DR. K. JEYAPALAN
California State University
Fresno, Calif. 93740

PETE VIVOLI
Photogrammetric Services Inc.
Columbus, Ohio 43216

Absolute Orientation from Independent-Model Data

The elements for stereoplotting (particularly for the Stereosimplex II and III instruments) can be computed during independent-model aerotriangulation.

INTRODUCTION

GENERALLY THE objective in aerial triangulation is to provide control for plotting by analog instruments. Prior to plotting, the analog instruments have to be relatively oriented, scaled and leveled, a process which takes approximately one to two hours of machine time. If the absolute orientation elements are determined *a priori* and set on the analog instrument, there is considerable saving of machine time. The objective of this paper is to describe a method of obtaining absolute orientation elements from independent-model triangulation.

In all triangulation methods, absolute orientation elements can be obtained as a by-product. In analog triangulation, these values can be obtained by recording the orientation elements after each model and then correcting these values for the propagation of errors. Prescott[3] assumes that propagation of errors in ϕ, ω and bx are linear. This method, apart from its simplicity, has the following disadvantages: (1) propagation of errors, particularly in ϕ is not linear, (2) the linearity is only valid for short strips, and (3) the method requires a special instrument with the facility of base-in and base-out.

ABSTRACT: *An independent-model triangulation program written by C. W. B. King was modified to give the absolute orientation elements. The absolute orientation elements for the Santoni Stereosimplex II can be obtained from independent model triangulation using the Stereosimplex III. The procedures described here are currently used by Photogrammetric Services, Inc., of Columbus, Ohio.*

In fully analytical methods, the triangulation is generally accomplished by either two methods based on a simultaneous adjustment or the Thompson method[1]. In the first method, the absolute orientation elements are obtained directly, whereas in the second* they are a by-products[2]. However, obtaining absolute orientation elements by analytical triangulation has the disadvantage that a one-to-one correspondence between the analytical results and the analog instrument needs to be established by careful calibrations of the camera and the analog instrument.

* The simultaneous adjustment method uses the collinearity equation as an observation equation and solves for the parameters directly. The Thompson method using the coplanarity condition to determine the relative orientation elements, uses relative orientation elements to form model coordinates and then transforms these to ground coordinates.

In semi-analytical methods, the absolute orientation elements can be obtained as a by-product. The advantage here is that the same instrument can be used for plotting the model. If a different instrument is used for plotting, then a one-to-one correspondence between the two instruments has to be established by proper calibration. The method suggested in this paper is being used by the Photogrammetric Services Inc. (PSI), of Columbus, Ohio, in their normal production work and it results in a saving of \$20.00 to \$30.00 per model of plotting.

The second section of this paper gives the theory of obtaining absolute orientation elements. The third section gives the theory of modifying a computer program written by King[2] which is based on model by model adjustments. This program is now used by the Photogrammetric Services, Inc., Columbus, Ohio, who uses Santoni's Stereosimplex III to obtain the model coordinates. In the fourth section, the modifications needed to use this program for Stereosimplex II are stated and the final section outlines the conclusion and recommendation.

Theory

In independent-model triangulation, each model is formed by relative orientation. The models are transformed into the first model system, by rotation, translation and scaling. The models are then transformed into a ground system. Generally the relative orientation is done by either of two methods (a) single-projector method and (b) double-projector methods.

Single-projector method

Suppose in Figure 1, s_1 and s_2 are the exposure stations: (x_1, y_1, z_1) is the first photo system, and (x_2, y_2, z_2) is the second photo system. Then

$$\begin{pmatrix} G_x \\ G_y \\ G_Z \end{pmatrix} = S_1 R_1(\kappa_1, \phi_1, \omega_1) \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + \begin{pmatrix} X_1(s_1) \\ Y_1(s_1) \\ Z_1(s_1) \end{pmatrix}$$

$$= S_2 R_2(\kappa_2, \phi_2, \omega_2) \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} + \begin{pmatrix} X_2(s_2) \\ Y_2(s_2) \\ Z_2(s_2) \end{pmatrix}$$

where (Gx, Gy, G_Z) is the ground system, $R(\kappa_1, \phi_2, \omega_1)$ are the rotation matrices, $X_1(s_1), Y_1(s_1)$, $Z_1(s_1)$ and $X_2(s_2)$, $Y_2(s_2)$, $Z_2(s_2)$ are the exposure stations coordinates. S_1, S_2 are scale factors, and $(\kappa_1, \phi_1, \omega_1)$ and $(\kappa_2, \phi_2, \omega_2)$ are the absolute orientation elements. In this process of relative orientation, the elements $\Delta\kappa$, $\Delta\phi$, $\Delta\omega$, By, Bz are determined where

$$B_y = \Delta y = Y_2 - Y_1$$

$$B_z = \Delta z = Z_2 - Z_1$$

$$R_1 \cdot \Delta R(\Delta\kappa, \Delta\phi, \Delta\omega) = R_2.$$

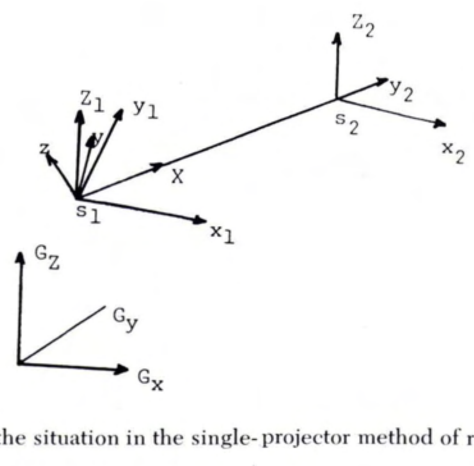

FIG. 1. Geometry of the situation in the single-projector method of relative orientation.

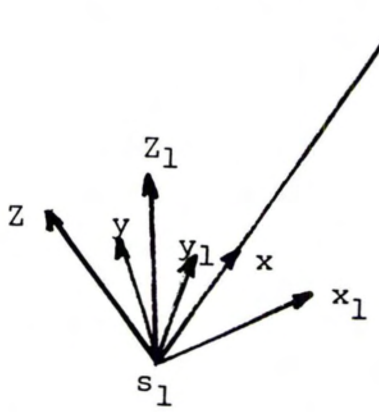

FIG. 2. Geometry of the situation in the double-projector method of relative orientation.

Thus, after absolute orientation, during which process R_1, X_1, Y_1, Z_1 and scale are determined, it is possible to determine R_2, X_2, Y_2, Z_2.

DOUBLE PROJECTION METHOD

Suppose in Figure 2, if we choose a system of axes (x, y, z) such that its origin is s_1, its x axis passes through s_2 and the y axis lies in the plane of axes x_1, y_1, then by the process of relative orientation, we determine $\Delta\kappa_1$, $\Delta\kappa_2$, $\Delta\phi_1$, $\Delta\phi_2$, $\Delta\omega_2$ such that

$$R \cdot \Delta R_1 (\Delta\kappa_1, \Delta\phi_1) = R_1$$

$$R \cdot \Delta R_2 (\Delta\kappa_2, \Delta\phi_2, \Delta\omega_2) = R_2.$$

Thus after absolute orientation, during which process R, X_1, Y_1, Z_1 and scale are determined it is possible to determine R_2, R_1, X_2, Y_2, Z_2 and scale.

The second method of relative orientation is commonly used in independent-model triangulation. The purpose of independent-model triangulation is to determine R, X_1, Y_1, Z_1 and scale directly or indirectly for all the models in a strip or a block so as to determine the ground coordinates of pass points, etc.

In the usual application the ground coordinates are then used to get an absolute orientation of the model in order to plot the topographic information. The elements ω_1, ω_2, ϕ_1, ϕ_2, Bx, Bz, $\Delta\kappa_1$, $\Delta\kappa_2$ are determined directly from the instruments by analog methods and the angle κ is determined on the plotting board such that

$$\Delta R_1 (\Delta\kappa_1) \cdot R(\kappa) = R_1 (\kappa_1)$$

$$\Delta R_2 (\Delta\kappa_2) \cdot R(\kappa) = R_2 (\kappa_2).$$

However, these elements can be determined from R, X_1, Y_1, Z_1 and the scale given for each model after independent-model triangulation using the following method:

- Given $B'x$ at the time of independent-model triangulation, the required Bx at the plotting scale is determined from $Bx = (s'/s) B'x$ where s' is the scale of the model at the independent-model stage and s is the required scale of the model at the plotting stage;
- Knowing $\Delta\omega_2$, $\Delta\phi_1$, $\Delta\phi_2$ at the time of independent-model triangulation the required elements ω_1, ω_2, ϕ_1, ϕ_2 for plotting are determined from

$$R_1 (\omega_1\ \phi_1, \kappa_1) = R (\omega, \phi, \kappa) \cdot \Delta R_1 (\Delta\omega_1, \Delta\phi_1, \Delta\kappa_1)$$

$$R_2 (\omega_2\ \phi_2, \kappa_2) = R (\omega, \phi, \kappa) \cdot \Delta R_2 (\Delta\omega_2, \Delta\phi_2, \Delta\kappa_2)$$

In practice, because the photographs are nearly vertical, ω, ϕ are small. Also, at this stage we do not require κ_1, κ_2, therefore, the above equations can be written as:

$$\begin{pmatrix} 1 & 0 & -\phi_1 \\ 0 & 1 & \omega_1 \\ \phi_1 & -\omega_1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -\phi \\ 0 & 1 & \omega \\ \phi & -\omega & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -\Delta\phi_1 \\ 0 & 1 & \Delta\omega_1 \\ \Delta\phi_1 & -\Delta\omega_1 & 1 \end{pmatrix}$$

That is, for small angles

$$\phi_1 \simeq \phi + \Delta\phi_1$$

$$\omega_1 \simeq \omega + \Delta\omega_1$$

As $\Delta\omega_1 = 0,\ \omega_1 \simeq \omega$

And similarly,

$$\phi_2 = \phi + \Delta\phi_2$$

$$\omega_2 = \omega + \Delta\omega_2$$

- Bz required for setting the model for plotting can be computed from

$$Bz = Bx \cdot \phi.$$

Modifications to the Independent-Model Triangulation Program

The independent-model triangulation program written by King[2] gives the ground coordinates obtained from model-by-model adjustment. Here each model is treated as a unit which is rotated, translated and scaled to fit the adjoining models as well as the ground control points.

Each model is first transformed into the first model system and then scaled to a nominal scale, S_o. The transformation equations converting the model coordinates to the ground coordinates are

$$X = AA \cdot x + BB \cdot y + C_{xn}$$

$$Y = -BB \cdot x + AA \cdot y + C_{yn}$$

$$Z = z_n - x\,\phi_n + y\,\omega_n + D_n$$

where $(X, Y, Z)^T$ is the ground system, $(x, y, z_n)^T$ is the model system, AA, BB are constants for the strip, C_{xn}, C_{yn}, D_n are constants of the nth model.

The observation equations for the control points are

$$A_n x_n + B_n y_n + C_{xn} = X$$

$$-B_n x_n + A_n y_n + C_{yn} = y$$

$$-\phi_n x_n + \omega_n y_n + z_n + D_n = Z$$

where (x_n, y_n, z_n) are the model coordinates in the nth model, A_n, B_n, C_{xn}, C_{yn}, ϕ_n, ω_n, D_n are the model constants. Using these observation equations and similar equations for the tie points, the seven constants for each model are determined by least squares. From the seven constants the strip constants

$$KZ_1^2 = (A_1^2 + B_1^2) / S_o^2$$

$$AA = A_1 / KZ_1$$

$$BB = B_1 / KZ_1 \quad (\text{note, } AA^2 + BB^2 = S_o^2)$$

are computed. From these strip constants, constants

$$KZ_n^2 = (A_n^2 + B_n^2) / S_o^2$$

$$\alpha_n = (A_n \cdot AA + B_n \cdot BB) / S_o^2$$

$$\beta_n = (B_n \cdot AA - A_n BB) / S_o^2$$

$$(\text{note, } \alpha_n^2 + \beta_n^2 = A_n^2 + B_n^2$$

are computed. Then using the following equations,

$$X_{new} = \alpha_n\left[x(1 - \phi_n^2 / 2) + \phi_n \cdot z\right] + \beta_n\left[y(1 - \omega^2 / 2) - \omega_n \cdot z + \phi_n\omega_n x\right]$$

$$Y_{new} = \beta_n\left[x(1 - \phi_n^2 / 2) + \phi_n \cdot z\right] + \alpha_n\left[y(1 - \omega_n^2 / 2) - \omega_n z + \omega_n\phi_n x\right]$$

$$Z_{new} = KZ_n \left[z (1 - \phi_n^2 / 2) - \phi_n x + \omega_n y\right],$$

the coordinates of each model are modified so as to be parallel and of the same scale as the strip. The procedure given above can be expressed in terms:

$$\overbrace{\begin{pmatrix} AA & BB & 0 \\ -BB & AA & 0 \\ 0 & 0 & 1 \end{pmatrix}}^{\mathbf{R}_k} \overbrace{\begin{pmatrix} A_n & B_n & 0 \\ -B_n & A_n & 0 \\ 0 & 0 & KZ \end{pmatrix}}^{\mathbf{R}_{\Delta\kappa}} \overbrace{\begin{pmatrix} 1 - \phi_n^2 / 2 & -\phi_n\omega_n & \phi_n \\ \phi_n\omega_n & 1 - \omega_n{}^2 / 2 & -\omega_n \\ -\phi_n & \omega_n & 1 - \phi_n^2 - \omega_n^2 / 2 \end{pmatrix}}^{\mathbf{R}_\theta} \begin{pmatrix} x_n \\ y_n \\ z_n \end{pmatrix} + \begin{pmatrix} C_{xn} \\ C_{yn} \\ D_n \end{pmatrix} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \quad (1)$$

where the matrix $\mathbf{R}_\theta$ levels the model parallel to the ground system, $\mathbf{R}_{\Delta\kappa}$ rotates and scales the model to the strip system and $\mathbf{R}_\kappa$ rotates and scales the strip to the ground system.

Now the values ϕ_n, ω_n, $B_n/A_n = \Delta\kappa$ are small and hence we will have the absolute orientation elements of the nth model as

$$Kappa = tan^{-1}\left(\frac{BB}{AA}\right) + B_n/A_n$$

$$Phi = \phi n$$

$$Omega = \omega_n$$

$$Scale = \left\{ (AA^2 + BB^2)^{1/2} \cdot (A_n^2 + B_n^2)^{1/2}\right\} = S_o \cdot S_n.$$

In practice, the program is designed for a number of iterations, γ, whereupon we can write the Equation 1 as

$$(R_{\kappa_\gamma} \ldots R_{\kappa_2} R_{\kappa_1}) \cdot (R_{\Delta_{\kappa_\gamma}} \ldots R_{\Delta_{\kappa_2}} R_{\Delta_{\kappa_1}}) \cdot (R_{\theta_\gamma} \ldots R_{\theta_2} R_{\theta_1}) \begin{pmatrix} x_n \\ y_n \\ z_n \end{pmatrix} + \begin{pmatrix} C_{xn} \\ C_{yn} \\ D_n \end{pmatrix} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

Because $\theta_1, \theta_2 \ldots \theta_\gamma, \Delta\kappa_1, \Delta\kappa_2 \ldots \Delta\kappa_\gamma, \kappa_1, \kappa_2 \ldots \kappa_\gamma$ are small, we can write the absolute elements as

$$Kappa = tan^{-1}\left(\frac{BB}{AA}\right)_1 + \left\{\left(\frac{B_n}{A_n}\right)_1 + \left(\frac{B_n}{A_n}\right)_2 + \ldots + \left(\frac{B}{A}\right)_\gamma\right\} + \left\{\left(\frac{BB}{AA_2}\right) + \ldots + \left(\frac{BB}{AA_\gamma}\right)\right\}$$

$$Phi = \phi_\gamma^n + \ldots + \phi_1^n$$

$$Omega = \omega_\gamma^n + \ldots + \omega_1$$

$$Scale = (So_1^n \cdot So_2^n \ldots So_\gamma^n) \cdot ((S_n)_1 \ldots (S_n)_\gamma).$$

From these equations, we can compute the absolute orientation elements of each plate of the model as

$$\begin{aligned} &Kappa\ Left = Kappa + Rel\ (Kappa)\ Left \\ &Omega\ Left = Omega + Rel\ (Omega)\ Left \\ &Phi\ \ Left = Phi + Rel\ (Phi)\ Left \end{aligned}$$

and similarly, for the right plate. The relative orientation values, *Rel (Kappa)* etc., can be obtained during the independent-model triangulation.

The original program is modified so that for every iteration the program writes the values of $AA, BB, A_1 \ldots A_n, B_1 \ldots B_n, \omega^1 \ldots \omega^n, \phi^1 \ldots \phi^n$ on a disk. Then a subroutine is added to the program which reads the relative orientation elements, the base components, and retrieves the information from the disk to form the absolute orientation elements. The base components for a given plotting scale is computed from

$$B_x\,(plotting) = \frac{B_x\,(Independent\ model)}{Scale} \times Scale\ (plotting)$$

$$B_y = B_y' + B_x \cdot Kappa$$

$$B_z = B_z' + B_z \cdot Phi$$

where B_y' and B_z' are the base components of the independent model.

Because in plotting, *Kappa* is set on the plotting table, the relative orientation values of Kappa, B_y', can be set on the plotting instrument.

Modifications to Stereosimplex II from Stereosimplex III

The main difference between Stereosimplex II and III (apart from the fact that Stereosimplex III is more accurate than II) is that III has the facility of b_z setting, unlike the II. However, the common ϕ can be set on II by lowering and lifting the ends of the bar carrying the projectors by $Z_1 = D \cdot (Phi)$ (see Figure 3). Thus, in Stereosimplex II the *Phi* settings will be the same as in relative orientation. Table I shows a typical computer output.

Inasmuch as Stereosimplex II is cheaper but less accurate than III, it is advantageous to use III for triangulation and II for plotting.

Conclusions and Recommendation

The operators found the absolute orientation elements invaluable in setting the model for plotting, especially if the model is undulating. The time spent on careful relative orientation during independent-model triangulation is gained during plotting.

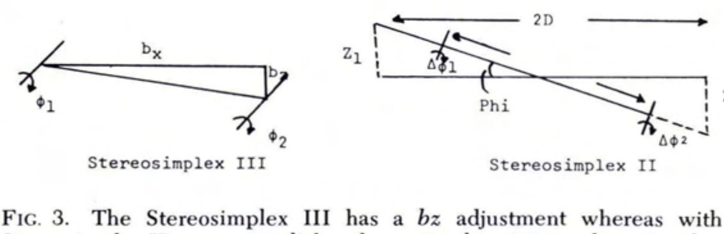

FIG. 3. The Stereosimplex III has a *bz* adjustment whereas with the Stereosimplex II one accomplishes the motion by raising or lowering the ends of the bar that supports the projectors.

TABLE 1. COMPUTER OUTPUT

Model 2				
Simplex III	Model Scale	1600	Bx	199.658
Plate		27	Plate	28
By		50.000	by	50.000
BZL		50.000	BZR	46.507
κ_1		98.200	κ_1	98.430
Phi		99.854	PHI	101.454
Omega		201.794	Omega	201.354
κ_2		54.898	κ_2	55.128
Simplex II	Model Scale	1600	Bx	199.658
Plate		27	Plate	28
By		50.000	By	50.000
PH2L		17.494	PH2R	−17.494
κ_1		98.200	κ_1	98.430
PHI		98.740	PHI	100.340
Omega		201.794	Omega	201.354
κ_2		54.898	κ_2	55.128

Similar modifications can be applied to any independent-model triangulation to give the absolute orientation elements. The absolute orientation elements obtained by analytical methods can also be used.

REFERENCES

1. Jeyapalan, K., "Some Methods of Obtaining Approximate Coordinates for Simultaneous Block Adjustment" American Society of Photogrammetry, Fall Convention, 1972.
2. King, C. W. B., "Programming Considerations for Adjustment of Aerial Triangulation", *Photogrammetria*, 1968.
3. Presscott, W. G. and Ghosh, S. K., "Global Adjustment of Aerial Triangulation" Report No. 69, Department of Geodetic Science, Ohio State University, 1966.

PHOTOGRAMMETRIC SURVEYS DIVISION

PURPOSE

THE PURPOSE of the Photogrammetric Surveys Division is to facilitate the leadership, promotion, sponsorship of, and consultation service for, the development and improvement of the scientific technology, applications, procedures, instruments, and processes of interpretation and photogrammetry in making surveys; and to facilitate the use and acceptance of photogrammetrically made surveys.

STRUCTURE

The interests, the activities, and the responsibilities of the Photogrammetric Surveys Division encompass the entire spectrum of acquisition, metric and interpretative use of all types of imagery, the conventional as well as the nonconventional, for the photogrammetric accomplishment of surveys.

The range of Division interests, activities, and responsibilities is divided into several areas each of which is entrusted to a Committee. The total scope and structure of the Photogrammetric Surveys Division include its Committees, and each Committee is guided by the purpose of the Division as applicable to its particular responsibilities.

Cadastral Surveys Committee. Accomplishing selected tasks within the broad scope of cadastral surveying by photogrammetric methods.

Charting Committee. Constructing or compiling charts by employment of photogrammetric principles, techniques, and procedures, and stimulating the use of charts by new applications.

Close-Range Photogrammetry Committee. Applying close-range photogrammetry in such areas as architecture, bio-medicine, engineering, law enforcement, etc.; stimulating new applications of close-range photogrammetry and assuring that proper attention is given to calibration and evaluation of geometric quality of close-range photogrammetric systems.

Computational Photogrammetry Committee. Applying computational and digital techniques in all photogrammetric and associated surveys.

Extraterrestrial Applications Committee. Using advances in space technology for accomplishing surveys by photogrammetric methods of the earth and other celestial bodies.

Mapping Committee. Applying photogrammetric and cartographic principles to achieve acceptable standards for all forms of mapping the natural and cultural features on the earth's surface, and to stimulate the development and use of photogrammetric methods of map compilation for all users of maps.

Standards Committee. Working closely with the Standards Committee of the American Society of Photogrammetry, and providing a member and an alternate member from the Standards Committee of the Division to serve as a member of the Standards Committee of ASP. Compiling standards applicable to the interests and work of all Committees within the Division and presenting them to the Standards Committee of ASP for its consideration and adoption for use by all concerned.

Transportation and Engineering Surveys Committee. Utilizing photogrammetry in making surveys for transportation and all other types of engineering works and facilities, including surveys for their reconnaissance, location, and design, for obtaining their right-of-way, and for their construction, maintenance, improvement, and reconstruction wherever necessary.

Functions

Each committee and subcommittee of the Division will perform the following functions:

- Maintain a current roster of the individuals and organizations engaged in activities of its field;
- Recruit authors for preparation of technical papers and organize, when appropriate, technical programs for the annual and regional conventions of ASP;
- Organize and conduct special symposia, when appropriate, and/or cooperate with other organizations in promoting and conducting symposia;
- Solicit or otherwise encourage the submission of technical papers for publication in the ASP Journal;
- Serve as an aid to the Review Board of the staff of *Photogrammetric Engineering* in judging papers relative to their suitability for publication;
- Encourage and guide educational and research institutions in the field of mutual interest;
- Promote correlation of effort with other committees of the Society and with other societies on matters of mutual interest; and
- Develop and appropriately define standards, as deemed necessary, for better functioning and coordination of the efforts and work of specialists in each of their fields of endeavor and areas of mutual interest.