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Linear Transformation for Data Rejection*

The rejection criterion concept applied to the normal distribution of accidental errors can also be used for quasi-systematic errors.

Abstract: *Almost all present procedures for error detection in aerotriangulation adjustment are based on the anaysis of residuals. The general practice to reject mistakes is to remove from the data processing, the ground control that has the largest residual error after the adjustment of the aerotriangulation. This procedure does not mean that the rejected data are in fact blunders if a polynomial adjustment is used in aerotriangulation; it means only that the rejected points differ from the mathematically created polynomial surface by a maximum amount. The characteristics of the modern connections and strip adjustment by polynomials of second or third degrees are not mutually independent but strongly correlated. Thus the actual control blunders often are not manifested in the residuals. Therefore a method of data rejection is developed here with statistical and mathematical conceptions. The rejection criterion of aerotriangulation is also established. According to numerous tests, this method is found to be most accurate for detection of blunders in strip adjustments. This method results in hand analysis and a saving of computer time.*

Introduction

A major problem faced by personnel responsible for manipulating aerotriangulation data through polynomial adjustment programs in an electronic computer is that of detecting and isolating blunders and other erroneous data. These mistakes originate from both the photogrammetric observations and also from the measurements of the ground control.

At present almost all procedures for the error detection in aerotriangulation adjustments are based on analysis of the residuals, which are directly or indirectly associated with the measured data being adjusted. The general practice to reject mistakes and other data at the Washington State Highway Department, as well as in many other organizations, is to remove from the data processing the ground control that has the largest residual error after the adjustment of the aerotriangulation. This procedure does not mean that the rejected data are in fact blunders if a polynomial adjustment is used in aerotriangulation; it means only that the point to be rejected differs from the mathematically created polynomial surface by a maximum amount.

The characteristics of the model connections and strip adjustment by polynomials of second or third degree are not mutually independent but are strongly correlated; thus the actual control blunders often are not manifested in the residuals, and the largest residual may in fact be a *correct* point, and

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smaller residual may be an *error* point. Therefore it seemed important to develop a practical method of data rejection, with statistical and mathematical conceptions to check the consistency of input data prior to aerotriangulation adjustment. Such a method should be able to detect the mistakes in photogrammetric or in ground control survey data, and such detection should be applied prior to the polynomial or aerotriangulation adjustment.

Error Analysis in Aerotriangulation

The polynomial adjustment of aerotriangulation deals with errors introduced during the measurement of the ground control, the photogrammetric processing, and the error propagation of the bridging process. Four types of errors are considered: blunders, systematic, accidental and quasi-systematic errors. These *errors* are defined as follows:

- ★ *Blunders* are caused by faults and mistakes during the working procedure such as misrecordings, misinterpretation, misidentification, etc.
- ★ *Systematic errors* under the same conditions will be of the same size and sign, such as lens distortion, systematic film shrinkage, instrument errors, and refraction.
- ★ *Accidental errors* are due to the imperfection of the instrument, the film, the flight height of the aircraft, and the observer. These errors can be tested statistically for the theory of probability.
- ★ *Quasi-systematical errors* are accidental errors that have an effect similar to systematic errors introduced by double summation of accidental errors, such as longitudinal bending of a strip, scale transfer, etc.

The accidental errors of transfer of scale in aerotriangulation affect the strip coordinates of the points as double summation errors. The relationship between the single and double-summed errors in aerotriangulation may be written as shown next, and in order to see the effect of the double-summation errors, the general study of the relation between the simple and double-summed errors is advisable.

Suppose E_n are accidental errors, EE_n are double-summed errors. Thus, relations are:

$$E_n = E_1 + E_2 \ldots + E_n \text{ in the } n\text{-th model.}$$

Because the scale change of each model in a strip manifests itself as double-summation error, the quasi-systematic error may be written as:

$$EE_N = nE_1 + (n-1)\,E_2 + \ldots + 2E_{n-1} + E_n \text{ in the } n\text{-th model.}$$

According to Moritz[1], the probability that the accidental error has a value between E and $E + dE$ where it is assumed that the standard error is equal to 1, is given by:

$$P(E) = \frac{1}{\sqrt{2\pi}}\, e^{-E^2/2}\, dE \qquad (1)$$

and the probability of the double-summed error is also given by a Gaussian distribution:

$$P(EE) = \frac{1}{\sigma k\,\sqrt{2\pi}}\, e^{-EE^2/2\sigma^2}\, dEE \qquad (2)$$

To show this more completely, the Gaussian distribution for accidental and double-summed errors are shown in Figure 1.

In summary from this theoretical model:

- The Gaussian curve for double-*summed* accidental errors is much flatter than the curve for the linear transformation of accidental errors.
- The residuals after polynomial adjustment for double-summed errors are not mutually independent but are strongly correlated. Thus, the strip adjustment by polynomials of second or higher degrees will fit to the error curves (including blunders from the ground control points), the largest residual may be a *good* point, and a smaller residual may be a *bad* point.
- Because the double-summed error is the linear combination of the normally distributed accident error, the former is also normally distributed. It seems that direct linear transformation can be employed for the rejection criterion of aerotriangulation.

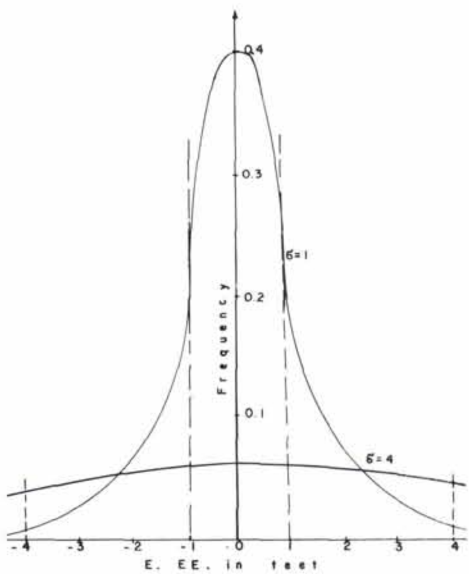

FIG. 1. Comparison of Gaussian distribution of accidental errors (E) and double summation errors (EE).

Procedure for Error Detection

The mathematical solution suggested for data rejection in aerotriangulation in a previous section was to use linear transformation equations with a least-squares adjustment. The final working equation of the rejection criterion will be given in the following sections.

LINEAR TRANSFORMATION FOR DATA REJECTION

The linear transformation can be employed, which consists of scale change and rotation as shown as:

$$E = a_{11}\,x + a_{12}\,y + E_o$$
$$N = a_{11}\,y - a_{12}\,x + N_o$$
$$H = a_{31}\,x + a_{32}\,y + a_{33}\,z + H_o \qquad (3)$$

in which E, N are horizontal coordinates of ground control points, H is the elevation of control points; x, y, z are strip coordinates; E_o, N_o, and H_o are unknowns of translation terms in E, N, and H; $a_{11} \ldots, a_{33}$ are coefficients of the transformation as unknowns. In matrix form as observation equations,

$V = AX - L$ (for horizontal ground points)

$V' = A'X' - L'$ (for vertical ground points) (4)

where

$$V = \begin{bmatrix} V_{E_1} \\ V_{N_1} \\ \cdot \\ \cdot \\ \cdot \\ V_{E_n} \\ V_{N_n} \end{bmatrix}; A = \begin{bmatrix} 1, 0, (x_1 - xs), (y_1 - ys) \\ 0, 1, (y_1 - ys), -(x_1 - xs) \\ \cdot \quad \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \quad \cdot \\ 1, 0, (x_n - xs), (y_n - ys) \\ 0, 1, (y_n - ys), -(x_n - x_s) \end{bmatrix}$$

$$X = \begin{bmatrix} E_o \\ N_o \\ a_{11} \\ a_{12} \end{bmatrix}; L = \begin{bmatrix} E_1 - Es \\ N_1 - Ns \\ \cdot \\ \cdot \\ \cdot \\ E_n - Es \\ N_n - Ns \end{bmatrix}$$

where X_s, Y_s, Z_s, E_s, N_s, H_s are the gravity centers of x, y, z, E, N, and H, respectively.

$$V' = \begin{bmatrix} V_{H_1} \\ \cdot \\ \cdot \\ \cdot \\ V_{H_n} \end{bmatrix}; A' = \begin{bmatrix} x_1, y_1, z_1, 1 \\ \cdot \quad \cdot \\ \cdot \quad \cdot \\ \cdot \quad \cdot \\ x_n, y_n, z_n, 1 \end{bmatrix}$$

$$X' = \begin{bmatrix} a_{31} \\ a_{32} \\ a_{33} \\ H_o \end{bmatrix}; L' = \begin{bmatrix} H_1 - H_s \\ \cdot \\ \cdot \\ \cdot \\ H_n - H_s \end{bmatrix}$$

The normal equation of unit weight is:

$$NX = U \qquad (5)$$

where $N = A^TA$; $U = A^TL$

and the unknowns can be found by computing the inverse of N thus:

$$X = N^{-1}U. \qquad (6)$$

A similar equation is formed for the unknowns X:

$$X' = N'^{-1}\,U'. \qquad (7)$$

The residuals of all ground control points after linear transformation adjustment can be computed as follows:

$$VdE = a_{11}\,x + a_{12}\,y + E_o - E$$
$$VdN = a_{11}\,y - a_{12}\,x + N_o - N$$
$$VdH = a_{31}\,x + a_{32}\,y + a_{33}\,z + H_o - H \qquad (8)$$

where VdE, VdN, VdH are residuals of transformed values in E, N, and H and the standard errors are

$$\sigma_E = \sqrt{\frac{[VdE \times VdE]}{n}}$$
$$\sigma_N = \sqrt{\frac{[VdN \times VdN]}{n}}$$
$$\sigma_H = \sqrt{\frac{[VdH \times VdH]}{n}} \qquad (9)$$

The Limits of Rejection Criterion

A rejection rule in aerotriangulation adjustment ideally reject all truly *error points* from the ground control. As a second- or higher-order polynomial equation in aerotriangulation cannot be used for data rejection, a suggested linear transformation adjustment of all ground control points should be employed for data rejection criteria. The limits of rejection criterian are:

Firstly, according to Hou[2], the expected standard error for variations in flight height may be obtained as:

$$e = K \times H \qquad (10)$$

where e is the expected standard error, $K = 0.00012$ (Empirical constant for system of RC8 6-inch camera and Wild A7 Autograph), and H is the flight height in feet. Based on Equation 10, 25 test flight strips were independently adjusted in order to determine the smallest detectable error.

The 25 test strips were taken at flight heights of 1500 ft, 3000 ft, 6000 ft, and 12,000 ft with a Wild RC-8, f=6 inch camera during the period from January to October by the Washington State Highway Department. All horizontal ground control points were determined by traverses, and all vertical control points were obtained by precise levels of second-order accuracy. All control points were targeted with white bars in a dark background of Washington State Highway Department's standard targets (Hou[4]).

After a strip adjustment of the 25 strips with all ground control and check points, it was found that the all residuals were within the range of three times the expected standard error in various flight heights.

The statistical data shown in Figure 2, indicated that they follow the same mathematic function as predicted in the least-squares adjustment; that is, the residuals that are larger than three times of standard error of the total data can be regarded as mistaken and must be rejected. In this research, however, the standard error is replaced by the expected standard error given by Equation 10 for various flight heights. Using this criterion, any residual error having values outside of this limit must be rejected.

The value of three times the determined residual standard errors for various flight heights are given only as the minimum detected errors in rejection criterion for

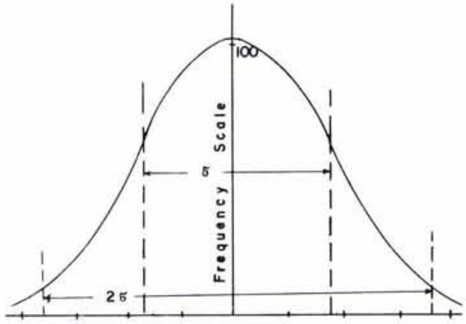

FIG. 2. Gaussian distribution of errors of the flight strips of 1500 feet.

residual errors in aerotriangulation. However, if a blunder exists in the strip adjustment, all the residual errors may lie outside of these limits; therefore, a procedure of rejection criterion must be established.

Secondly, based on the computed standard errors of the 25 test strips in linear transformation adjustments with all ground control and check points, it was found that 99 percent of errors are within the range of two times the computed standard errors of the adjustment. The probability of the above criteria therefore permit an automatic rejection criterion which can be established as:

$$dE > 2\sigma E > 3e$$

$$dN > 2\ \sigma N > 3e$$

$$dH > 2\ \sigma H > 3e$$

where dE, dN, dH are residuals in E, N, and H (x, y, z), σ is the standard error of adjustment in linear transformation adjustments, e is an empirical standard for acceptable accuracy, equal to 0.012 percent times the flight height (later it will be referred to as *empirical standard error*).

Any residuals lying outside of these limits are to be rejected in aerotriangulation, and the rejected points will then be remeasured in the field, or if the control is sufficient, the data will be excluded from the adjustment of aerial triangulation.

A relevant computer program was developed by the author. The data-rejection computer program and aerotriangulation adjustment are performed in two steps. These steps are performed one after the other without operator intervention.

As a first stage, the strip with all ground control points is subjected to a linear transformation to the ground coordinate system for detection and rejection of the blunders. Having rejected all predicted data errors, the strip adjustment is performed in the second stage. The generalized flow chart for data rejection and adjustment is shown in Figure 3.

Extended Use of the Rejection Method

It was shown in Figure 1 that the quasi-systematic errors also follow the Gaussian or normal distribution of errors and the rejection criterion of probability of errors. This is an indication that the use of the rejection criterion can be extended to detect and reject errors of quasi-systematic type originating from any phase of the

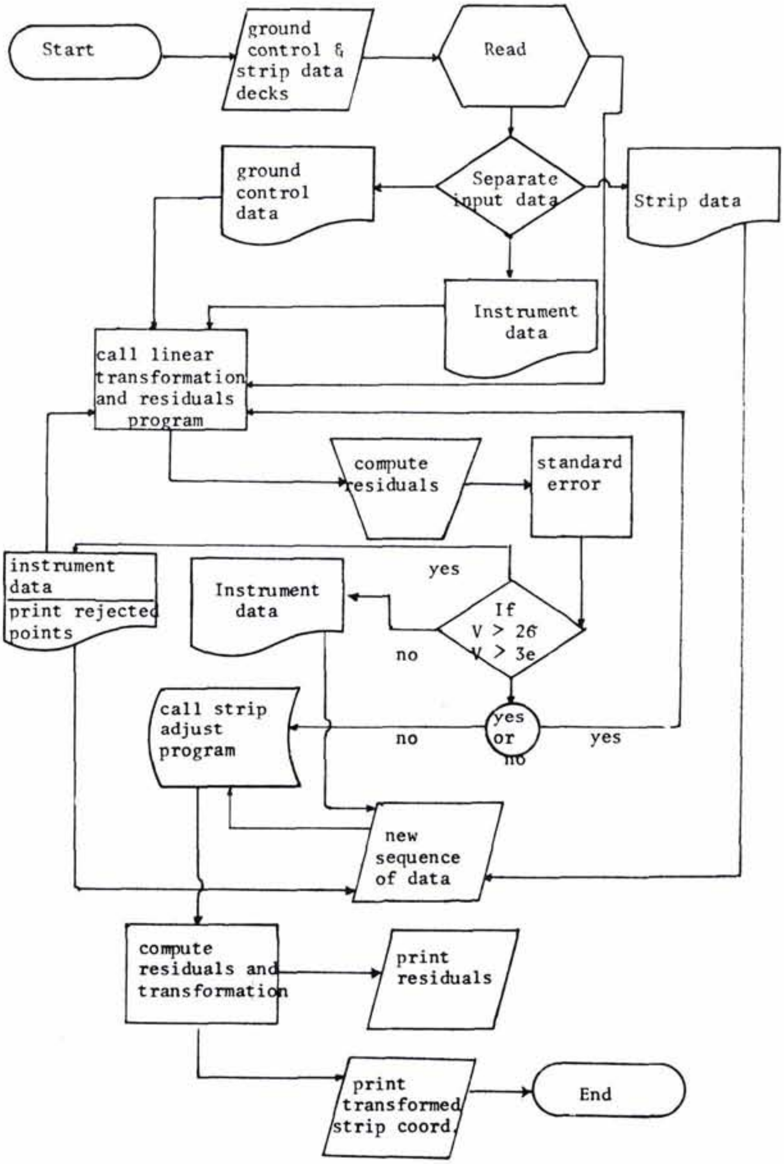

FIG. 3. Strip-adjustment and data-rejection flow chart.

photogrammetric process, even though the error may be introduced by means other than double summation.

The quasi-systematic type errors can occur in photogrammetry if the error is systematic as far as a single photograph is concerned but where each of the photographs has a systematic error which changes from photograph to photograph. As these photographs are connected into a strip, the different systematic errors of individual photographs are double summed in a manner somewhat similar to that shown for the accidental errors. Therefore, the final result is quasi-systematic type of errors. This is an indication that the established rejection criterion can be used for the rejection of the non-uniform type of systematic errors. However, the standard for accepted accuracy may be different from that given by Equation 10. The capability of establishing the rejection method may be demonstrated by the following example.

A very important factor in photographs is the distortion of film due to dimen-

TABLE 1. RESIDUALS IN FEET AFTER STRIP ADJUSTMENT DUE TO THE ERRORS OF FILM DISTORTION.

R E S I D U A L S			
POINT	EAST	NORTH	ELEVATION
30219			−0.12
30220			0.29
30221			0.76
30230			−0.57
30231			0.07
30232			0.02
30233			−0.21
30234			−0.07
30235			0.97
30236			−0.33
30027			−0.61
40217	−0.51	−0.58	−0.53
40218	0.31	0.64	0.27
40238	−0.08	0.46	−0.07
66239	0.28	−0.52	0.12
R.M.S.E.	0.33 ft.	0.56 ft.	0.44 ft.

sional change, which may be uniform or non-uniform. The systematic distortions may be corrected for scale by linear transformation, but the non-uniformity of dimensional change in different directions of areas of the film will result in non-correctable errors if only four fiducial marks are measured. For example, a flight strip at 1500 feet of seven models for the Grandview project was bridged and adjusted; the residuals are shown in Table 1 and Figure 4.

All residuals of the adjusted points in this flight are larger than three times that of the empirical standard errors at a flight height of 1500 feet, and smaller than twice the standard errors. Thus, the photographs may contain a non-uniform type of systematic errors.

The following procedure of the rejection method was used in this example. Firstly, the measurements of the four corner fiducial marks of the flash plate and eight diapositives were recorded. Then, each example was used to fit the calibrated positions of four fiducial marks as shown in Figure 5. The deformation of four fiducial marks of the flash plate and eight diapositives are shown in Table 2 and displayed in Figure 5 where the solid lines indicate the calibrated positions, and the dashed lines indicate the residual deformation of the film.

It can be shown from Figure 5 that each of the photographs deform during the flight or during the photographic processing and this deformation is different for photograph to photograph. Such a criterion for quasi-systematic type of error was described previously.

The deformation of the photographs taken during this flight was larger than that of the directly exposed photographic glass plate. Thus, as all the residuals of the adjusted points in a flight are three times larger than the standard error, it is indicated that the photographs may contain non-systematic film distortion which causes the model deformation.

In this particular instance, the rejection criterion can only be established by increasing the standard for accepted accuracy or, as mentioned before, by increasing the empirical standard error. The relation between standard error of normally distributed accidental error that of quasi-systematic error was found to be 1:4, (see Figure 1). Consequently the standard for accepted accuracy may also be increased accordingly for the rejection of blunders in aerotriangulation which contains quasi-systematic type of errors other than accidental origin. It must be emphasized that in order to determine the empirical standard error for such a strip, more statistical data are required, which is beyond the scope of this research.

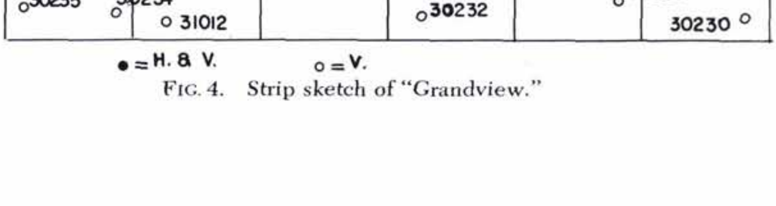

FIG. 4. Strip sketch of "Grandview."

TABLE 2. RESIDUALS IN MM AFTER LINEAR TRANSFORMATION OF THE FOUR FIDUCIAL MARKS OF A TEST STRIP

Film No.	Point No.	Error, X	Error, Y	Remark
0	1	.010	.008	
0	2	−.011	−.012	
0	3	.015	.011	(FLASH PLATE)
0	4	−.014	−.007	
1	1	−.040	.022	
1	2	.017	.027	
1	3	−.012	.004	
1	4	.035	.001	
2	1	.008	.004	
2	2	.024	−.016	
2	3	−.011	.049	
2	4	−.021	−.036	
3	1	.002	.026	
3	2	−.039	−.046	
3	3	.059	.009	
3	4	−.022	.010	
4	1	.009	.018	
4	2	−.036	−.042	
4	3	.060	.015	
4	4	−.033	.009	
5	1	.003	.020	
5	2	−.043	−.049	
5	3	.073	.010	
5	4	−.033	.020	
6	1	.012	.009	
6	2	−.048	−.047	
6	3	.087	.011	
6	4	−.050	.028	
7	1	.009	.010	
7	2	−.041	−.038	
7	3	.069	.006	
7	4	−.037	.023	
8	1	.005	.016	
8	2	−.041	−.044	
8	3	.069	.008	
8	4	−.033	.021	

However, in many instances the residual errors found in a strip contain a non-uniform type of systematic errors which are in general larger than that acceptable for highway engineering, such as shown by this example. In this particular situation, the rejection criterion can be used to determine that the area must be reflown in order to obtain photographs free from the non-uniform type of systematic errors, where the ground control surveys should not be remeasured.

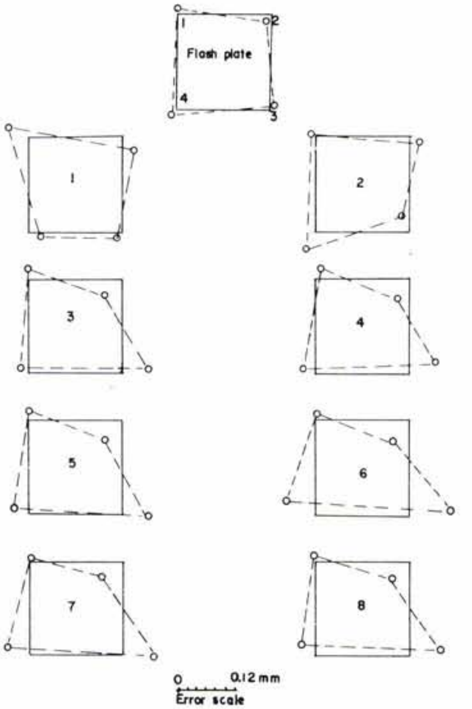

FIG. 5. Film distortions after a linear transformation of the four fiducial marks of a test flight strip.

In summary, it can be said, that the rejection method established to reject blunders of photogrammetric or ground surveys of individual points can also be used to reject series of data containing a non-uniform type systematic errors. From a practical point of view, it can be used to determine which data (photogrammetry, photography or ground survey) should be re-established, thereby minimizing the expenses of new surveys.

COMPARISON OF THE RESIDUALS OF HELMERT AND AFFINE TRANSFORMATIONS

In this research two linear transformations are used simultaneously—Helmert transformation for x, y coordinates and an affine transformation for elevations. It therefore needs to be shown that the rejection method is applicable for both of these transformations. The results slightly differ for the two types of transformations; however, this difference is negligible from a practical point of view.

In both types of transformation, least-squares adjustments are used. The nature of the least-squares adjustment tends to distribute the errors (including blunders) equally between x and y coordinates which are adjusted separately so that the residuals are also

TABLE 3. COMPARISON OF THE RESIDUALS AFTER HELMERT AND AFFINE LINEAR TRANSFORMATION

	Affine		Helmert	
Point	x	y	x	y
5	−2.25	0.27	−1.32	0.00
6	0.65	−0.05	0.58	0.26
7	0.79	−0.16	0.19	0.17
18	−1.69	0.21	−0.29	−0.07
19	−0.78	−0.05	0.00	0.00
20	0.98	−0.16	0.30	−0.15
21	0.74	−0.06	0.54	−0.20

separated. If a blunder of relatively small amount lies in the direction of bisector of the x and y coordinates, and the residuals after the least-squares adjustment is distributed through a similarity transformation. The rejection method may not detect this small amount of blunder; however, a very small possibility exists for such a situation.

In general, the difference between two kinds of transformation using the rejection method is that the residual errors should be somewhat larger using affine transformation due to the effect that component of the blunder may be larger in the x or y direction.

In order to show the above analysis, a mathematical example was computed. The comparison of the residuals after Helmert and affine transformation adjustment with a given error in east of 2.00 feet at Point 5 are shown in Table 3. The largest residual 2.25 feet indicated the true error of Point 5 of 2 feet for the affine transformation and 1.32 feet for the Helmert transformation; thus both methods can be used for data rejection.

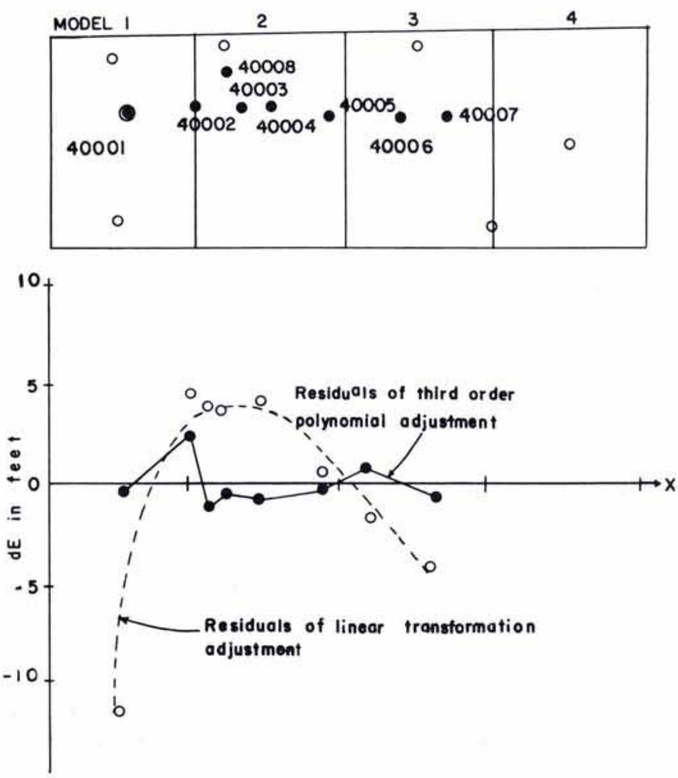

FIG. 6. (Upper) Strip sketch of Test Strip No. 1 for data rejection. (Lower) The residuals of adjusted points by using third-order polynomials and a linear transformation.

TABLE 4. THE RESIDUALS OF THE ADJUSTED POINTS BY USING A THIRD-ORDER POLYNOMIAL EQUATION.

Point No.	Residuals (ft) East	North	Remarks
45	0.21	−0.40	
47	−0.73	0.51	
48	−1.55	−0.73	
42	2.96	0.36	Rejected a "good" point (largest residual in E)
43	−0.50	0.55	
44	−0.96	0.51	
46	1.36	−0.84	
41	−0.80	−0.01	(True error point)
R.M.S.E.	1.39 (0.42m)	0.55 (0.17m)	

PRACTICAL TESTING FOR DATA REJECTION

It was pointed out earlier that polynomials of a certain degree can be adjusted to an aerotriangulated strip surface which contains mistaken points. This is due to the mathematical flexibility inherent to the polynomial adjustment. In order to demonstrate this phenomena an aerotriangulation strip was selected. The test Strip 1 of the 25 strips (R. Street in Anacortes), ground control Point 40001 has an error in East *(y)* of 22 feet (field checked). However, after strip adjustment, the strip fits well to the error curves with blunders from ground control surveys as shown in Table 4. It can be found that the largest residual is a *good* point (No. 40002) and smaller residual is a *bad* point (No. 40001). However, by using the suggested method, Point No. 40001 was rejected as shown in the following.

Figure 6 (upper) represents the planimetric distribution of control points as related to the individual models. Figure 6 (lower) shows two profiles as a function of the residual error in X, or in Eastings, along the strip axis. Profile 1 is represented by the dashed line which shows the residuals after linear adjustment (Helmert transformation). Profile 2 exhibits the residuals after polynomial adjustment of third degree shown by the solid lines. In addition, the third-order polynomial is also shown at its approximate location as fitted to the linear adjustment in form of a parabolic curve. It can be shown from this figure that the maximum deviation occurs at Point 40002 due to the curvature in the polynomial surface. Point 40001, which contains the 22-foot blunder, is located at the portion of the polynomial surface that diverges to infinity, therefore providing a *good fit!*

TEST STRIP 1

As mentioned in the previous section, the ground control Point 40001 in Test Strip 1 is in error in easting of 22 feet (remeasurement). After the computation of Equation 2 to 11, Point 40001 in this example indicated an error of 12 feet. Therefore, Point 40001 should be rejected ($dE > 2\,\sigma E > 3e'$ or $12.42 > 2 \times 5.8 > 0.54$) (see Table 5 for details).

TEST STRIP 2

A test strip at a flight height of 1500 feet (upon remeasurement of Point No. 33 in error of 30 feet and Point No. 34 in error of 5 feet) was selected. The amount of blunder has been checked by field measurement. Residuals of the ground control points after linear transformation adjustment are shown in Table 6 and Figure 7. Point No. 33 in this example indicated a mistake of 17 feet ($16.74 > 2 \times 7.52 > 3 \times 0.18$ or $16.74 > 15.04 > 0.54$). Therefore, Point No. 33 was rejected and the strip was recomputed. Point No. 34 indicated an error of 2.60 feet and extended over the accepted range ($2.60 > 2.58 > 0.54$); therefore, Point No. 34 was rejected.

If now the linear transformation without

TABLE 5. THE RESIDUALS OF THE ADJUSTED POINTS BY USING A LINEAR TRANSFORMATION. (TRUE ERROR POINT WAS DETECTED).

Point No.	Residuals (ft) dE	dN	Rejected Errors
45	0.61	0.31	
47	−4.34	1.03	
48	4.51	−2.04	
42	6.40	−0.19	
43	4.41	−0.20	
44	3.15	0.04	
46	−2.32	0.08	
41	−12.42	0.23	♦ rejected point (true error)
R.M.S.E.	5.80	0.18	

TABLE 6. RESIDUALS OF GROUND CONTROL POINTS AFTER DATA REJECTION BY USING POINT NO. 33 IN ERROR OF 30 FEET AND POINT NO. 34 IN ERROR OF 5 FEET.

Point No.	Elevation (feet)		
	Iteration 1	2	Final adjustment
31	−5.00	−1.41	0.02
33	−16.74		(−30.40)
46	2.73	1.53	0.04
42	3.89	0.85	0.05
47	5.58	0.12	−0.22
43	7.92	−0.02	0.11
32	−6.82	0.40	−0.03
34	5.76	−2.60	(−5.08)
44	2.66	1.13	0.03
Standard Error	7.52	1.29	0.10

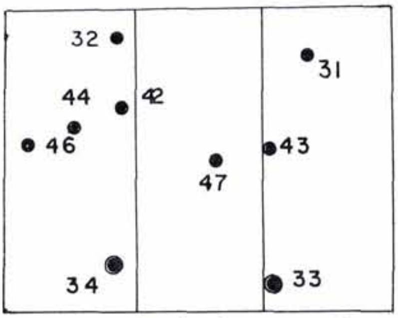

FIG. 7. Sketch of Test Strip 2.

Point Nos. 33 and 34 in this strip is computed, the residuals will become small (dE, dN, dH $< 2 \times$ standard error $< 3 \times 0.18$). In other words, no residuals of any other points in this strip lie outside of these limits; thus, no point should be rejected as shown in Table 6.

TEST STRIP 3

This test strip was taken from the project "Study of Traffic in Seattle," which consists of 14 models. The flight height was 1500 feet above the ground elevation of 100 feet. Ten horizontal control points and 23 vertical control points were established and targeted, where Points 30048 and 40034 had elevation errors of 56.5 feet and −6.2 feet, respectively, as confirmed by field measurement and shown in Figure 8. The residuals of all ground control points after first linear transformation are shown in Table 7. The accepted limit is 2 × standard error or 34.4. Thus, point 30048, with the value of 43.1, extended over the accepted range and was rejected.

For the second linear transformation (which was performed after the rejection of Point 20048), the residuals are shown in Table 7, Column 2; the accepted range is 2.80 feet. Thus, Point 40034, with the value of 5.35 feet, was removed. In the third linear transformation, the residuals are shown in Table 7, column 3. The minimum rejected limit is 0.54 feet. However, all deviations are within the limits. The residuals of the final adjustment are also in Table 7.

TEST STRIP 4

This test strip was taken from the job of "Alpowa CR to Clarkston" photographed on September 26, 1973, which consists of 5 models. The flight height was 1800 feet above the mean ground elevation of 700 feet. In this Strip, 9 horizontal control points and 13 vertical control points were established and targeted as shown in Figure 9.

In order to obtain the acceptable results, this strip was adjusted with a second-order polynomial equation five times. It seems that all ground control points had coordinate errors as shown in Table 8. However, by using the newly developed method, "data rejec-

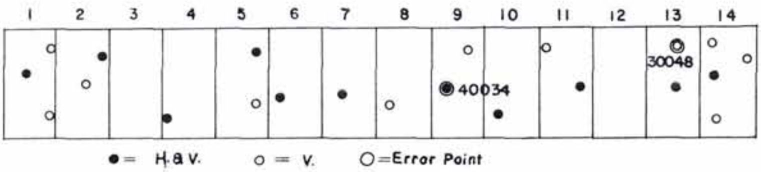

FIG. 8. Sketch of Test Strip 3.

TABLE 7. RESIDUALS AFTER DATA REJECTION OF STRIP 3.

Point No.	Linear Adjustment Residuals in feet			Strip Adjustment
Iteration	1	2	3	
30025	−8.74	0.01	0.12	0.10
30027	−1.97	−0.06	−0.32	−0.08
30033	3.28	0.09	0.02	−0.06
30035	6.06	0.05	0.17	0.02
30040	−13.17	0.20	0.16	−0.09
30046	−4.97	1.02	0.13	0.09
30048	43.08	*		(56.20)
30050	−7.76	0.55	−0.09	−0.08
30052	−6.77	−0.61	−0.26	−0.07
40031	−8.21	0.20	0.13	0.10
40032	−2.50	0.39	−0.41	−0.34
40033	0.71	1.06	0.12	0.21
40034	−6.48	−5.25	*	(−6.21)
40035	1.87	0.97	−0.01	−0.01
40036	1.50	1.15	0.30	0.28
40037	7.56	0.57	−0.04	−0.16
40038	−2.04	0.00	−0.13	−0.09
40039	−1.46	−0.33	0.11	0.17
R.M.S.E. =	17.2	1.40	0.19	

* should be rejected.

tion," this strip adjustment was performed by only one pass through the computer, the true error of control Point No. 40094 was detected, and the residuals of all ground control were within the limits of 0.66 feet as shown.

CONCLUSIONS AND RECOMMENDATIONS

- It has been shown that the distribution of the quasi-systematic errors follow the Gaussian distribution of errors. Consequently, the rejection criterion concept applied to the normal distribution of accidental errors can also be used for quasi-systematic errors. This is theoretically correct if the amount of the standard error is known.

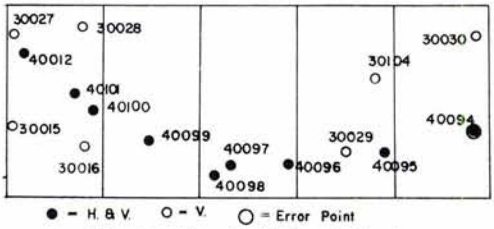

FIG. 9. Sketch of Test Strip 4.

TABLE 8. RESIDUALS OF TEST STRIP 4.

Point No.	2nd order polynomials Residuals in East feet				Data Rejection Final
Iteration	1	4	5	1	
40101	−1.74	2.37	1.51	−8.76	0.22
40102	11.19			−17.78	−0.44
40100	−5.24	−0.80	4.80	−5.75	0.14
40099	−10.23	−4.52		2.37	0.08
40098	−7.29	0.95	−0.56	13.40	0.15
40097	−5.83	2.39	0.95	13.51	−0.05
40096	7.52			21.32	−0.26
40095	34.02			31.08	0.16
40094	−22.40	−0.39	−0.10	−49.38	(−89.05)
R.M.S.E.	15.14	2.36	1.16	22.74	0.22

This theoretical standard error, however, can effectively be replaced by the *accepted standard error,* which is an empirical value based on a statistical analysis of a large number of aerotriangulations. The advantage of this method is that the computation of the final strip adjustment is performed automatically if the ground control point errors are detected and rejected by using a linear transformation.

- Strip adjustment by polynomials of second degree or higher will fit to the error curves, including blunders from the ground control points. Thus, the largest repeating residual may be a *good* point, and the smaller residual may be a *bad* point.
- A suggested method shows that the linear transformation adjustment can be employed for the rejection criterion of aerotriangulation. The rejection criterion should be established as:

$$dE > 2\sigma_E > 3e$$

$$dN > 2\sigma_N > 3e$$

$$dH > 2\sigma_H > 3e$$

where dE, dN, dH are residuals in E, N, and H, σ is the standard error in linear transformation adjustment, and e is the empirical standard error equal to 0.012 percent times the flight height. Any residuals lying outside of these limits may be repeatedly rejected in aerotriangulation.

The minimum detected error of $3e$ may be established in the horizontal and vertical directions separately, and also the constant of $3e$ may be changed to $2e$ or $1e$ or $4e$, and 2σ may be changed to 1.35, 1.55, or 1.85, as required accuracies in each individual organization.

- The suggested method can be employed for data rejection in semi-analytical, and analytical aerotriangulation. However, in the simultaneous adjustment the basic mathematical model consists of collinearity or coplanarity equations. The complete linearized normal equations contain unknowns of the elements of exterior orientation and the ground coordinates of pass points or points to be aerotriangulated in each photo. In this form of analytical aerotriangulation each photograph and, consequently, each point, is handled individually. Thus no model connection, numerical or mechanical is performed. This means no double summation of errors is introduced into the strip or block. Having no double summation, there is no need for polynomial adjustment in the process of aerotriangulation. Further, the standard errors are computed for each individual point. As a consequence there is no need for a rejection method as it is shown in this research for this type of aerotriangulation because the rejection can be directly based on the individual standard error. However, it must be pointed out that such a method is not used routinely in the practice due to the fact that a very large number of equations (not unusual a 1500 × 1500 matrix) must be solved simultaneously. Therefore, the importance and application of the rejection method developed is not restricted by the above limitations.
- According to numerous tests, this method is found to be most accurate for detection of blunders in the strip adjustment. This method results in a manual analysis and a saving of computer time.

Acknowledgment

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Errata

On Page 854 of the July issue, the Photogrammetric Award was given to Prof. H. M. Karara for his leadership and development of techniques and equipment for close-range photogrammetric applications, and the utilization of analytical processes and non-metric cameras to free the systems from the restrictions of conventional hardware. The reason stated was erroneous.

Also, on page 855, the names are reversed in the upper right and lower left photos of the recipients of the Abrams Award; in the text (right column near the center of the page) Mr. Davies name is misspelled; and the title of the article should read, "Mariner 9: Primary Control Net".