stand, to be connected to it and be removed at the completion of the survey. Due to the constant and limited amount of thread, the height of the casing and the location remains the same at the repeated occupation of the station. The casing has been used in connection with the Wild P-30 phototheodolite and BC-4 ballastic camera, and it performs equally well in both applications.

FIG. 4. A pipe stand arrangement can be used for repeatedly setting a phototheodolite over the same ground point.

CONCLUSION AND SUMMARY

Terrestrial photogrammetry can provide valuable applications in engineering and photogrammetric practice. If these applications are to be exploited, the instrumentation and methodology of terrestrial photogrammetry must change according to the added requirements.

First of all, the present calibration data of the phototheodolites should be extended substantially. The calibration should include the Eccentricity Constant or Constants. These data should be calibrated by the manufacturer and should have the same accuracy as the focal length, which by present standards is ± 0.010 mm.

The sensitivity of the plate levels of a phototheodolite is ± 20 sec of arc. Such a sensitivity is sufficiently accurate to level the instrument, but is inadequate for precise analytical work. Therefore, a modern phototheodolite should be equipped with a striding level in order to be able to obtain the ± 5-sec accuracy in the κ rotation angle.

In addition, the instrument should be provided with auxiliary equipment which would provide the rapid measurement of instrument height. A better solution would be to have auxiliary equipment, or a device incorporated into the phototheodolite which would provide the height of the frontal nodal point of the objective, thus giving the vertical coordinate of the camera station.

According to the previously presented study, the accuracy of the ϕ and ω angles from ± 2 to 5 sec of arc is sufficient. Thus, the presently used T-2 types of angle measurement are adequate, provided that the vertical and horizontal angles of the camera can also be determined with the same accuracy. If the ϕ and ω angles are not measurable, as for certain phototheodolites, but instead have preset values, then the calibrated values for the preset angles should be given and be used. This would enable the user to obtain the orientation elements within the abovementioned accuracies.

If the above data and devices are provided with a phototheodolite, then one can assume that the application of terrestrial photogrammetry will be expanded. This expansion will mainly be in the field of computational photogrammetry.

There were two basic methods presented here; the first obtains the orientation elements of the camera indirectly from the measured control points, and the second is the direct measurement of orientation elements. The first method does not need the additional calibration and auxiliary equipment, and in this respect is more advantageous than the second one. However, the limitation of the method is given by the location of the control points. Under certain geographic conditions it is nearly impossible to locate these points properly, and thus the use of the second method is required.

The advantage of the second method is that it is independent of control points, but the auxiliary equipment and calibrations, as mentioned earlier, are required.

It is evident from the above that for certain projects, under certain circumstances, a combination of the methods is the most desirable.

The achievable accuracy in each method properly executed with proper equipment is about the same. The accuracy can be substantially increased from that mentioned in this paper by using more than two camera stations to determine points in space, as is the present practice at the U.S. Army Corps of Engineers, Seattle District.[3]

Finally, it may not strictly belong to the subject of this paper, but the economy of these methods must be mentioned. The conventional survey for structural measurement is extremely point-dependent; i.e., the cost of the survey depends upon the number of points to be determined on the structure. On the other hand, the terrestrial photogrammetric method is nearly independent from the number of points to be determined. Therefore, a direct comparison is difficult because it is different for each individual project. As a guideline, however, it has been found that if the number of points to be monitored is between 10 and 20 on a structure, then photogrammetry costs approximately half as much as a classical survey. The advantage increases with the number of points and equalizes at around 5 points.

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