Bruce C. Forster*
Papua New Guinea Univ. of Techn.
35 Lewisham Road
Windson, 3181, Australia

Aerotriangulation Accuracy

An examination of the planimetric errors of several systems as shown in international reports.

Abstract: *The various factors affecting the output coordinate accuracy of block triangulation are examined. The separate effects of the geometry of the block and the relative precision of the various components of aerial triangulation systems are determined by using results published by various researchers. The effect on accuracy of a block geometry is then formulated. Using the relative precisions determined, field test results from various sources are related to a common system, which allows examination of the effect of photograph scale on the final coordinate accuracy. A formula is derived enabling the calculation of the standard error of the coordinates determined at a point in a block. This formula can be modified to apply to a particular aerial triangulation system.*

Introduction

The high internal uniform accuracy of block triangulation is now well known. Many theoretical and practical studies have been made that justify this concept. Ackermann[1] probably illustrates this principle best. In discussing blocks with perimeter control, he wrote: "The average accuracy of the block remains in the order of magnitude of the accuracy of a fully controlled single model."

He further suggested that, for planimetry at least, blocks with perimeter control can be used up to any size.

It is difficult however, from the varying tests cited in literature to estimate the accuracy possible for a particular control pattern and a particular photograph scale, given a particular system of block adjustment. This estimate is necessary if one is to show that a method is capable of producing coordinates adequate to allow mapping at a certain standard. The determination of this estimate forms the basis for this paper. The well-documented independence of horizontal and vertical error propagation allows planimetric accuracy to be examined separately.

Standard Error

The standard error, σ_i of the co-ordinates determined at a point in a block is given by[2]:

$$\sigma_i = q_i \, \sigma_{oxy}.$$

Both q_i and σ_{oxy} are relatively independent and provide a means for the comparison of tests conducted under varying conditions.

The variance q_i is affected by the geometry of the block, i.e., number of photographs, number and distribution of control points, etc. The standard error of unit weight σ_{oxy}, is a measure of the precision of the system being used, and is affected by camera, operational techniques, overlap, premarking of control, etc.

If the factors affecting σ_{oxy} are determined, then their effects can be taken into account when comparing various test results, so that σ_i will depend only on q_i.

Factors Affecting the Standard Error of Unit Weight

The effect of these different factors are described below.

- The use of superwide-angle photography

* Now with the School of Surveying, University of New South Wales, Kensington, N.S.W. 2033, Australia.

(s.w.a.) as opposed to wide-angle (w.a.) would not seem to have any beneficial effects on planimetric accuracy[3]. The favorable base-height ratio ensures, however, an increase in the vertical accuracy. Thus,

$$\frac{\sigma_{oxy\ w.a.}}{\sigma_{oxy\ s.w.a.}} = 1$$

for all other effects being constant.

- An increase in the sidelap between strips from the normal 20 to 25 percent up to 60 percent has a marked effect on planimetric accuracy. Basically this is due to the increased *mechanical* strength of the block and because this strength is now comparable along and between strips. Ackermann[1] indicates an increase in the planimetric accuracy of about 40 percent, that is,

$$\frac{\sigma_{oxy\ 20\text{-}25\%}}{\sigma_{oxy\ 60\ \%}} = 1.7$$

- The method of analytical triangulation adjustment can be subdivided into three[4] categories: (a) semi-analytical independent-model method, (b) fully analytical sequential method, and (c) fully analytical simultaneous method.

Figures tabled by Lortz[5] allow extraction of the following information: for the same block, i.e., q_i constant,

$$\frac{\sigma_{\text{oxy independent}}}{\sigma_{\text{oxy sequential}}} = \frac{3.00}{2.19} = 1.37$$

and for another block,

$$\frac{\sigma_{\text{oxy independent}}}{\sigma_{\text{oxy sequential}}} = \frac{18.9}{7.8} = 2.43$$

an average figure,

$$\frac{\sigma_{\text{oxy independent}}}{\sigma_{\text{oxy sequential}}} = 1.9$$

can thus be obtained.

Ackermann[1] confirms this ratio. He obtained $\sigma_{oxy} \doteq 2.0$ cm for the sequential method and $\sigma_{oxy} = 3.8$ cm by the independent method, from tests on the same block, i.e.,

$$\frac{\sigma_{\text{oxy independent}}}{\sigma_{\text{oxy sequential}}} = \frac{3.8}{2.00} = 1.9 .$$

In 1969 Anderson[6] estimated a 37-percent improvement in error by the use of the simultaneous method if compared to the sequential method. This estimate, however, was based on one test only. Anderson, in conjunction with Ramey[4], later indicated a greater reduction. From various tests on a simulated block the average change was noted for various constant q_i as shown in Table 1. Thus,

$$\frac{\sigma_{\text{oxy sequential}}}{\sigma_{\text{oxy simultaneous}}}$$

for each test was:

1A, 3.89 ÷ 1.12 = 3.47
2A, 5.39 ÷ 2.06 = 2.62
1B, 2.46 ÷ 0.70 = 3.52
2B, 1.91 ÷ 1.61 = 1.19
1C, 1.46 ÷ 0.50 = 2.92
2C, 1.81 ÷ 0.69 = 2.62 .

The results of Test 2B would seem suspect. The standard deviation of the set excluding Test 2B gives a value of 0.4. The result shown by Test 2B is outside the 99.73 percent confidence level from the mean of this sample and for this reason the result was rejected.

Thus the average proportion,

$$\frac{\sigma_{\text{oxy sequential}}}{\sigma_{\text{oxy simultaneous}}} = 3.0 .$$

Considering all three triangulation systems then, for one unit of error occurring in a simultaneous method, 3.0 units will occur in the sequential method and 5.7 units in the independent method for the same block.

These proportions are supported by Tewinkel[2]. He indicates that the value of σ_0 has been found to vary between 2 and 10 μm at plate scale, depending on the techniques used.

The number of tie points (including pass points) has a marginal effect on the standard error of unit weight. Ackermann[1] indicates a reduction of about 20 percent if 60 tie points per model are used as compared to the more normal four tie points.

Anderson *et al.* indicate a 10-percent decrease in error if 25 points are used as compared to 9 points.[4]

As there is a much larger increase in the number of tie points in the former instance, a 20 percent reduction would seem reasonable if comparing strongly tied models (i.e., in excess of 50) compared to the normal situation. Thus,

$$\frac{\sigma_{\text{oxy normal ties}}}{\sigma_{\text{oxy 50 ties}}} = 1.25$$

$$\frac{\sigma_{\text{oxy normal}}}{\sigma_{\text{oxy 25 points}}} = 1.1 .$$

TABLE 1. AVERAGE VALUE OF VARIANCE FACTOR q_i

	Average Planimetric Error in Micrometers	
Test	*Sequential*	*Simultaneous*
1A	3.89	1.12
2A	5.39	2.06
1B	2.46	0.70
2B	1.91	1.61
1C	1.46	0.50
2C	1.81	0.69

VARIANCE (PLANIMETRIC) AS A FUNCTION OF PERIMETER CONTROL AND BLOCK SIZE

From adjustments conducted by various researchers on simulated blocks, the manner in which q varies with changing control patterns and number of models can be determined. Perimeter-controlled blocks only were considered.

TABLE 2. CHANGE IN VALUE OF VARIANCE WITH BLOCK SIZE (KUNJI)

No. of Models	q_{av}
8	0.90
18	1.05
32	1.10
50	1.24
72	1.33
98	1.47

TABLE 3. CHANGE IN VALUE OF VARIANCE WITH NUMBER OF PERIMETER CONTROL POINTS (KUNJI)

No. of Control Points	*No. of Models*	q_{av}
28	98	1.85
24	72	1.85
20	16	1.85
16	32	1.85

TABLE 4. CHANGE IN VALUE OF VARIANCE WITH NUMBER OF CONTROL POINTS AND NUMBER OF MODELS (TALTS)

Number of Models	q_{av}
For 4 Control Points	
2	1.4
8	2.0
32	3.4
72	4.75
For 72 Control Points	
4	3.25
6	1.90
8	1.30
12	1.05
24	0.85

TABLE 5. VARIANCE AS A FUNCTION OF MODELS AND CONTROL

C-1	*M*	q_{av}
7	8	0.90
7	18	1.05
7	32	1.10
7	50	1.24
7	72	1.33
7	98	1.47
3	2	1.40
3	8	2.00
3	32	3.40
3	72	4.75
3	72	3.25
5	72	1.90
7	72	1.30
11	72	1.05
23	72	0.85
27	98	1.85
23	72	1.85
19	50	1.85
15	32	1.85

M = Number of Models
C-1 = Number of Control Points less One.

For eight perimeter control points Kunji[3] conducted various tests by varying the block size. His results were shown in graphs; the data in Table 2 were taken from them. For the purpose of analysis, it was assumed that $\sigma_x = \sigma_y$ and the average of q_x, q_y was used. For dense perimeter control, Kunji further indicated data as shown in Table 3.

From graphs derived by Talts[7], the data shown in Table 4 was extracted.

These data are summarized in Table 5. The value *C-1* was used because if *C* is less than 2, then q must be indeterminate or, alternatively, approach infinity.

A relationship of the form

$$q = a(C - 1)^{B_1}(M)^{B_2}$$

was assumed because of the shapes of the curves derived by Talts and Kunji. Linearizing by logarithms, a linear regression curve was fitted to the data with following results:

$N = 19$
$R = 0.7012$
$S = 0.1511$
$F = 7.7378$
$A = 0.2268 = \log a$
$B_1 = -0.3880$ $S_1 = 0.1211$
$T_1 = -3.1222$
$B_2 = 0.1981$ $S_2 = 0.0871$
$T_2 = 2.2733.$

The tests for significance F, T_1 and T_2 show that the formula derived was highly significant. The formula derived was:

$$q_{av} = 1.6858(C - 1)^{-0.3780}\, M^{0.1981}$$

The information from which the formula was derived took no account of the scale of the photographs although it should affect the final value of σ in real blocks. This would seem reasonable on the basis of the reduction of image resolution due to atmospheric effects at higher altitudes.

To test this assumption the results of a number of tests at various scales were examined. Each test must first be related to a common procedure so that a comparison is valid. The variation of σ_{oxy} with different procedures, as determined earlier, was applied directly to σ.

The *standard procedure* adopted was as follows:

- ★ Use of wide-angle panchromatic photography.
- ★ Use of 20-percent sidelap.
- ★ Use of the independent analytical method of adjustment.
- ★ Use of normal number of tie points per model.
- ★ Use of pre-targeted perimeter control points.

Source A. Ackermann[1] gave the results for blocks with perimeter control as shown in

TABLE 6. STANDARD ERROR ON THE GROUND AT VARIOUS SCALES

Test	*Photo Scale*	*No. of Models*	*No. of Control Points*	*Error (cm)*
1	1: 6,000	32	42	8.0
2	1: 7,500	170	32	8.0
3	1:10,000	33	13	12.0
4	1:28,000	200	40	35.0
5	1:28,000	32	16	35.0
6	1:60,000	85*	19	64
7	1:90,000	64	13	299
8	1:90,000	64	10	308
9	1:90,000	64	6	347
10	1:90,000	64	6	366
11	1:90,000	64	4	518
12	1:41,500	105	4	383†
13	1:41,500	105	28	124†

* Equivalent number of models at 20 percent sidelap.
† The average of σ_x and σ_y was used.

Table 6 where the conditions for Tests 1 to 3 were wide-angle photography, independent triangulation adjustment, medium number of ties per model, 20-percent sidelap, pretargeted perimeter control.For this standard procedure,

Test 1, $\sigma = 1.1 \times 8 = 9$ cm
Test 2, $\sigma = 1.1 \times 8 = 9$ cm
Test 3, $\sigma = 1.1 \times 12 = 13$ cm

At the photograph scale the errors were:

Test 1, $\sigma = 15$ μm
Test 2, $\sigma = 12$ μm
Test 3, $\sigma = 12$ μm.

The conditions for Tests 4 and 5 were superwide-angle photography, sequential triangulation adjustment, normal number of ties per model, 20-percent sidelap, pretargeted perimeter control. For this adopted standard procedure,

Test 4, $\sigma = 35 \times 1.9 = 67$ cm
Test 5, $\sigma = 28 \times 1.9 = 53$ cm

At photograph scale,

Test 4, $\sigma = 24$ μm
Test 5, $\sigma = 19$ μm.

Source B. From a test block with perimeter control, Eichert and Eller[8] published the result shown for Test 6 for which the conditions were: superwide-angle photography, sequential triangulation adjustment, normal number of ties per model, 60-percent sidelap, pretargeted perimeter control. For this adopted standard procedure, $\sigma = 64 \times 1.7 \times 1.9 = 207$ cm, and at the photograph scale, $\sigma = 34$ μm.

Source C. Lortz[5] in 1953 published the results for a single block with varying perimeter control as shown in Table 6 for Tests 7 through 11. The conditions were: wide-angle photography, sequential triangulation adjustment, 20-percent sidelap, pretargeted perimeter control. For this adopted standard procedure,

Test 7, $\sigma = 1.9 \times 299 = 658$ cm
Test 8, $\sigma = 1.9 \times 308 = 585$ cm
Test 9, $\sigma = 1.9 \times 347 = 659$ cm
Test 10, $\sigma = 1.9 \times 366 = 695$ cm
Test 11, $\sigma = 1.9 \times 518 = 984$ cm.

TABLE 7. COMPARISON OF ACTUAL AND THEORETICAL VARIANCE (HORIZONTAL)

Test	*Photo Scale*	*M.*	*C-1*	*q (actual)*	*q (theor.)*	q_a/q_t
1	1: 6,000	32	41	1.5	0.8228	1.8
2	1: 7,500	170	31	1.2	1.0492	1.1
3	1:10,000	33	12	1.2	1.3172	1.0
4	1:28,000	200	39	2.4	1.2055	2.0
5	1:28,000	32	15	1.9	1.2034	1.6
12	1:41,500	105	3	9.2	2.7890	3.3
13	1:41,500	105	27	3.0	1.2194	2.5
6	1:60,000	85	18	3.4	1.3630	2.5
7	1:90,000	64	12	6.3	1.5020	4.2
8	1:90,000	64	9	6.5	1.6746	3.9
9	1:90,000	64	5	7.3	2.0912	3.5
10	1:90,000	64	5	7.7	2.0912	3.7
11	1:90,000	64	3	10.9	2.5366	4.3

At the photograph scale,
Test 7, σ = 63 μm
Test 8, σ = 65 μm
Test 9, σ = 73 μm
Test 10, σ = 77 μm
Test 11, σ = 109 μm.

Source D. Tests 12 and 13 by Soehngen[9] were conducted on a simulated block. However, they were so conducted that the results related to a particular photograph scale, and for all purposes could be considered as an actual block. The two particular tests selected from his many results were chosen because they used the greatest and least perimeter control for the same size block.

The conditions for these tests were: wide-angle photography, independent triangulation adjustment, normal number of ties per model, 20-percent sidelap, and pretargeted perimeter control. The sigma values for the adopted standard procedure were: Test 12, σ = 383 cm; Test 13, σ = 124 cm. At photograph scale the errors were: Test 12, σ = 92 μm; Test 13, σ = 30 μm.

The combined data for all tests related to the adopted standard procedure are shown in Table 7. The value of q_{actual} was determined assuming the standard error of unit weight to be 10 μm. The value for $q_{theoretical}$ was determined from the empircal formula:

$$q = 1.6858\,(C - 1)^{-0.3780}\,(M)^{0.1981}.$$

The value for $q_{actual}/q_{theoretical}$ is also tabulated for each test.

From the data it seems reasonable to assume a linear relationship between scale and q_a/q_t. This also follows from Tewinkel[10] who suggests that resolution error increases linearly with flying height.

A linear regression curve was fitted to the data using q_a/q_t as a function of scale denominator divided by 10,000. The following result was obtained:

$$q_a/q_t = 1.1077 + 0.3123\ \frac{(S)}{10{,}000}.$$

The following statistical information was also obtained:
N = 13
R = 0.9349
S = 0.4319
F = 76.3006
and, for the coefficients,
S = 0.0358
T = 8.7114.
F and T are significant at the 1-percent level. The correlation is therefore, highly significant.

Thus, for aerial triangulation using the adopted standard procedure, a reasonable estimate of the standard planimetric error at photograph scale of points determined can be obtained from:

$$\sigma_{xy} = 17\,\left(1.1 + \frac{0.31S}{10{,}000}\right)\,(C - 1)^{-0.38}\,(M)^{0.20}\ \mu\text{m}$$

assuming σ_{oxy} = 10 μm.

Conclusion

A formula has been derived which enables the calculation of the standard error of the planimetric coordinates determined at a point in a block for varying photograph scale, number of control points and number of models in the block. Further, by applying the factors that affect the standard error of unit weight, the derived formula can be modified to apply to a particular aerial triangulation system.

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