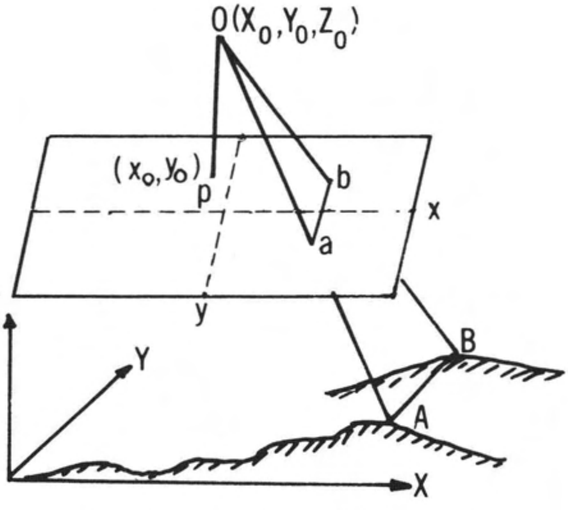

FIG. 1. Geometry of an aerial photograph.

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Optimum Ground Control for Camera Calibration

(Abstract on next page)

How to overcome the limitations presented by flat terrain.

INTRODUCTION

THE PROBLEM of camera calibration over nearly flat terrain is well known. This problem arises due to strong correlation between the exposure station coordinates X_o, Y_o, Z_o (Figure 1) and the calibration elements x_o, y_o, c. Consider a vertical photograph of nearly flat terrain with the ground and photo axes as shown in the figure. The relationship between the linear elements of inner and exterior orientation is given by:

$$\frac{X_A - X_o}{x_a - x_o} = \frac{Z_A - Z_o}{c} \quad (1)$$

$$\frac{Y_A - Y_o}{y_a - y_o} = \frac{Z_A - Z_o}{c} \quad (2)$$

$$\frac{\partial X_o}{\partial x_o} = \frac{Z_A - Z_o}{c} \quad (3)$$

$$\frac{\partial Y_o}{\partial y_o} = \frac{Z_A - Z_o}{c} \quad (4)$$

$$\frac{\partial Z_o}{\partial c} = \frac{-(Z_A - Z_o)}{c}. \quad (5)$$

With perfectly flat terrain we get

$$\frac{Z_A - Z_o}{c} = \frac{H}{c}$$

where H is the flying height above the terrain and therefore we have

$$\frac{\partial X_o}{\partial x_o} = \frac{\partial Y_o}{\partial y_o} = K, \frac{\partial Z_o}{\partial c} = -K$$

with $K = H/c$ being a constant. Because the differentials of X_o, Y_o, Z_o become a scaler times the differentials in x_o, y_o, c we get into the obvious difficulty of inverting a singular matrix **N** if we try to determine the parameters x_o, y_o, c over flat or nearly flat terrain. So long as we use the well-known projective equations of the type

$$x - x_o = cp/r \quad (6)$$

$$y - y_o = cq/r \quad (7)$$

where

$$p = a_{11}(X-X_o) + a_{12}(Y-Y_o) + a_{13}(Z-Z_o)$$

$$q = a_{21}(X-X_o) + a_{22}(Y-Y_o) + a_{23}(Z-Z_o)$$

$$r = a_{31}(X-X_o) + a_{32}(Y-Y_o) + a_{33}(Z-Z_o)$$

the only way to break the singularity of **N** matrix is either to use oblique photography or calibrate over mountaneous terrain thereby making the partial derivatives $\partial X_o/\partial x_o$, $\partial Y_o/\partial y_o$, $\partial Z_o/\partial z_o$ functions of image coordinates x, y and no longer a constant K. The former approach is that of D. Brown[1] and the latter of Merchant[2] using a mixed model of flat and mountainous terrain.

An attempt is made in this paper to show how the strong correlation between X_o, Y_o, Z_o and x_o, y_o, c can be removed by a suitable selection of ground control. The problem may be stated as follows: Given a ground control point P_1 with coordinates X_1, Y_1, Z_1, where should the second ground point P_2 be so as to make

$$\partial X_o/\partial x_o = \partial Y_o/\partial y_o = \partial Z_o/\partial z_o = O?$$

ABSTRACT: *Camera calibration over nearly flat terrain presents difficulties. A new mathematical model is derived to obtain constraints on the ground control in order to overcome the strong correlation between the exposure station coordinates and calibration elements. A numerical example illustrates the use of simulated optimum ground control location.*

TWO-POINT PROJECTIVE EQUATION

In order to solve the above-mentioned problem we seek a projective equation connecting two ground coordinates and their photo coordinates. Projective Equations 6 and 7 are of no help here as they relate to a single point.

Referring to Figure 1 and using the collinearity condition, we can imagine point a and A to be connected rigidly by a straight rod hinged at O, the exposure station. Let the space rod move an element of distance dS on the ground so that the corresponding distance on the photo is ds. The relationship between dS and ds is given by:

$$dS = \left[\frac{(X-X_o)^2 + (Y-Y_o)^2 + (Z-Z_o)^2}{(x-x_o)^2 + (y-y_o)^2 + c^2}\right]^{1/2} ds \quad (8)$$

$$\frac{dS}{\left[(X-X_o)^2 + (Y-Y_o)^2 + (Z-Z_o)^2\right]^{1/2}} = \frac{ds}{\left[(x-x_o)^2 + (y-y_o)^2 + c^2\right]^{1/2}} \cdot \quad (9)$$

Integrating both sides we get:

$$\int_A^B \frac{dS}{\left[(X-X_o)^2 + (Y-Y_o)^2 + (Z-Z_o)^2\right]^{1/2}} = \int_a^b \frac{ds}{\left[(x-x_o)^2 + (y-y_o)^2 + c^2\right]^{1/2}} \cdot \quad (10)$$

To perform the integration on the left side we use the equation of a straight line in three dimensions and that of on the right the equation in two dimensions, i.e.,

$$\frac{X-X_A}{X_B-X_A} = \frac{Y-Y_A}{Y_B-Y_A} = \frac{Z-Z_A}{Z_B-Z_A} = T \quad (11)$$

where T is a variable whose value lies between zero and one.

Making use of Equation 11 in Equation 10, and using the following notations,

$$X_B-X_A = \Delta X \qquad X_A-X_o = \Delta X_1$$
$$Y_B-Y_A = \Delta Y \qquad Y_A-Y_o = \Delta Y_1$$
$$Z_B-Z_A = \Delta Z \qquad Z_A-Z_o = \Delta Z_1$$

we get

$$\int_A^B \frac{dS}{\left[(X-X_o)^2 + (Y-Y_o)^2 + (Z-Z_o)^2\right]^{1/2}} = ln\left(\frac{1 + D_3 + D_5}{D_6}\right) \quad (12)$$

with

$$D^2 = \Delta X^2 + \Delta Y^2 + \Delta Z^2$$
$$D_2^2 = \Delta X_1^2 + \Delta Y_1^2 + \Delta Z_1^2$$
$$D_1^2 = \Delta X \Delta X_1 + \Delta Y \Delta Y_1 + \Delta Z \Delta Z_1$$
$$D_3 = D_1^2/D^2$$
$$D_4 = D_2/D\,,\ D_5 = (1 + 2D_3 + D_4^2)^{1/2}\,,$$
$$D_6 = D_3 + D_4\,.$$

Similarly we have:

$$\int_a^b \frac{ds}{[(x-x_o)^2 + (y-y_o)^2 + c^2]^{1/2}} = ln\left(\frac{1 + d_3 + d_5}{d_6}\right) \tag{13}$$

where

$$\Delta x = x_b - x_a,\ \Delta_y = y_b - y_a,\ \Delta x_1 = x_1 - x_o,$$
$$\Delta y_1 = y_1 - y_o$$
$$d^2 = \Delta x^2 + \Delta y^2,\ d_2^2 = \Delta x_1^2 + \Delta y_1^2 + c^2$$
$$d_1^2 = \Delta x \Delta x_1 + \Delta y \Delta y_1,\ d_3 = d_1^2/d^2$$
$$d_4 = d_2/d,\ d_5 = (1 + 2d_3 + d_4^2)^{1/2},\ d_6 = d_3 + d_4.$$

From Equations 10, 12 and 13 we get

$$\frac{1 + D_3 + D_5}{D_6} = \frac{1 + d_3 + d_5}{d_6}\,. \tag{14}$$

In the functional notation we can write Equation 14 as

$$F = (1 + D_3 + D_5)\,d_6 - (1 + d_3 + d_5)\,D_6 = 0\,. \tag{15}$$

This, then, is the two-point projective equation containing the ground and photo coordinates of two points A and B and also the exposure station coordinates X_o, Y_o, Z_o and the calibration elements x_o, y_o, c. The equation is obviously independent of the rotations, κ, ϕ and ω elements.

Conditions for Optimum Control Distribution

In order to study the effect of changes in exposure station coordinates on the changes of calibration elements x_o, y_o, c we have to determine $\partial X_o/\partial x_o$, $\partial Y_o/\partial y_o$, $\partial Z_o/\partial z_o$, and to break the correlation between X_o, Y_o, Z_o and x_o, y_o, c we must set these partial differentials equal to zero. It can be shown that using Equation 15 we get:

$$\frac{\partial X_o}{\partial x_o} = \left(\frac{D}{d}\right)^2\left(\frac{D_6}{d_6}\right)^2\left[\frac{(1+d_3+d_5)\,(\Delta x + \Delta x_1 \cdot \frac{d}{d_2}) - {}^{d_6}/{}_{d_5}\,\{\Delta x_1 + \Delta x(1+d_5)\}}{(1+D_3+D_5)\,(\Delta X + \Delta X_1 \cdot {}^{D}/{}_{D_2}) - {}^{D_6}/{}_{D_5}\,\{\Delta X_1 + \Delta X(1+D_5)\}}\right] \tag{16}$$

$$\frac{\partial Y_o}{\partial y_o} = \left(\frac{D}{d}\right)^2\left(\frac{D_6}{d_6}\right)^2\left[\frac{(1+d_3+d_5)\,(\Delta y + \Delta y_1 \cdot \frac{d}{d_2}) - {}^{d_6}/{}_{d_5}\,\{\Delta y_1 + \Delta y(1+d_5)\}}{(1+D_3+D_5)\,(\Delta Y + \Delta Y_1 \cdot \frac{D}{D_2}) - {}^{D_6}/{}_{D_5}\,\{\Delta Y_1 + \Delta Y(1+D_5)\}}\right] \tag{17}$$

$$\frac{\partial Z_o}{\partial c} = c\left(\frac{D}{d}\right)^2\left(\frac{D_6}{d_6}\right)^2\left[\frac{{}^{d_6}/{}_{d_5} - \frac{d}{d_2}\,(1+d_3+d_5)}{(1+D_3+D_5)\,(\Delta Z + \Delta Z_1 \cdot \frac{D}{D_2}) - {}^{D_6}/{}_{D_5}\{\Delta Z_1 + Z\,(1+D_5)\}}\right] \tag{18}$$

D_6 seems to play an important role in making the partials vanish. Setting $D_6 = 0$ we get

$$D_6 = D_3 + D_4 = \frac{\Delta X \Delta X_1 + \Delta Y \Delta Y_1 + \Delta Z \Delta Z_1}{\Delta X^2 + \Delta Y^2 + \Delta Z^2} + \left[\frac{\Delta X_1^2 + \Delta Y_1^2 + \Delta Z_1^2}{\Delta X^2 + \Delta Y^2 + \Delta Z^2}\right]^{1/2} = 0\,. \tag{19}$$

Equation 19 reduces to:

$$\Delta Z^2(\Delta X_1^2 + \Delta Y_1^2) - 2\,\Delta Z\,\Delta Z_1(\Delta X\,\Delta X_1 + \Delta Y\,\Delta Y_1) + \Delta Z_1^2(\Delta X^2 + \Delta Y^2) - (\Delta X\,\Delta Y_1 - \Delta X_1\,\Delta Y)^2 = 0\,. \tag{20}$$

Equation 20 can be written as:

$$\Delta Z^2(\Delta X_1^2 + \Delta Y_1^2 + \Delta Z_1^2) - 2\,\Delta Z\,\Delta Z_1(\Delta X\,\Delta X_1 + \Delta Y\,\Delta Y_1 + \Delta Z\,\Delta Z_1) + \Delta Z_1^2(\Delta X^2 + \Delta Y^2 + \Delta Z^2) - (\Delta X\,\Delta Y_1 - \Delta X_1\,\Delta Y)^2 = 0\,. \tag{21}$$

If we let

$$\Delta X \Delta Y_1 - \Delta X_1 \Delta Y = 0 \qquad (22)$$

we get

$$D_2 \cdot \Delta Z - D \cdot \Delta Z_1 = 0 \, . \qquad (23)$$

Equations 22 and 23 can be written as:

$$\frac{\Delta X}{\Delta Y} = \frac{\Delta X_1}{\Delta Y_1}$$

$$\frac{\Delta Z}{\Delta Z_1} = \frac{D}{D_2} \qquad (24)$$

Making use of Equation 19, we get the following simple relations:

$$\Delta X = \frac{\Delta X_1}{\Delta Z_1} \, \Delta Z \qquad (25)$$

$$\Delta Y = \frac{\Delta Y_1}{\Delta Z_1} \, \Delta Z \qquad (26)$$

$$\Delta Z = \left(\frac{\Delta X^2 + \Delta Y^2}{\Delta X_1{}^2 + \Delta Y_1{}^2} \right)^{1/2} \Delta Z_1 \, . \qquad (27)$$

The geometrical interpretation of Equations 25 to 27 is shown in Figure 2. It simply means that, given a control point A with coordinates X_A, Y_A, Z_A the second control point B must lie on the line joining the exposure station 0 and the ground plot A. Thus Equations 25 to 27 can be used to locate B if A is given. This, then, is the optimum location of the ground control which will make the partial derivatives $\partial X_o/\partial x_o$, $\partial Y_o/\partial y_o$, and $\partial Z_o/\partial z_o$ and vanish, and hence make the determination of x_o, y_o, c independent of X_o, Y_o, Z_o. This location of point B in fact solves for the exposure station coordinates X_o, Y_o, Z_o so that once they are known independently they have no more correlation with x_o, y_o, and c. The effect in the solution of the calibration problem is same as if X_o, Y_o, Z_o had been observed independently.

Numerical Example

An example illustrates the situation with the data of Photo No. 67 of Casa-Grande Range. Table 1 gives the ground and photo data. Using single-photo resection and using Equations 6 and 7, the following values were obtained by a least-squares adjustment using Dept. of Geodetic Science computer program RESEC 1.

$$\kappa = 3.08310 \text{ radians}, \varphi = 0.02409 \text{ radians},$$
$$\omega = 0.01719 \text{ radians}$$

$$X_o = 90.6221 \text{ meters}, Y_o = 89.4537 \text{ meters}$$
$$Z_o = 5559.9082 \text{ meters}.$$

The values used for initial calibration elements were:

$$x_o = 0.0, \, y = 0.0, \, c = -152.01 \text{ mm}.$$

Figure 3 shows the distribution of ground control. Table 2 gives the values of the partial derivatives using Equations 3 to 5 and Table 3 shows the values of the partial derivatives with the second control point generated with the contraints as defined by Equations 25 to 27 for each of the ground points except for Point No. 4 which is very close to the expos-

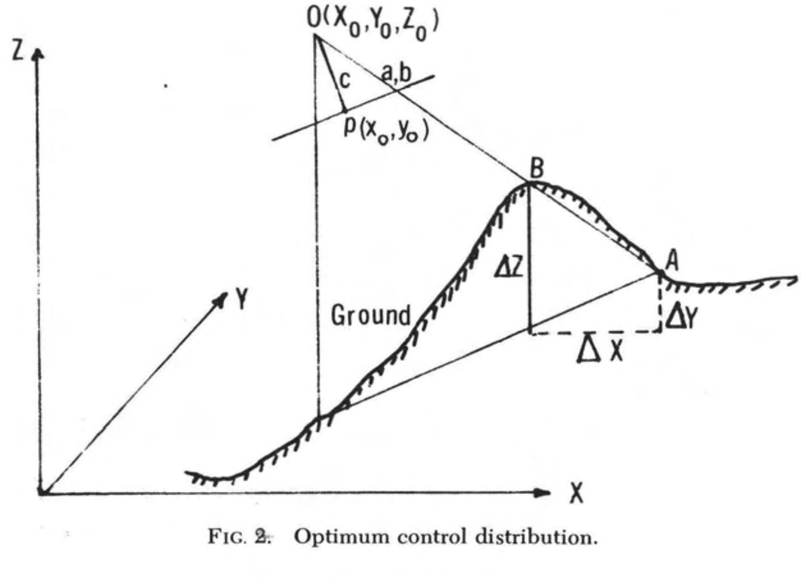

FIG. 2. Optimum control distribution.

TABLE 1.

Point No.	Name	X(m)	Y(m)	Z(m)	x(mm)	y(mm)
1	AK-46	− 78.791	−1630.666	1015.161	− 1.498	60.519
2	AL-46	1609.440	*−1636.380*	*1017.219*	−58.770	57.933
3	AJ-45	−1629.679	60.520	1009.826	53.044	6.662
4	AK-45	0.0	0.0	1012.783	− 0.953	5.558
5	AJ-44	−1562.837	1612.218	1007.535	53.492	− 44.501
6	AL-43	1598.843	3230.481	1007.641	−47.825	−104.879

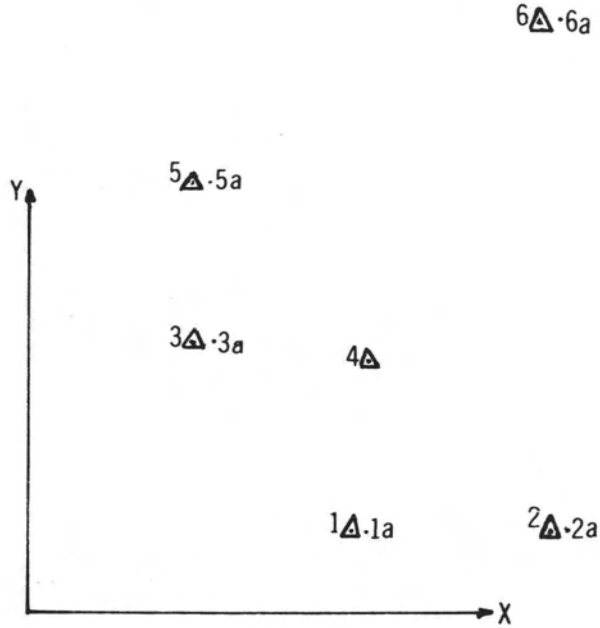

FIG. 3. The Casa Grande range control.

TABLE 2.

Point No.	$\frac{\partial X_o}{\partial x_o}$	$\frac{\partial Y_o}{\partial y_o}$	$\frac{\partial Z_o}{\partial c}$
1	29,897.687	29,897.687	−29,897.687
2	29,884.148	29,884.148	−29,884.148
3	29,932.783	29,932.783	−29,932.783
4	29,913.330	29,913.330	−29,913.330
5	29,947.855	29,947.855	−29,947.855
6	29,947.157	29,947.157	−29,947.157

TABLE 3.

Point No.	$\frac{\partial X_o}{\partial x_o}$	$\frac{\partial Y_o}{\partial y_o}$	$\frac{\partial Z_o}{\partial c}$
1 and 2	39,397.972	30,267.165	29,438.625
3 and 4	25,800.380	90,756.062	29,965.446
5 and 6	33,617.782	29,225.889	30,643.835

TABLE 4.

Point No.	$\frac{\partial X_o}{\partial x_o} \times 10^{-7}$	$\frac{\partial Y_o}{\partial y_o} \times 10^{-7}$	$\frac{\partial Z_o}{\partial c} \times 10^{-7}$
1 and 1a (simulated)	0.31967247	0.047802818	−0.31735248
2 and 2a (simulated)	1.4987150	1.5857954	−5.6320685
3 and 3a (simulated)	0.066108212	0.061397500	−0.50224209
5 and 5a (simulated)	1.6577006	0.7108945	−4.6541826
6 and 6a (simulated)	0.0	0.0	0.0

ure station and therefore would require a very large ΔZ. The photo coordinates were simulated with $\Delta Z = 15$ m and with the ground coordinates of the simulated point selected so as not to place the simulated point exactly on the exposure station and the ground point line $0A$ as shown in Figure 2.

It is clear from the numerical values in Table 4 that the partial derivatives become extremely small. In the ideal situation where the constraints of Equations 25 to 27 are fully satisfied, we would have both the points A and B imagined at a single point and, under these conditions, the partial derivatives vanish completely. This can also be seen mathematically. Under these conditions the integeral on the right hand side of Equation 10 is zero because $ds = 0$ and the integeral on the left reduces to $(1+D_3)/D_3$ as $D_4^2 = D_3^2$. Hence using Equation 15 we get

$$\frac{\partial F}{\partial X_o} = \frac{1}{D_3^2 D^2} \Delta X$$

and $\partial F/\partial x_o = 0$ so that

$$\frac{\partial X_o}{\partial x_o} = \frac{\partial F}{\partial x_o} \Big/ \frac{\partial F}{\partial X_o} = 0 \ .$$

Similarly we have

$$\frac{\partial F}{\partial Y_o} = \frac{\Delta Y}{D_3^2 D^2}, \ \frac{\partial F}{\partial y_o} = 0$$

so that $\partial Y_o/\partial y_o = 0$ and also $\partial Z_o/\partial_c = 0$.

Conclusions

- The difficulties of camera calibration over nearly flat terrain can be minimized by the optimization of ground control locations. If necessary, signals of desired height and location as obtained from the Equations 25 to 27 can be erected in the vicinity of an already existing ground control.
- For points slightly off the ideal location, the mathematical model used for the solution of the calibration problem may have to be modified as the well-known projective equations of a single point do not help. One such model for the two points and the corresponding partial derivatives has been obtained as given in Equations 15 to 18.
- The numerical example above has been studied for simulated points. For further investigation a designed flight is necessary.

Acknowledgments

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(Continued from page 100)

The company also markets under its SPECTRAL DATA trademark:

- ★ SPECTRAL electronic density slicing equipment for analysis of film images.
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