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Ground Dimension from Slant-Range Radar

A nomograph is developed as an aid to image analysts and interpreters for converting image measurements.

Introduction

Radar, in the remote-sensing field, is becoming more important to interpreters and analysts as a source of intelligence, with much-improved resolution in current systems. Because radar-imaging geometry is different from that of a photograph, the mensuration principles are also different, particularly for radars which image in terms of slant range. Most imaging radars use slant-range sweeps because of their greater accuracy. With slant-range imagery, determining the actual distance between two points lying in a horizontal plane, but at an angle with respect to the ground track of the aircraft, requires several time-consuming calculations involving conversion of the measured slant range component of the distance to an equivalent ground distance, then measurement of the along-track scale to determine the ground component along-track. Then by using these components, the resultant distance between the two points is determined.

The purpose of this article is to present a derivation of an expression and a nomograph of scale reciprocals which can be used to convert any measurement at any angle across the imagery to an equivalent ground distance in the horizontal plane — without the tedious calculations normally required.

Abstract: *Determining the horizontal ground distance between two points oriented at an angle on slant-range radar imagery requires several steps involving conversion of the slant-range component to a component in the horizontal plane, as well as applying the scale reciprocals to determine the values of the along-track and range components in terms of ground distance. Once these ground components have been calculated from the image measurements, then the resultant straight-line ground distance between the two points can be determined. A derivation with a nomograph reduces all the necessary conversion steps into one simple operation of merely multiplying the slant-range image measurement by a scale reciprocal selected from the nomograph to determine the ground distance between any two points oriented at any angle across the slant-range image, as a function of radar depression angle. The nomograph can easily be used for any imagery scale, and as such is a convenient timesaver for image analysts and interpreters.*

Mathematical Basis

The basic geometry and definitions regarding the slant-range film and ground-plane geometry are shown in Figure 1, where two imaged points, *C* and *D*, are to be considered. A definition of the terms is as follows:

- y_s — Component of slant range between points *C* and *D* on the image.
- x_i — Along-track components between *C* and *D* on the image.
- α' — Angle on film between the along-track direction and a line between *C* and *D*.
- I — Image distance between *C* and *D*.
- y_g — Range component of ground distance between *C* and *D*.

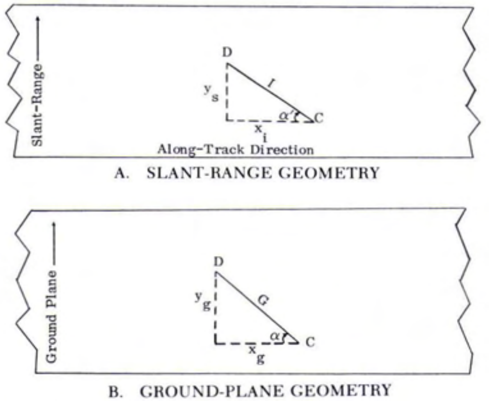

FIG. 1. Comparison of slant-range and ground-range images.

- x_g — Along-track component of ground distance between C and D.
- G — Direct ground distance between C and D.
- α — True angle on the ground between the along-track direction and a line on the ground between C and D.

The slant range component y_s must be converted into a ground-plane component y_g before

$$G = \left(x^2_g + y^2_g\right)^{1/2}$$

can be determined. The geometry illustrating this conversion is shown in Figure 2 where only the range *components* y_s and y_g are considered. From the figure, it can be seen that

$$y_g = Ry_s/\cos\theta \qquad (1)$$

where R is the image scale reciprocal in the slant-range direction, and where θ is the depression angle to the further point D, determined by

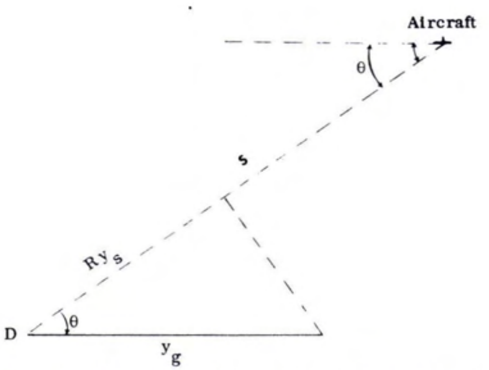

FIG. 2. Geometry of imagery slant-range and ground-range components.

$$\theta_D = \text{arc sin}\left(\frac{H}{S}\right) \qquad (2)$$

where H is aircraft vertical clearance above the plane in which points C and D are located, S is the slant range from the radar to point D.

The ground component x_g is determined by

$$x_g = Ax_i \qquad (3)$$

where A is the along-track scale reciprocal. Normally the along-track and slant-range scales are the same and, thus, normally $R = A$. (These are distinguished for the purpose of a general derivation in this article.)

Thus the ground distance between points C and D becomes

$$G = \left(x^2_g + y^2_g\right)^{1/2} \qquad (4)$$

To save the interpreter from the preceding measurements and calculations, it is desirable to derive a scale reciprocal nomograph which will allow him to determine the true ground distance by merely *measuring directly* on the image between any two points (*e.g.*, I in Figure 1A), and multiplying by the appropriate scale reciprocal. The following is a derivation of the scale reciprocal necessary for direct image measurements as a function of depression angle, and orientation angle α' of the measurement being made on the image with respect to the along-track direction.

Consider (Figure 3) the geometry *on the ground* of an actual physical distance, or length, L_{ACT} oriented at an angle α with respect to the along-track direction. Then, on slant-range imagery, the *along-track* component is:

$$\frac{L_{ACT}\cos\alpha}{A} \qquad (5)$$

where A is the along-track scale reciprocal, and the slant-range component becomes

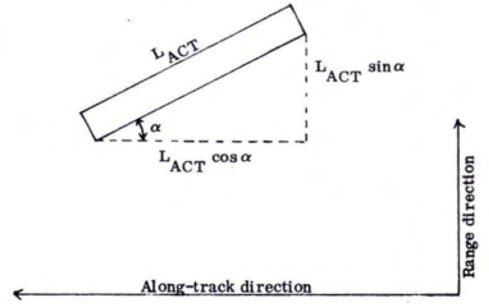

FIG. 3. Geometry of along-track and ground-range components of a physical length on the ground.

$$\left(\frac{L_{ACT}\sin\alpha}{R}\right)\cos\theta \qquad (6)$$

where R is the slant-range scale reciprocal and θ is the depression angle to target.

Thus, on the slant-range imagery, the measured length becomes

$$L_{Film} = L_{ACT}\left[\frac{\cos^2\alpha}{(A)^2} + \frac{\sin^2\alpha\cos^2\theta}{(R)^2}\right]^{1/2} \qquad (7)$$

where L_{Film} is the length measured directly on the film, in film dimensions. The actual physical distance, L_{ACT} becomes

$$L_{ACT} = \frac{L_{Film}}{\left[\frac{\cos^2\alpha}{(A)^2} + \frac{\sin^2\alpha\cos^2\theta}{(R)^2}\right]^{1/2}} \qquad (8)$$

$$= F \ . \ L$$

where

$$F = \frac{1}{\left[\frac{\cos^2\alpha}{(A)^2} + \frac{\sin^2\alpha\cos^2\theta}{(R)^2}\right]^{1/2}} \qquad (9)$$

The factor F is the scale reciprocal required at depression angles θ and orientations α, *on the ground.* In most applications the orientation α on the ground is not known. However, the *on-the-film* the angle of the length measurement with respect to the along-track direction is readily measured from the film. This length measurement orientation angle α' is related to the true ground orientation angle α by the following (Figure 4):

$$\tan\alpha' = \frac{(L_{ACT}\sin\alpha\cos\theta)/(R)}{(L_{ACT}\cos\alpha)/(A)} \qquad (10)$$

$$\tan\alpha' = \frac{A}{R}\tan\alpha\cos\theta \qquad (11)$$

or, solving for α,

$$\alpha = \tan^{-1}\left(\frac{R}{A}\ \frac{\tan\alpha'}{\cos\theta}\right) \qquad (12)$$

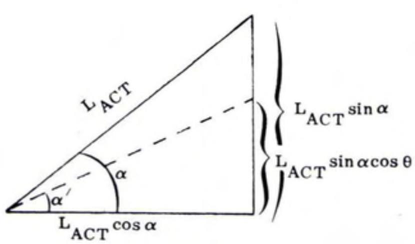

FIG. 4. Relation of angle α on the ground with the corresponding angle α' in the slant-range plane.

Now by substituting the expression for α from Equation 12 into Equation 9, a set of scale reciprocals can be generated as a function of measurement orientation angle α' on the film, radar depression angle θ and scale reciprocals R and A of the slant-range and along-track directions, respectively, on the film.

A computer program was written to generate scale reciprocals F as a function of the above mentioned variables; the resulting nomograph is given in Figure 5. The derivation presented here assumes that the points, between which the distance (or length) is being measured, lie in the same horizontal plane. Also note that this derivation, and consequently the factor F, are good for only relatively short dimensions on the film, where the depression angle does not change appreciably from one end of the object (or length) to the other. For example, a slant component of 200 feet in the range direction at an altitude of 50,000 feet and depression angle of 56 degrees results in an error from the true ground component of less than 0.5 per cent. However, a slant *component* of several thousand feet can result in errors of 10 per cent or more at high depression angles. Thus, unless the measurement is made only for a fast estimate, this procedure should be used for short dimensions. Note also that the error in determing the true, or actual length of an object or distance on the ground depends on the accuracy with which the orientation angle, the depression angle, and the along-track and slant-range scales are determined.

If the radar imagery scales in the slant-range direction and in the along-track direction are equal, the ration of (R/A) in Equation 12 becomes unity. This assumption is made in the generation of the nomograph of Figure 5, where the scale reciprocals R and A were chosen to be 100,000. This figure was chosen to allow easy use of the nomograph for radar imagery with a different scale. For example, if a slant-range radar image is a scale of 1:200,000, then the values of F, read from Figure 5, are merely doubled. Or, in general, the values of F are multiplied by (Z/100,000) where Z is the scale reciprocal of the slant range radar system or image in question.

SUMMARY

A nomograph of scale reciprocals F has been generated to allow direct conversion of image film measurements at any angle across slant-range imagery to actual physical (ground) dimensions. This nomograph obviates the necessity of numerous different

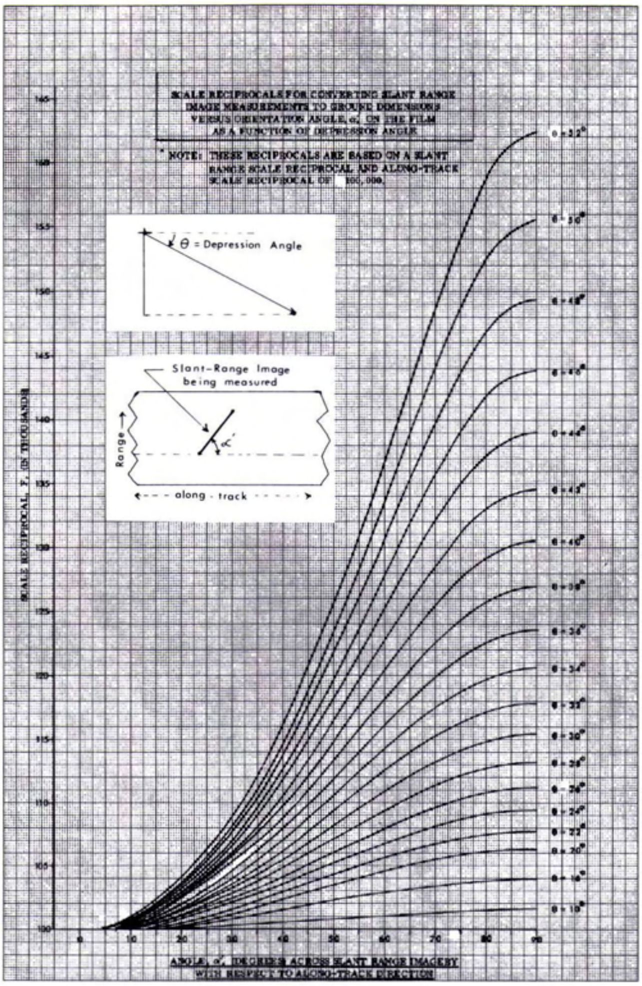

FIG. 5. Scale reciprocals for converting slant range image measurements to ground dimensions versus orientation angle α' *on the film* as a function of depression angle.

measurements and trigonometric calculations normally required to convert slant-range image measurements to true ground or physical dimensions (in the horizontal plane). Thus, the procedure is condensed to the following brief steps:

- Read the film distance between two points, which may be oriented at any angle α' on the film with respect to the along-track direction (the edge of the film.)
- Determine the depression angle θ to the area in question. $\theta = \arcsin(H/S)$ where H is the terrain clearance between the aircraft and target and S is the slant range to the target.
- Measure angle α' on the film (α' is the angle between edge of film and a line through the two points in question).
- Read the scale reciprocal F from the nomograph at the appropriate θ and α' values.
- Multiply F by the film distance between the two points of interest. This will give the actual ground distance between the two points.

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