XIII Congress of the
International Society for Photogrammetry
Helsinki, 1976

Commission V
Working Group V/1
Invited Paper

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Calibration of Close-Range Photogrammetric Systems: Mathematical Formulation

An analytical method for the self calibration of non-metric, close-range cameras, employing the coplanarity condition and minimum object space control, is described.

INTRODUCTION

According to Woodhead[23] a system can be defined as a "collection of components which are connected together in a specific way described as the system structure so that collectively the components perform or satisfy the system purpose". In photogrammetric systems, components such as camera, film, compilation, and data processing equipment can be readily identified together with their structural—often sequential—arrangement such that they fulfill the purpose of obtaining quantitative (metric) information about an object via photography. The prefix "close range" merely indicates a shorter object distance. Such a system encompasses several subsystems, namely photographic data acquisition, photographic processing, mensuration, data processing, and data presentation systems.

Calibration, as far as photogrammetrists are concerned, applies to the data acquisition and mensuration systems or, more precisely, to the camera equipment and to the plotter or comparator. When performing a calibration, numerical values are extracted from a photograph and processed according to certain mathematical procedures. May I therefore briefly consider analytical photogrammetric systems? As mentioned by Jaksic[13], a photograph is nothing but an analogue memory storage of the "read only" type. The processing unit is an electronic computer, while the reading device consists of a comparator (or plotter with numerical read-out) which converts some of the analogue information of the photograph into digital form. In a simple one-way flow this constitutes what is commonly considered as analytical photogrammetry. If there is a feed back between the computer and the comparator type unit, we have an analytical type plotter system capable of extracting additional analogue information. The digital computer can of course be replaced by an analogue instrument which is "programmed" for a certain configuration—central perspective, in our case—which satisfies the majority of photogrammetric production purposes, is simple, and is less expensive in operation. In return, for accurate results it requires photography that is obtained with a close approximation of central perspective as is the case with metric cameras.

In close-range applications, there are numerous cases where this condition cannot be fulfilled for various reasons, and non-metric cameras are used for data acquisition. Calibration provides the necessary data to process this general type of photography with high accuracy. Although it is basically regarded as camera calibration, errors such as affinity and non-perpendicularity of axes of the comparator can be included into the model, which means that in effect a calibration of the system is accomplished.

Calibration

Calibration provides the link between metric and non-metric cameras, considering that I have defined the latter as being cameras whose interior orientation is completely or partially unknown and potentially instable[10]. Interior orientation, of course, encompasses in this context the basic parameters principal point and principal distance (calibrated focal length or camera constant) as well as radial (symmetric) lens distortion, decentering (frequently considered in form of its components asymmetric and tangential) lens distortion, film deformation, and affinity. In addition, non-perpendicularity of the comparator axes is included for the system calibration.

Calibration is commonly carried out in three forms: laboratory, on the job, and self calibration.

Abstract: *In this paper a method of self calibration applicable to non-metric cameras is presented and discussed in connection with various other calibration approaches. The method is extremely general and includes radial symmetric and decentering lens distortions, affinity, and non-perpendicularity of axes. Although it provides interior orientation parameters for each photograph separately, the minimum control requirement remains at two horizontal and three vertical control points.*

Résumé: *Cette étude expose une méthode d'auto-étalonnage des chambres non-métriques. Également l'étude délibère sur d'autres methodes d'étalonnages. Cette méthode est bien génerale, elle inclus la distortion symétrique radiale et de décentrement de l'objectif, l'affinité et la non-perpendicularité des axes. Même si cette méthode fournit les paramètres de l'orientation intérieure de chaque cliché, la quantité minimum de levées demeure deux repères horizontaux et trois verticaux.*

Zusammenfassung: *Im Zusammenhang mit anderen Kalibrierungsmethoden wird im vorliegenden Artikel ein Selbstkalibrierungsverfahren für Amateurkameras erläutert. Der Ansatz des Verfahrens ist äusserst generell und berücksichtigt symmetrisch und asymmetrisch radiale sowie tangentielle Verzeichnungen, Affinität und Nichtortogonalitat der Achsen. Obleich die Parameter der inneren Orientierung für jede Photographie getrennt berechnet werden, reichen zwei Lage—und drei Höhenpasspunkte zur Lösung aus.*

Laboratory calibration

Several types of calibrations appear under this heading, the common fact being that calibration is completely separated from object photography. Therefore metric cameras are ideally suited for this approach. Besides goniometers, collimator banks, and similar arrangements, test areas of various sophistication have been used for laboratory calibration as described in the literature (e.g., [2,7,8,15,18,21,22]). The mathematical formulation is normally based on the collinearity equations, with each object space control point providing two equations. As pointed out by Linkwitz[17] five control points are required to solve for the principal point and principal distance with a slight overdetermination (three interior plus six exterior orientation parameters determined with ten equations).

With the inclusion of additional parameters, the number of object space control points has to increase accordingly. The mathematical formulations for laboratory calibration are well documented, and again I would like to refer to the literature e.g., [2,8,12,14,21] The basic resection approach fails for telelenses with cone angles of 2° or less. Merrit[20] has overcome this problem applying the Hartman method.

"ON THE JOB" CALIBRATION

When performing a calibration utilizing object photography, the object space has to be controlled to meet the requirements stated above, e.g., at least one full control point for every two unknown parameters.

In close-range photogrammetry often photo scales of 1:10 or larger are encountered, which means that 10 μm in photo scale represents 0.1 mm in the object. To achieve and maintain this accuracy for control by conventional surveying procedures is time consuming and often difficult even when maintaining a laboratory testfield. Often, however, accuracy requirements permit the construction of a special control frame[4,5,9] which maintains its geometric configuration sufficiently well to be used for control purposes.

The mathematical formulation is usually the same as for the laboratory calibration. Although it is not explicitly a calibration method, I would like to mention the "Direct Linear Transformation" approach, developed at the University of Illinois[1] in this context. It has been modified recently to incorporate radial symmetric and asymmetric lens distortions[19]. These require five parameters in addition to the original eleven, which means a minimum requirement of eight object space control points. Over-determination is, of course, highly desired. As pointed out by Abdel-Aziz and Karara[2], the method is highly economical as far as computer costs are concerned, and provides accuracies similar to the more conventional space resection type approach.

SELF CALIBRATION

This approach differs significantly from the previous ones in that it does not require object space control as such for the calibration. According to Kölbl[15] three convergent photographs are taken of the same object with the same camera and unchanged interior orientation. Utilizing the coplanarity condition and well identified object points, the basic parameters of interior orientation are computed. Recently this approach has been extended to include radial symmetric, asymmetric, and tangential lens distortions. These are modelled somewhat differently to what has been the general practice[16]. Brown[6] also has been applying self calibration in close-range photogrammetry using multiple station arrangements.

As mentioned before, these self calibration approaches do not require object space control, except for the actual object evaluation, which like any other photogrammetric evaluation requires two horizontal and three vertical control points. It has to be noted here that for these methods the interior orientation is considered as unchanged between photographs, which might not always be the case for non-metric photography, where the interior stability of the camera can be rather weak.

At the Department of Surveying Engineering at the University of New Brunswick a self calibration method was devised by the author and his collaborators[9,11] which permits the determination of the interior orientation parameters for each photograph while using the coplanarity condition and minimum object space control.

The UNB Self Calibration Approach

Based on the coplanarity condition, the method requires at least one overlapping stereomodel, but is however more general and can be applied to multiple stereo models as well as photogrammetric blocks. Due to expected changes in interior orientation the working unit is always the individual photograph.

According to Figure 1 the base vector between two camera stations C' and C'' is

$$\boldsymbol{B} = \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} X_{C''} - X_{C'} \\ Y_{C''} - Y_{C'} \\ Z_{C''} - Z_{C'} \end{bmatrix} \tag{1}$$

while the vector $\boldsymbol{U}'$ from C' to an object point $P(X,Y,Z)$ is expressed as

$$U' = \begin{bmatrix} U_{X'} \\ U_{Y'} \\ U_{Z'} \end{bmatrix} = \begin{bmatrix} X - X_{C'} \\ Y - Y_{C'} \\ X - Z_{C'} \end{bmatrix} \tag{2}$$

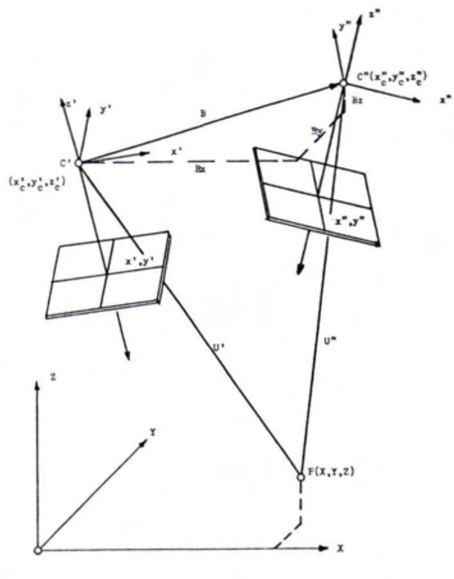

FIG. 1. Geometry of photography.

The angular orientation of the axes of the photograph with respect to the object space system as described by the rotational matrix $\boldsymbol{R}'$, the photo coordinate vector $\boldsymbol{u}'$, and the scale factor λ' again describe $\boldsymbol{U}'$:

$$\boldsymbol{U}' = \begin{bmatrix} X - X_{C'} \\ Y - Y_{C'} \\ Z - Z_{C} \end{bmatrix} = \lambda' \boldsymbol{R}' \begin{bmatrix} x' - x_{C'} \\ y' - y_{C'} \\ 0 - z_{C'} \end{bmatrix} \tag{3}$$

where x' and y' are the photo coordinates of point P, and x_C', y_C', z_C' are the image coordinates of the exposure centre, which means the (unknown) basic parameters of interior orientation (principal point and principal distance).

Similarly the vector $\boldsymbol{U}''$ from the second exposure station C'' is expressed as

$$\boldsymbol{U}'' = \begin{bmatrix} U_x'' \\ U_y'' \\ U_z'' \end{bmatrix} = \lambda'' \boldsymbol{R}'' \begin{bmatrix} x'' - x_c'' \\ y'' - y_c'' \\ 0 - Z_c'' \end{bmatrix}. \tag{4}$$

The coplanarity condition is expressed as

$$\boldsymbol{G} = \begin{vmatrix} B_x & U_x' & U_x'' \\ B_y & U_y' & U_y'' \\ B_z & U_z' & U_z'' \end{vmatrix} = \lambda' \lambda'' \begin{vmatrix} B_x & u_x' & u_x'' \\ B_y & u_y' & u_y'' \\ B_z & u_z' & u_z'' \end{vmatrix} = 0. \tag{5}$$

Although distortion parameters could be introduced into Equations (3) and (4) at this stage, a first iteration with only the basic parameters of interior orientation was chosen for computational reasons. These parameters are then held fixed while a subsequent iteration includes distortion and affinity parameters. Inspired by the iterative approach in PAT-M-43[3] this sequence is repeated.

The inclusion of distortion and affinity parameters into Equations (3) and (4) leads to:

$$\boldsymbol{u}' = \boldsymbol{R}' \begin{vmatrix} (x' - x_c') + dr_x' + dp_x' + dg_x' \\ (y' - y_c') + dr_y' + dp_y' + dg_y' \\ - z_c' \end{vmatrix} \tag{6}$$

$$\boldsymbol{u}'' = \boldsymbol{R}'' \begin{vmatrix} (x'' - x_c'') + dr_x'' + dp_x'' + dg_x'' \\ (y'' - y_c'') + dr_y'' + dp_y'' + dg_y'' \\ -z_c'' \end{vmatrix} \tag{7}$$

$$\text{where } dr_x = (x - x_c)(k_o + k_1 r^2 + k_2 r^4 + k_3 r^6)$$
$$dr_y = (y - y_c)(k_o + k_1 r^2 + k_2 r^4 + k_3 r^6) \tag{8}$$

$$dp_x = p_1(r^2 + 2(x - x_c)^2) + 2p_2 (x - x_c)(y - y_c)$$
$$dp_y = p_2(r^2 + 2(y - y_c)^2) + 2p_1 (x - x_c)(y - y_c) \tag{9}$$

$$dg_x = A\,(y - y_c)$$
$$dg_y = B\,(y - y_c)\quad . \tag{10}$$

Note the constant term k_o in Equation (8) which is necessary due to the fact that z_c remains fixed for this iteration step. The coefficients $k_o \ldots k_3, p_1, p_2, A$, and B are unknowns along with the elements of relative orientation, while r is expressed as

$$r = ((x - x_c)^2 + (y - y_c)^2)^{1/2}$$

To include absolute orientation into the approach, control points have to be utilized. Rather than using collinearity equations—which is an option in the program and avoids step by step iterations but leads to a simultaneous solution at the expense of requiring more object space control—a control restraint condition is used. Prior to its use, the scale factors λ' and λ'', which do not affect the coplanarity equations, have to be approximated for the control points. To do so, the space coordinates of point P (see Figure 1) are expressed in both photo systems:

$$\begin{bmatrix} X_p' \\ Y_p' \\ Z_p' \end{bmatrix} = \begin{bmatrix} X_c' \\ Y_c' \\ Z_c' \end{bmatrix} + \lambda' \begin{bmatrix} u_x' \\ u_y' \\ u_z' \end{bmatrix}$$
$$\begin{bmatrix} X_p'' \\ Y_p'' \\ X_p'' \end{bmatrix} = \begin{bmatrix} X_c'' \\ Y_c'' \\ Z_c'' \end{bmatrix} + \lambda'' \begin{bmatrix} u_x'' \\ u_y'' \\ u_z'' \end{bmatrix}. \tag{11}$$

Then coplanarity is expressed as

$$\Phi = (X_p' - X_p'')^2 + (Y_p' - Y_p'')^2 + (Z_p' - Z_p'')^2 = \min \tag{12}$$

or

$$\frac{\partial \Phi}{\partial \lambda'} = \frac{\partial \Phi}{\partial \lambda''} = 0 \tag{13}$$

which leads to

$$(u_x'^2 + u_y'^2 + u_z'^2)\,\lambda' - (u_x'u_x'' + u_y'u_y'' + u_z'u_z'')\,\lambda'' = (B_x u_x' + B_y u_y' + B_z y_z')$$
$$(u_x'u_x'' + u_y'u_y'' + u_z'u_z'')\,\lambda' - (u_x''^2 + u_y''^2 + u_z''^2)\,\lambda'' = (B_x u_x'' + B_y u_y'' + B_z u_z'')\quad . \tag{14}$$

Equations (14) are solved for λ' and λ'', expressing the base components according to Equation (1). These values for λ' and λ'' are used in the control restraint:

$$\begin{aligned} C_x &= X_c - \lambda u_x - X_G = 0 \\ C_y &= Y_c - \lambda u_y - Y_G = 0 \\ C_z &= Z_c - \lambda u_z - Z_G = 0 \quad . \end{aligned} \tag{15}$$

In a least squares solution this means:

$$\begin{aligned} &\frac{\partial C_x}{\partial X_c} dX_c + \frac{\partial C_x}{\partial \lambda} d\lambda + \frac{\partial C_x}{\partial \omega} d\omega + \frac{\partial C_x}{\partial \phi} d\phi + \frac{\partial C_x}{\partial \kappa} d\kappa + V_{cx} = 0 \\ &\frac{\partial C_y}{\partial Y_c} dY_c + \frac{\partial C_y}{\partial \lambda} d\lambda + \frac{\partial C_y}{\partial \omega} d\omega + \frac{\partial C_y}{\partial \phi} d\phi + \frac{\partial C_y}{\partial \kappa} d\kappa + V_{cy} = 0 \\ &\frac{\partial C_z}{\partial Z_c} dZ_c + \frac{\partial C_z}{\partial \lambda} d\lambda + \frac{\partial C_z}{\partial \omega} d\omega + \frac{\partial C_z}{\partial \phi} d\phi + \frac{\partial C_z}{\partial \kappa} d\kappa + V_{cz} = 0 \quad . \end{aligned} \tag{16}$$

As mentioned before, the actual adjustment is done in a step by step iterative procedure.

In the first step the observation equations are:

$$\begin{aligned} V_G &= G\,(X_c', Y_c', Z_c', X_c'', Y_c'', Z_c'', \omega', \phi', \kappa', \\ &\qquad \omega'', \phi'', \kappa'', x_c', y_c', z_c', x_c'', y_c'', z_c'') \\ V_{F_1} &= F_1(X_c', Y_c', Z_c', \omega', \phi', \kappa') \\ V_{F_2} &= F_2\,(X_c'', Y_c'', Z_c'', \omega'', \phi'', \kappa'') \end{aligned} \tag{17}$$

where the first Equation of (17) is obtained by combining Equations (5) with (3) and (4) and linearizing it by developing into a Taylor series.

The other two equations in Equations (17) are obtained according to Equation (16). Then the weighted square sum of the corrections V is minimized.

In the second iteration step the function G in Equation (17) is replaced by

$$\begin{aligned} V_H = H\,(&X_c', Y_c', Z_c', X_c'', Y_c'', Z_c'', \omega', \phi', \kappa', \omega'', \phi'', \kappa'', \\ &k_o', k_1', k_2', k_3', k_o'', k_1'', k_2'', k_3'', p_1', p_2', \\ &p_1'', p_2'', A', B', A'', B''). \end{aligned} \tag{18}$$

Its linearized form is again adjusted together with V_{F_1} and V_{F_2} (see Equation (17)).

Iteration continues, using alternatively V_G and V_H, until the differences between subsequent results are less than 10^{-6} mm and 10^{-5} radians respectively. This usually takes approximately four iterations.

Due to the fact that the normal equation matrix is fully occupied, the program requires a considerable amount of cpu-time. This is the main reason why the ground control is considered presently as fixed rather than incorporated as an observed quantity in a combined parametric-condition adjustment as in the optional case where collinearity equations are used for the control points[9]. An expansion in this direction is planned.

Perhaps it should be pointed out that there is no need for full X,Y,Z control points, as the control restraint can utilize horizontal and vertical control points separately.

Practical Tests

Several test objects have been photographed using a Nikomat-FT 35 mm camera with a 50 mm Nikkor—4 lens and evaluated using 2, 3, 4, or 6 photographs, forming 1, 3, 6, and 13 (more practically : 11) photogrammetric models. The method does not necessarily require convergent photography which however provides better stereo coverage.

There are 17 unknowns per photograph in the second iteration step, which leads to the minimum number of common points to be measured in each photograph as shown in Table 1. It is quite apparent that more than four photographs become uneconomical as they require much more measuring and computing effort.

All these combinations were evaluated using ten each horizontal and vertical control points and many object points. Both a full calibration (including all distortions and affinity and using V_G and V_H alternatively) and a partial calibration (using V_G only) were carried out. Further-

TABLE 1. NUMBERS OF PHOTOGRAPHS, MODELS, AND CORRESPONDING MEASUREMENTS.

Photos	Number of Models	Unknowns	Minimum Number of Object Points	Total Point Measurements (e.g., mono-comparison)	Model Point Measurements (stereo plotter)
2	1	34	34	68	34
3	3	51	17	51	51
4	6	68	12	48	72
6	13(11)	102	8(10)	(60)48(60)	104(110)

TABLE 2. RESULTS OF TEST CASES.

Number of Photos	Models	Object Points	Control Points	Photo Scale	Iteration Sequence	RMS in μm (photo scale) for all points	RMS for 24 (x, y, z) Check Points in μm (photo scale) Horizontal	Vertical
2	1	42	10H, 10V	1:15	V_G, V_H	1.7	0.8	0.5
2	1	64	2H, 3V	1:30	V_G	5.3	45.0	27.8
2	1	64	2H, 3V	1:30	V_G, V_H	3.4	46.5	31.1
2	1	64	10H, 10V	1:30	V_G	2.9	2.6	0.3
2	1	64	10H, 10V	1:30	V_G, V_H	2.7	2.5	0.4
3	3	62	2H, 3V	1:30	V_G	9.3	27.4	18.5
3	3	62	2H, 3V	1:30	V_G, V_H	7.5	46.4	24.2
3	3	62	10H, 10V	1:30	V_G	7.0	2.2	0.5
3	3	62	10H, 10V	1:30	V_G, V_H	6.7	1.5	0.3
4	6	61	2H, 3V	1:30	V_G	9.7	38.5	44.9
4	6	61	2H, 3V	1:30	V_G, V_H	7.7	25.6	34.3
4	6	61	10H, 10V	1:30	V_G	6.7	2.0	2.6
4	6	61	10H, 10V	1:30	V_G, V_H	6.4	2.2	1.7
6	11	33	2H, 3V	1:30	V_G	8.4	26.4	60.8
6	11	33	2H, 3V	1:30	V_G, V_H	7.8	18.3	56.1
6	11	33	10H, 10V	1:30	V_G	7.2	5.5	1.0
6	11	33	10H, 10V	1:30	V_G, V_H	6.2	1.2	1.2

more, the same cases were computed with minimum control (two horizontal and three vertical control points). The results are listed in Table 2, based on PSK—measurements ($\sigma = 3\ \mu m$).

Following common photogrammetric practice, no variance-covariance matrix was obtained for computational reasons, which explains why no standard deviations for the unknowns are given. The quality of the method, however, is apparent from the RMS values for all points and for the check point residuals. The increase in RMS values for all points with more photographs was expected as all points were transformed in all models. Even so, they are very small.

The actual fitting into the control was somewhat weak when using minimum control, which might partly be caused by uncertainties of the control point coordinates. However, with ten control points in both planimetry and height an excellent fit has been obtained.

CONCLUDING REMARKS

Although computer intensive, the method described is an excellent tool for close-range photogrammetry, especially when using non-metric cameras, as it does not assume a constant interior orientation when taking different photographs. The step-by-step iteration procedure permits a simple basic calibration if the job at hand does not require more. As planimetric and vertical control are treated separately, the minimum control requirement can easily be met without much surveying work. If the object is placed on a plane surface, three bolts with predetermined height can be placed conveniently around it, and a ruler or similar object of known length, located on the plane surface is all that is needed for defining the object space coordinate system. Usually, this is sufficient for close range work, as any local system is permissible. The results proved that high accuracies can be obtained with this method.

In closing I would like to acknowledge the efforts of Mr. H. Moniwa, graduate student at U.N.B., in connection with this research, which in part was sponsored by the National Research Council of Canada.

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