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Ophthalmologic Stereophotography*

Geometric problems encountered in topographic mapping of a small area on the retina of the human eye.

Abstract: *Stereophotographs taken by ophthalmologic instruments do not meet basic photogrammetric requirements; their geometry is atypical and cannot be assessed by standard calibration procedures. This analysis of two basic categories of instruments shows the peculiarities of the single-image and stereoimage formations. Although parallax-type instruments and analog plotters can mainly provide approximate solutions, analytical methods (and especially analytical plotters) offer a more rigorous and reliable photogrammetric treatment. This includes the reconstruction of a model from parallel or nearly parallel projections which are typical for some ophthalmologic instruments. The basic theory of these unconventional procedures is illustrated by an example.*

Introduction

Stereophotogrammetric reconstruction of three-dimensional models usually requires the use of special metric cameras calibrated to provide all the necessaryinformation on the geometry of images. In some non-cartographic applications, however, this approach is not always feasible and cameras of a non-metric type are often accepted if the basic accuracy requirements are not too stringent, and if suitable analytical solutions ensure additional calibration of the individual photographs. This situation becomes more complicated if photographs of small objects cannot be taken directly but only from secondary optical images formed and relayed by special optical systems. This is particularly true in ophthalmology where stereophotographs are obtained with the use of standard or only slightly modified ophthalmologic photo-optical systems, such as a photo slit lamp, fundus camera, etc. The imaging geometry of these instruments is atypical from a photogrammetric point of view and can not be assessed with the use of regular calibration procedures. Fundamental difficulties arise from the fact that the human eye is a part of the imaging system.

Photogrammetric methods are used in ophthalmology mainly to study the optic disc topography and to detect, record and follow-up three-dimensional changes associated with eye diseases and visual disorders. Holm[6,7] bases his studies on the use of single photographs containing multiple slit projections on the optic disc. Most other investigators prefer stereophotographs and introduce the photogrammetric base by a plane-parallel plate (Allen,[1] Crock,[3,4] Jönsas,[10] Kottler[11]) or with the aid of a prismatic beam splitter (Bynke,[2] Saheb and Drance,[13] Schirmer[15]). The latter arrangement ensures simultaneous photography. The photogrammetric techniques used for the reconstruction of models range from simple parallax methods (Bynke,[2] Schirmer[15]) through an extensive use of analog plotters (Crock,[3,4] Jönsas,[10] Saheb and Drance[13]) to digital processing of scanned images in a computer (Kottler).[11] However, it should be pointed out

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that most of the reported investigations do not utilize procedures for a precise photogrammetric calibration. Consequently, they suffer from the lack of information on the image geometry and provide only approximate photogrammetric models.

The present article attempts to fill this gap and analyzes the geometry of ophthalmologic stereoimages. It is shown that their interior orientation is very unstable and sensitive even to very small, inevitable changes in practical conditions of taking the pictures. In some instances the effective projection center is transferred to infinity and perspective bundles of rays used in standard photogrammetric procedures must be replaced by parallel beams of rays. As this is not feasible with analog instruments, analytical photogrammetric solutions should be used. Basic features of analytical formulations suitable for this purpose, are discussed. The analytical plotter is considered as an instrument providing the most versatile and reliable solution for ophthalmologic applications.

Photo-Optical Systems in Ophthalmology

From an optical point of view, basically two types of photo-optical instruments are used in ophthalmology:

- Microscope-type instruments which can render close-up images of external or anterior portions of the eye (cornea, sclera, iris, lens), and
- Telescope-type instruments used for the examination of the eye fundus (retina, disc, macula etc.).

The first category is typically represented by a slit lamp which is a microscope combined with an independent illumination unit projecting a luminous slit of variable width at different angles onto and into the eye. Modern slit lamps are built as stereomicroscopes with a convergence angle of about 13°. A photorecording system is built-in, or standard cameras can be attached to the oculars. A slit lamp is often used in combination with a preset or contact lens of a negative power necessary to eliminate the refractive effect of the eye proper. This arrangement makes it possible to examine the eye fundus in a way similar to telescope-type instruments.

Retinal cameras and fundus cameras belong to the second group of instruments suitable for the examination of the rear portions of the eye. The photorecording unit is built-in and can be used during the observation. These cameras can be refocused in a range sufficient to compensate the refractive error of the examined eye. The illumination can be either separated or combined with the optical imaging system.

Some other general purpose medical cameras, such as Kowa RC-2, can be used for ophthalmologic purposes in both microscopic and telescopic mode.

In order to obtain good photographic images, the eye must be kept wide open and immobile. With the aid of a dilating agent, the eye pupil is dilated to a diameter of 7 to 8 mm, and a special chin rest and forehead support help in steadying the patient's head. A luminous fixation target ensures directional stability of the eye and helps to eliminate its accommodation.

Geometry of Opthalmologic Images

Photographs of the external parts of the eye represent a close-up of the object and, from the point of view of geometry, can be treated in a way similar to standard photogrammetric pictures. With reference to the generally adopted theory of W. Roos[12] the position of the effective projection center in the object space coincides with the center of the entrance pupil of the imaging system. The bundle of rays to be simulated in photogrammetric reconstructions is then identical with the bundle of chief rays passing through this center. This is justified for two main reasons:

- The chief rays are assured of passing through the optical system even if its aperture is decreased;
- Each point of a fixed image plane represents a locus for the center-of-confusion circles corresponding to all points located on a single chief ray in the object space.

The corresponding bundle of rays in the image space is analytically defined by the parameters of interior orientation. They can be derived using standard calibration procedures well-established in close-range photogrammetry.

Photography of the fundus is peculiar because portions of the eye participate in the process of forming the photographic images. Theoretically, it is impossible to define the geometry of the imaging process in a rigorous way because of natural biological variations in the eye build-up of different patients, as well as because of the inherent biological instability of any live tissue. Any metric analysis and simulation of the optical function of the eye must inevitably be based on simplifications and, therefore, represent an approximation of reality. In general, one neglects certain irregularities and all optical aberrations of an individual eye by substituting an ideal, the so-called *Gullstrand's simplified eye*. Considering the generally limited

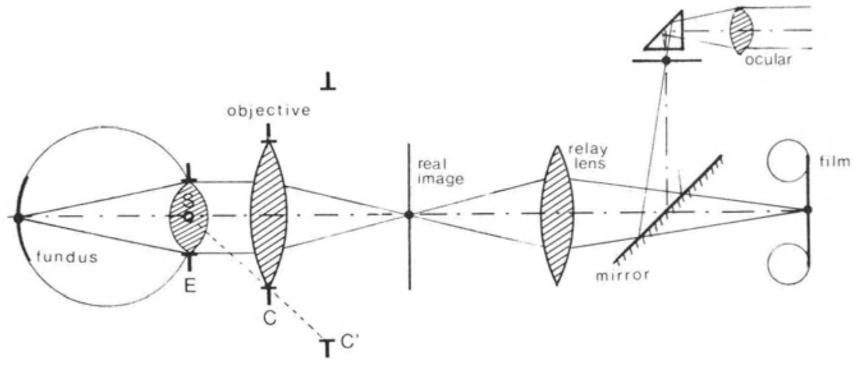

FIGURE 1

metric requirements in biomedical applications, the simplified approach is justified.

The inclusion of the eye in the photo-imaging system means that one has to deal with two-media photogrammetry. The eye fundus is the optical object and vitreous humor is the first refractive medium with an index of $n = 1.336$. The optical sections of the eye and the optical elements of ophthalmologic instruments must form a real image in the plane of the photographic film. Obviously, the second medium is air with the index $n' = 1$. The refractive power of the eye, which is 58.64 diopters, defines its primary focal distance as $f = 22.785$ mm for the object space and the secondary focal distance $f' = 17.055$ mm in the image space (cf. Havelka).[5] The differences in the media inside and outside the eye, unless compensated, cause an affine distortion of reconstructed bundles. The photogrammetric model appears to be compressed and an appropriate correction factor $n = 1.336$ should be applied in the course of photogrammetric processing.

EFFECTIVE PROJECTION CENTER OF A SINGLE PHOTOGRAPH

Fundus Camera. Single photographs can be taken by a fundus camera with its optical axis going through the center of the eye pupil. For an emmetropic eye the camera functions like a telescope, the optical rays between the eye and camera forming a parallel beam as shown in Figure 1.

For an eye with an abnormal refraction the fundus camera must be refocused. The effective projection center in the object space is a function of the eye-camera combination. The location of the center depends on the size and relative position of the entrance pupils E and C for the individual systems. If one derives C' as a virtual image of the pupil C obtained through the eye, then the smaller of the stops E and C' (as viewed from the object position [fundus]) defines the resulting entrance pupil. Because the pupil of the eye is smaller than the aperture of the camera the object projection center S coincides with the center of the former.

Photo Slit Lamp. The use of a preset or contact lens with strong negative power eliminates the refractive power of the eye, and the resulting eye-lens combination becomes afocal ($f = \infty$). In effect, the eye fundus G is optically transformed into a virtual image G' observed by the slit lamp system as if in the air medium. This is illustrated in Figure 2. The afocal combination works as a front stop for the slit lamp so defining the effective entrance pupil and object projection center, assuming that the objective is wider than the eye pupil.

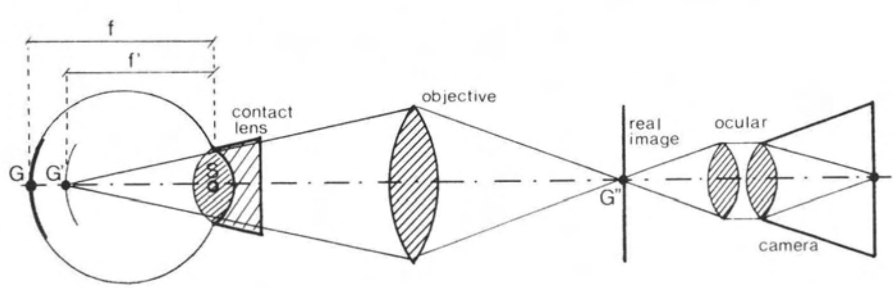

FIGURE 2

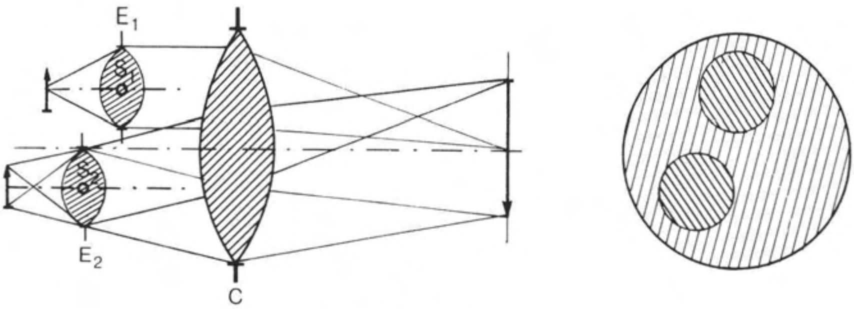

FIGURE 3

EFFECTIVE PROJECTION CENTERS FOR STEREO-PHOTOGRAPHS

Three practical ways are available to take stereophotographs of the eye fundus:

- Consecutive stereoseparation by a parallel shift,
- Simultaneous separation by a beam splitting prism, and
- Simultaneous separation by convergent photography.

The parallel shift of imaging optical rays is introduced with the aid of a tiltable plane-parallel plate unit such as the Allen stereoseparator. This system is used in combination with a fundus or retinal camera. Full-size pictures are obtained, maintaining unchanged orientation of the camera. In view of the limited reliability of the eye fixation, the consecutive mode of the operation may indirectly cause a disturbance in the relative orientation of the photographs.

Prismatic beam splitters are usually constructed as a result of off-shelf modifications and local improvements. Either single or twin prisms are attached in front of the fundus camera to obtain simultaneously a double image in two overlapping fields of a single frame. The angle of prisms used for this purpose is approximately 6 or 7°.

Simultaneous convergent photographs are produced by the stereomicroscope of a slit lamp. The convergence angle is fixed between 8 and 15°.

Stereoseparation by Parallel Shift. The effect of shifting a beam of optical rays is equivalent to a displacement of the camera. Such a change results in a forced optical decentration which will show up as a disturbance of the projection geometry. In optically centered systems the axes of individual elements coincide and the entrance and exit pupils are always uniquely defined by a single aperture stop of the system. It is obvious that in a system which becomes decentered, the unique pupil definition may be disturbed. As a result of the decentration, two different and well-separated stops can share the important function of limiting optical rays, which controls not only the free passage of light, brightness of the image and the field of view but, as will be shown, also the basic geometry of imaging.

It is important to realize that not every decentration automatically changes the geometry; depending on the conditions it may not always be critical. For instance, if the entrance pupil of a fundus camera is considerably larger than the eye pupil (see Figure 3), a lateral displacement of the camera does not cause, within a certain range, any change of the image formed by the objective in its focal plane. In this instance, one can not obtain the desirable stereoeffect even though the introduced base seems to be adequate. The stereoseparation is successful only if the decentration brings about a new definition

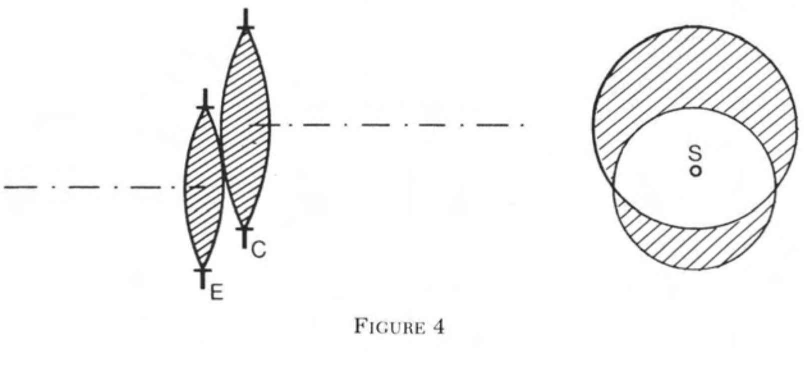

FIGURE 4

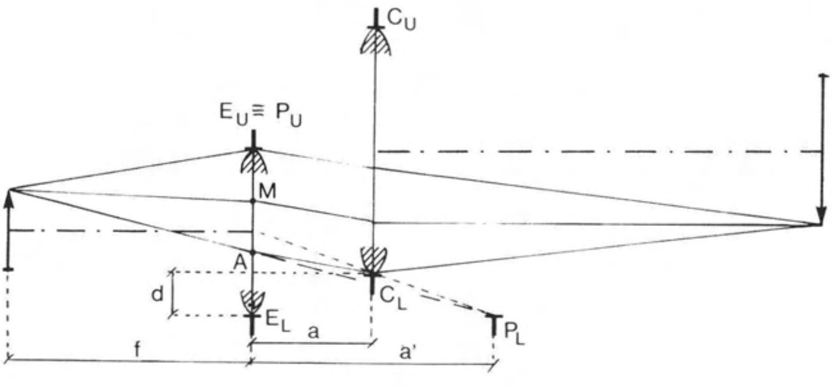

FIGURE 5

for the entrance pupil of the composite system. Obviously, this is not the situation in the previous example.

In an ideal application of two thin lenses which are in close contact (as shown in Figure 4) the resulting pupil has a form of a meniscus determined by the peripheries of both lenses with the chief ray passing through the centroid of the overlapping areas.

For a combination of thick lenses with an appreciable separation one must consider the relative position of both entrance pupils in the direction across, as well as along the optical axes. This situation is illustrated in Figure 5 where the symbols E_L, E_U, C_L, C_U designate the edges of the entrance pupils for the eye and objective of a fundus camera, respectively. The lower edges are mutually separated by the displacements d, a. Point C_L if projected into the object space through the eye, defines the virtual lower edge P_L of the effective entrance pupil for the combination of both lenses. Using the Gaussian lens formula one can derive the images distance a' for the position P_L and the lateral magnification m associated with the projection:

$$a' = \frac{fa}{f-a} \tag{1}$$

$$m = \frac{a'}{a} = \frac{f}{f-a}. \tag{2}$$

The symbol f denotes the focal length of the eye. The upper part of optical rays entering the system from the object space is limited by the edge $E_U \equiv P_U$ of the eye pupil, whereas the lower boundary of imaging rays coincides with the ray initially aimed at the virtual point P_L, but being optically refracted by the eye at A to pass through the real edge C_L.

Both edges P_U, P_L should define the object space position of the effective projection centre as a function of the parameters d, a. It is obvious that the bundle limiting edge A in the plane ϵ (Figures 5 and 6) moves depending on the position of the corresponding object point G. The chief ray of the limited cone of imaging rays passes midway between the points A and P_U through the point M which is also moving, with half the speed of A. As the changes in positions of the points G, A, M are linearly interdependent, the chief rays G_iM_i must all intersect at a single point S as shown in Figure 6. Consequently, this is the point that represents the effective projection center in the object space. Its distance s from the plane ϵ can be determined from the following relations:

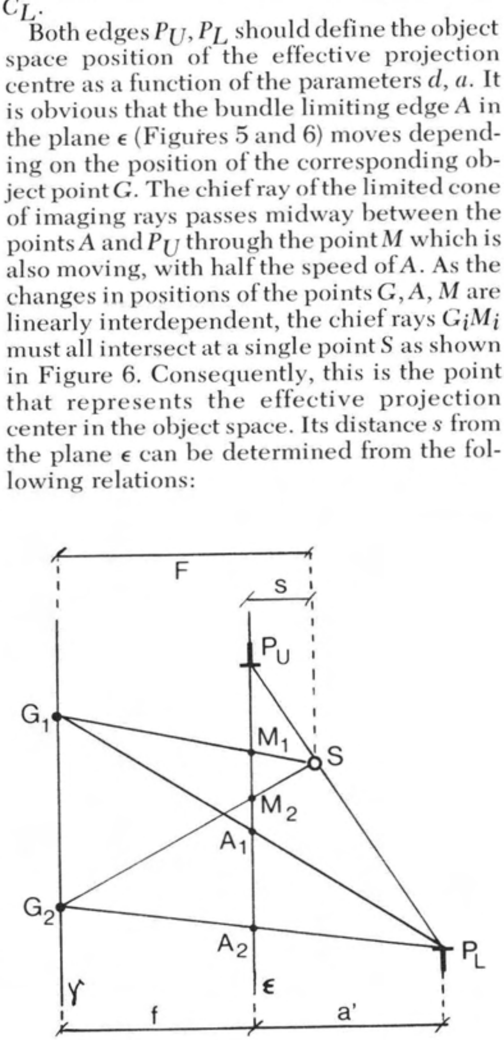

FIGURE 6

$$\overline{2M_1M_2} = \overline{A_1A_2}$$

$$\overline{G_1G_2} = \frac{f+a'}{a'}\,\overline{A_1A_2} = \frac{f+s}{s}\,\overline{M_1M_2}$$

yielding

$$\frac{f+s}{s} = 2\,\frac{f+a'}{a'}.$$

With the use of substitution a' from Equation 1 and, after formal rearrangements, one obtains

$$\frac{f+s}{s} = \frac{f}{s} + 1 = 2\,\frac{f}{a}$$

and eventually

$$s = \frac{fa}{2f - a}. \tag{3}$$

For the object distance F from the projection center S it follows

$$F = f + s = \frac{2f^2}{2f - a}. \tag{4}$$

In accordance with Figure 7 the projection center S is laterally offset from the optical axis of the eye by the distance r. Its magnitude is derived from the relations

$$r = 0.5D - t \text{ and } t = s \tan \alpha$$

where

$$\tan \alpha = [0.5D + m\,(0.5D - d)\,]/a'.$$

With the use of substitutions of Equations 2 and 3, we have first

$$t = \frac{Df - df - 0.5Da}{2f - a}$$

and finally

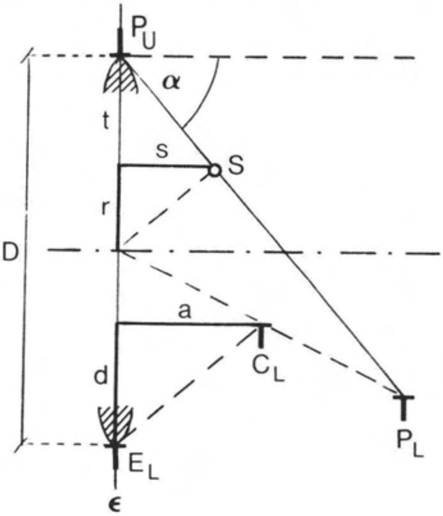

FIGURE 7

$$r = \frac{fd}{2f - a}. \tag{5}$$

Values s, r determine the position of the projection center S in the object space with respect to the center of the eye entrance pupil. The position of the geometrically corresponding projection center (S) in the image space must be reconstructed from the photographic image with the use of angles defined in the object projection center S. This will determine the positions of the center (S) and of the principal image point H' as illustrated in Figure 8. It should be emphasized that point (S) is not identical with S' optically conjugated to S. The positions (S) and H' refer to the first real image formed by the objective and any further image relay will introduce a linear change in these parameters.

In analytical terms, one derives for the effective principal distance of the image,

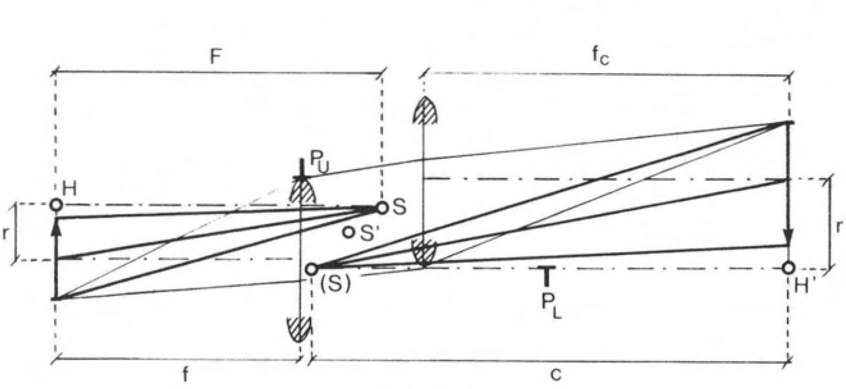

FIGURE 8

$$c = F\,\frac{f_c}{f} = \frac{2f\, f_c}{2f - a} \tag{6}$$

and for the displacement of the effective principal point H' in the image,

$$r' = \frac{f_c}{f}\, r = \frac{f_c d}{2f - a}\,. \tag{7}$$

Displacement r as given by Equation 5 defines a change necessary for the introduction of a photogrammetric base by two consecutive decentrations d_1, d_2 in opposite directions. The pertaining difference Δd determines the base

$$b = \Delta r = \frac{f \Delta d}{2f - a}\,.$$

One can now estimate the stereometric power of the system by computing the base-distance ratio,

$$\theta = \frac{b}{F} = \frac{\Delta d}{2f}\,. \tag{9}$$

If the diameter of the entrance pupil of the fundus camera is larger than that of the eye by a value of ΔD, the required decentration ΔC must include this amount to yield the effective change Δd:

$$\Delta C = \Delta d + \Delta D \qquad \Delta d = \Delta C - \Delta D\,.$$

A special situation arises if the camera distance a is twice the focal length of the eye. In this instance both projection centers are moved to infinity and the perspective bundles become affine. The effect of the stereobase is then substituted by a convergence angle γ of the two sets of projecting parallel beams of rays

$$\gamma \approx \frac{\Delta d}{2f}\,. \tag{10}$$

From the comparison of values s and r one derives a ratio $r/s = d/a$ showing that the projection center S is displaced from the center of the eye entrance pupil in a direction defined by connecting the points E_L, C_L (Figure 7). The resulting coefficient,

$$\rho = \frac{r}{d} = \frac{s}{a'} = \frac{F}{2f} = \frac{f}{2f - a}\,, \tag{11}$$

expresses the rate of change in the position of the projection center S, with respect to the basic parameters d, a underlying the change. With the use of this definition one can rewrite some of the previous formulas as follows

$\bar{s} = \rho a$	(3a)	$c = 2\rho f_c$	(6a)
$F = 2\rho f$	(4a)	$r' = f_c \rho d/f$	(7a)
$r = \rho d$	(5a)	$b = \rho \Delta d$.	(8a)

The meaning of the coefficient ρ is important. With the increased distance a between the eye and objective of the fundus camera, the coefficient ρ grows rapidly, and greatly affects the geometry of images. Table 1 gives a few examples of the changes involved.

The consequences of these findings are far-reaching. The position of the effective projection center is significantly changed by variations in the relative position of the eye and fundus camera. Because of the telescopic type of imaging, the sharpness of the image is not affected by errors in the camera positions. In effect, there is no reliable control of the geometry in practical operations, and the operator may not even realize that a slight shift of the camera changes the geometry in an appreciable way. If the entrance pupil of the camera is about 35 mm from the eye, the projection center moves to infinity and the projection becomes orthographic. Upon going farther from the eye, the effective projection center is transferred deep into the object space, thus defining negative bundles of projecting rays and a negative photogrammetric base. The resulting uncertainty can make it impossible to produce reliable photogrammetric evaluations with standard solutions.

TABLE 1. CHANGE OF IMAGE GEOMETRY WITH INCREASED CAMERA DISTANCE.

		Camera distance a				
		0	0.5f	f	2f	3f
Rate of change p (11)		0.5	0.67	1.0	∞	−1
Shift s of projection center	(3)	0	0.33f	f	∞	−3f
Offset r of projection center	(5)	0.5d	0.67d	d	∞	−d
Object-to-center distance F	(4)	f	1.33f	2f	∞	−2f
Image principal distance c	(6)	f_c	1.33f_c	2f_c	∞	−2f_c

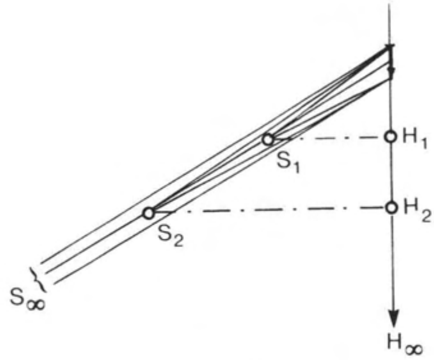

FIGURE 9

To avoid these major difficulties one should be extremely cautious in keeping the conditions of taking the pictures under good control. The appropriate distance between the eye and the objective should be established as required, or determined by direct measurement. For solutions where the length of the stereobase is important one should also measure the diameter of the dilated eye pupil.

It is interesting to note that the change in the distance a does not affect the base-distance ratio θ which, in accordance with Equation 9, depends only on the decentration difference Δd. Its maximum magnitude is limited by physical dimensions of both pupils. On the other hand, in changing the camera distance a, e.g., from 0 to $2f$, keeping at the same time the decentration difference Δd constant, one changes significantly the principal distance and the position of the principal point H' in the image plane. Basically the same image is defined by gradually narrowing the bundles of rays which eventually convert to parallel beams of rays as shown in Figure 9. This illustration indicates that too narrow bundles and the associated excessive displacement of the image principal point would not ensure good conditions for the photogrammetric reconstruction. However, if the projection centers vanish to infinity and perspective bundles become affine, the photogrammetric model can be reconstructed in a convenient and reliable analytical way, which is analyzed later on. With a negligible loss of rigor, this type of solution can be also used as a substitute for solutions with narrow bundles of rays which otherwise would not yield satisfactory results. One can even go so far as to suggest the use of parallel projection as a standard procedure for ophthalmologic images, making sure that the fundus camera is positioned at a suitable distance from the eye.

Prism Separation. A thin prism with an angle ϕ, placed close to the front of the objective of a fundus camera deviates the optical rays by a constant angle δ therefore causing a shift of the image in its plane. For a thin prism the deviation δ is half of the angle ϕ:

$$\delta = \phi/2 \ .$$

As long as the path of rays between the lenses is parallel there is no change in focusing in the image plane. The image is equivalent to one which would be formed without using any prism, with the camera tilted by the angle δ. Figure 10 illustrates the combined effect of a twin prism simultaneously forming two well-separated images in the same image plane (cf., Bynke).[2]

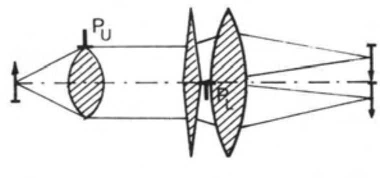

FIGURE 10

Optical rays participating in the projection of individual images are completely separated. They are limited by the half-periphery of the entrance pupil of the eye and by the base line of the prisms. From the geometric point of view this is a situation similar to that from the previous section. The difference is that now one of the effective semi-stops is a straight line. Thus, the shape of the limited pupil is defined as a semicircle or a segment close to it as shown in Figure 11.

Derivations from the previous section are also applicable here upon substituting the decentration d by the constant $D/2$. To be

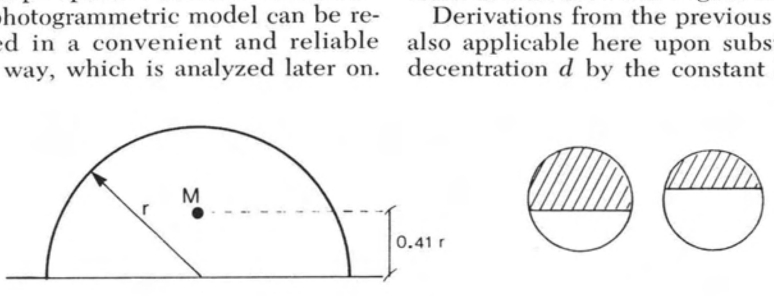

FIGURE 11

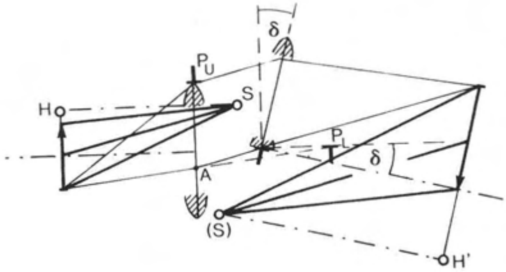

FIGURE 12

precise one should also define the chief ray as passing through the centroid of the virtual pupil. This would place point M at the distance equal to 41 percent of the pupil radius above the base line. With reference to Figure 6, it now holds true that

$$M_1M_2 = 0.59\ A_1A_2$$

and this relation leads to the following formulas characterizing the geometry of the prism separation:

$$\rho = \frac{f}{1.7f - a} \tag{12}$$

$$s = \rho a \tag{3b}$$

$$r = 0.5\rho D \tag{5b}$$

$$F = 2\rho f \tag{4b}$$

$$b = \rho D \tag{8b}$$

$$\theta = D/2f\ . \tag{9b}$$

Values s, r determine the position of the object projection center S.

Because of the prism effect, each stereoimage is tilted outwards, as in a pair of divergent photographs. By reverting the orientation of the prisms in the lens attachment one changes conditions to a convergent situation with interchanged images (cf., Saheb and Drance).[13] A single prism, as used by Schirmer, gives rise to a geometrically hybrid, normal-convergent setup. Due to the tilt a rigorous reconstruction of the image projection center *(S)* is more complicated than in the previous section and will be omitted here. A graphical illustration is given instead, in Figure 12.

The practical assessment of the metric instability of this type of stereoseparation follows the same reasoning as above for the parallel shift separation. However, one is now in a better position to control the camera distance by visually checking the quality of the stereoseparation in the viewing field of the fundus camera. The use of parallel projections will significantly simplify geometric relations if the camera can be positioned at the distance of $2.4f$ from the eye, which is roughly 42 mm. Prisms of a suitable power must be selected for this purpose in order to maintain a good image separation.

Convergent Photography is a typical arrangement for the stereosystem of a slit lamp. The convergence angle is fixed by the manufacturer and cannot be changed. Figure 13 illustrates geometric relations for one-half of the stereosystem. As the refractive power of the eye is eliminated by the use of a preset or contact lens, this optical combination represents an afocal system, and the eye pupil works only as a front stop for the objective of a slit lamp. Following the routine previously used in connection with Figure 6, one can now use Figure 13 to find

$$\frac{f+s}{2s} = \frac{f+a}{a}$$

and derive

$$s = \frac{fa}{2f + a}\ . \tag{13}$$

At the same time, it follows from Figures 13 and 14 that

$$t = s\,\frac{D - d}{a}\ \ and\ \ r = \frac{D}{2} - t$$

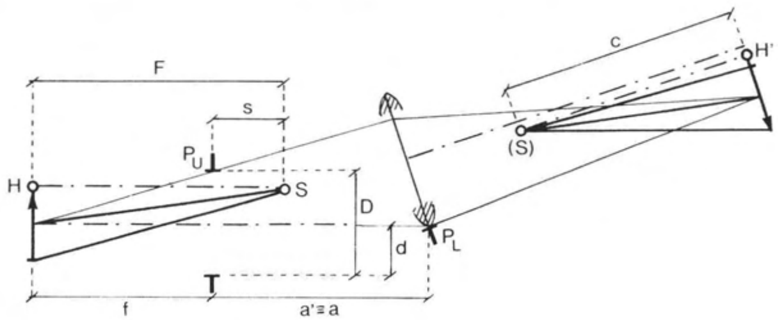

FIGURE 13

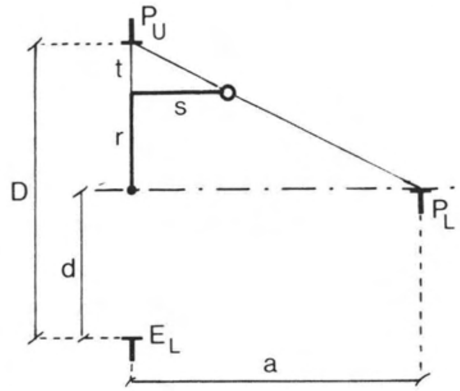

FIGURE 14

so that

$$r = \frac{0.5Da + df}{2f + a} \cdot \quad (14)$$

For the stereobase one derives

$$b = \frac{Da + 2df}{2f + a} \cdot \quad (15)$$

Because the offset d must be at least equal to $0.5D$, the minimum base is

$$b_{min} = D\,\frac{f + a}{2f + a} \cdot \quad (16)$$

Finally, the distance F of the effective projective center S from the examined eye fundus is given by

$$F = f + s = 2f\,\frac{f + a}{2f + a} \cdot$$

The base-distance ratio θ and the convergence ratio γ are identical and their minimum value is

$$\theta_{min} = \gamma_{min} = \frac{D}{2f} \cdot \quad (17)$$

Table 2 lists some of the parameters as functions of the distance a. The position of the projection center varies slowly and the practical range of change is rather limited. Unlike those systems with parallel shift or prism separation of images, there is no possibility for projection centers to be transferred to infinity. The position of the principal point and the magnitude of the principal distance for the first real image must be defined in agreement with the fixed tilt of individual microscopes as graphically indicated in Figure 13. As for the parallel or prism-modes of stereoseparation, the position of the effective principal point in the image can be well off the image center.

Calibration and Photogrammetric Evaluation

Photogrammetric calibration of an ophthalmologic photo-optical system includes the determination of all metric parameters that are necessary or important for photogrammetric processing. The extent and accuracy of the calibration depends on the type of instrument (microscope, telescope), on the type of stereoseparation (parallel, prism or convergent mode) and on the method used for the photogrammetric reconstruction (simple parallax method, three-dimensional analog or digital solution).

The simplest conditions are ensured if a fundus camera is used in combination with the stereoseparation by a parallel shift. Although inconvenient for practical reasons because of the need for the consecutive exposure, this type of separation preserves the stability of principal axes and image planes. Parallax measurements with the use of a stereoscope and parallax bar, or with the use of stereocomparator are adequate for spot heighting on discrete points, for construction of profiles and for computations of simple metric parameters, such as distances, areas and volumes. For direct plotting of form lines and profiles as well as for three-dimensional digitization, analog plotters may be used. In this instance, the evaluation can be improved by auxiliary leveling of models.

Disregarding some practical difficulties associated with the small size and rather low resolution of ophthalmologic photographs, a great advantage of the parallel stereosepara-

TABLE 2. CHANGE OF IMAGE GEOMETRY WITH INCREASED DISTANCE OF SLIT LAMP.

		Slit lamp distance a				
		0	0.5f	f	2f	3f
Shift s of projection center	(13)	0	0.2f	0.33f	0.5f	0.6f
Min. offset r of projection center	(14)	0.25D	0.3D	0.33D	0.37D	0.4D
Object-to-center distance F	(16)	f	1.2f	1.33f	1.5f	1.6f

tion lies in preserving the simple *normal* model with minimum control requirements; estimates of the horizontal and vertical scales or of the base-height ratio are sufficient. Rigorous reconstruction of the interior orientation in the plotter is unnecessary in this instance. The estimates of controlling parameters are based on clinical experience and calibration measurements of the pupils of the eye and the objective as well as of the parallel displacement for the stereoseparation. The reliability and accuracy of the parallax-type evaluation is limited and can be assessed only by statistical means.

If the orientation of the eye between consecutive exposures changes in an appreciable way, the reconstruction may still be feasible. An analog plotter can be used in some applications but, in general, an analytical solution is required. Additional calibration parameters to be determined at the time of photography include the camera distance that is used to derive the principal distance and positions of principal points in the images. If the principal distance is too long, the solution may become ill-conditioned and the model reconstruction fails. In this instance the non-rigorous substitution of perspective bundles by parallel beams of rays is helpful. An analytical description of this type of model formation is presented in the following section. The main advantage of using parallel projections is that the number of calibration and orientation parameters is fewer than for bundle solutions. On the other hand, the solution is feasible only in a digital mode or with the use of the analytical plotter.

Prism-separated and convergent stereopairs suffer from the fact that photographs are tilted and their principal points offset. Besides, the principal distance of prism-separated photographs may be very long. However, one often neglects these facts and applies simplified methods from previous paragraphs. The reconstruction then proceeds in a distorted spatial system with the use of unrelated, undefined units for measurements. Although very crude, these methods are being extensively used, and yield useful results as long as the conditions for taking of pictures and for their evaluation are maintained constant. The absolute metric value of such evaluations is low and they are mostly limited to consecutive comparisons on identical subjects. Rigorous solutions in these categories are not always possible with analog plotters and one should rather rely on analytical treatment. The model reconstruction is again very sensitive to ill-conditioned situations with narrow bundles and excessive offset of principal points. This reflects in an inadequate convergence of iterative solutions determining the parameters.

Therefore, it is advisable to use the geometrically simpler parallel projection in all nearly critical cases, and even for slit-lamp pictures for which projection centers cannot vanish to infinity. The substitution is possible because the fundus relief is very low in comparison with the distance of the projection center, so that the scale changes are practically negligible. Photogrammetric calibration should include the knowledge of the optical parameters of the camera or slit lamp, measurements of pupil diameters and determination of the prism power, or of the convergence angle for the stereomicroscope.

More efficient calibration procedures must use physical models substituting the real object, the human eye, in its basic metric and optical properties. For a slit lamp which applies close-up focusing, an appropriate model is a tiny metal block with two or more plane surfaces at different levels. Groups of at least four cross-marks are engraved into the surfaces conforming to the dimensions of the eye. A hollow hemisphere with circular opening at the top simulates the eye pupil in its function as a front stop for the slit lamp. Spatial coordinates are determined for all control points and the block can then be used for single and double resections of photographs taken under variable conditions in order to test methods, assumptions and other aspects of interest.

Fundus cameras must be tested with the use of a model which exhibits a simulation of the eye's refractive power. The base of the model is structured at different levels carrying groups of tiny marks serving as control points. The upper part of the model contains a lens with a power and dimensions corresponding to those of the human eye. Experiments with the use of such an *artificial eye* are described by Jönsas.[10]

Model Reconstruction from Parallel Projections

Collinearity Equations

Basic collinearity conditions, as used in analytical solutions dealing with projective bundles, consist of two well known equations,

$$F_x \equiv \Delta X z' - \Delta Z x' = 0 , \quad F_y \equiv \Delta Y z' - \Delta Z y' = 0 \tag{18}$$

where ΔX, ΔY, ΔZ are object coordinates with origin in the projection center and x', y', z' are image coordinates in the same coordinate system.

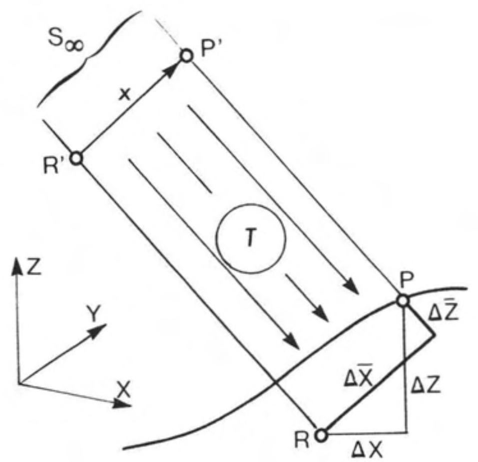

FIGURE 15

Parallel or orthographic projection can be considered as a special case of the perspective projection with the projection center vanishing to infinity. Thus, coordinates of the projection center are eliminated as unknowns from analytical considerations and complete definition of a beam of parallel rays is expressed by six parameters: dx', dy', m for the interior orientation, and α, β, γ for the exterior orientation.

The coordinates dx', dy' replace the function of a principal point in the bundle formulation, and here represent the image coordinates of an object reference point R defined within the range of the object by arbitrarily selected coordinates X_O, Y_O, Z_O. This is an auxiliary point that replaces the projection center and its image position does not have to be identified. The parameter m stands for the scale factor associated with the orthographic projection. In ophthalmic photogrammetry the m-factor is smaller than unity. Parameters α, β, γ define the rotation matrix **T** describing the spatial attitude of the parallel beam.

Equations 18 can be modified for the parallel projection through division by z'. In so doing the vertical dimension in both the image and the object spaces is eliminated and replaced by the scale factor $m = \Delta Z/z'$ and by unity, respectively. One obtains

$$Fx \equiv \Delta\bar{X} - x = 0 \ , \ Fy \equiv \Delta\bar{Y} - y = 0 \qquad (18a)$$

where

$$\begin{pmatrix}\Delta\bar{X}\\ \Delta\bar{Y}\\ \Delta\bar{Z}\end{pmatrix} = \boldsymbol{T}^{\mathrm{T}}\begin{pmatrix}\Delta X\\ \Delta Y\\ \Delta Z\end{pmatrix}, \begin{pmatrix}\Delta X\\ \Delta Y\\ \Delta Z\end{pmatrix} = \begin{pmatrix}X - X_O\\ Y - Y_O\\ Z - Z_O\end{pmatrix}$$

$$\begin{pmatrix}x\\ y\end{pmatrix} = m\begin{pmatrix}x'\\ y'\end{pmatrix}, \begin{pmatrix}x'\\ y'\end{pmatrix} = \begin{pmatrix}x_O' - dx'\\ y_O' - dy'\end{pmatrix}$$

As illustrated in Figure 15 the rotation is applied to object coordinates which are then compared with scaled and reduced image coordinates. The x_O', y_O' image coordinates have an arbitrary origin and, consequently, no fiducial marks are required in the images.

For any practical solution Equations 18a must be linearized; this process requires that matrices **B** and **A** of partial derivatives of the collinearity functions be formed, with respect to parameters **g** or with respect to image coordinates **p**'

$$B = \partial\begin{pmatrix}F_x\\ F_y\end{pmatrix}/\partial\,\mathbf{g} \quad , \quad A = \partial\begin{pmatrix}F_x\\ F_y\end{pmatrix}/\partial\mathbf{p}$$

In two sets of converging parallel beams the rotational parameter β is more significant than α and γ and therefore should not be neglected when linearizing the functions Fx, Fy. The resulting matrices and parameters are defined as follows, with respect to measured image coordinates,

$$\mathbf{A} = \begin{bmatrix}m & 0\\ 0 & m\end{bmatrix}$$

$$\mathbf{v}^{\mathrm{T}} = (v_x \, v_y) \qquad (19)$$

with respect to orientation parameters,

$$\boldsymbol{B}_O = \begin{bmatrix}m & 0 & -x' & -\Delta\bar{Y} & -\Delta\bar{Z} & 0\\ 0 & m & -y' & \Delta\bar{X} & 0 & -\Delta\bar{Z}\end{bmatrix}$$

$$d\boldsymbol{g}_O^T = (dx' \; dy' \; dm \; d\alpha \; d\beta \; d\gamma)$$

and with respect to object coordinates,

$$\boldsymbol{B}_x = \begin{bmatrix}1 & -\alpha & -\beta\\ \alpha & 1 & -\gamma\end{bmatrix} \approx \begin{bmatrix}1 & 0 & -\beta\\ 0 & 1 & 0\end{bmatrix}$$

$$d\,\mathbf{g}_x^{\,T} = (dX \; dY \; dZ).$$

PROJECTION EQUATIONS In accordance with Figure 15 one can project any object point into the image plane if he knows the six orientation parmeters of the parallel beam referred to the given auxiliary point R. The projection is expressed by

$$\begin{pmatrix} x'_o \\ y'_o \end{pmatrix} = \begin{pmatrix} dx' \\ dy' \end{pmatrix} + $$

$$+ \frac{1}{m} \begin{bmatrix} t_{11} & t_{21} & t_{31} \\ t_{12} & t_{22} & t_{32} \end{bmatrix} \begin{pmatrix} X - X_o \\ Y - Y_o \\ Z - Z_o \end{pmatrix} \qquad (22)$$

where t_{ij} are elements of the rotation matrix constructed by known methods from the parameters α, β γ.

INTERSECTION EQUATIONS

For the intersection of two corresponding rays two auxiliary projection centers S_1, S_2 should be determined first. They are located on the rays to be intersected at the minimum distance from the reference point R. The intersection proper then follows with the use of the same formulas as for perspective bundles. Whereas in bundle intersections the base is fixed and the directions of intersecting rays are variable, in beam intersections the base is variable and the ray directions are stable. In accordance with Figure 16 the auxiliary base is computed from scaled image coordinates as

$$\begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix} = \mathbf{T}_2 \begin{pmatrix} x_2 \\ y_2 \\ 0 \end{pmatrix} - \mathbf{T}_1 \begin{pmatrix} x_1 \\ y_1 \\ 0 \end{pmatrix} \qquad (23)$$

and the direction cosines of projection rays are

$$\mathbf{o} = \begin{pmatrix} \lambda \\ \mu \\ \nu \end{pmatrix} = \mathbf{T} \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} = - \begin{pmatrix} t_{13} \\ t_{23} \\ t_{33} \end{pmatrix} \qquad (24)$$

In the next step the lengths of the intersecting vectors are determined,

$$n_1 = \frac{\begin{vmatrix} b_x & b_z \\ \lambda'' & \nu'' \end{vmatrix}}{\begin{vmatrix} \lambda' & \nu' \\ \lambda'' & \nu'' \end{vmatrix}} \qquad n_2 = \frac{\begin{vmatrix} b_x & b_z \\ \lambda' & \nu' \end{vmatrix}}{\begin{vmatrix} \lambda' & \nu' \\ \lambda'' & \nu'' \end{vmatrix}} \qquad (25)$$

and eventually.

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} X_o \\ Y_o \\ Z_o \end{pmatrix} + \tfrac{1}{2}\boldsymbol{T}_1 \begin{pmatrix} x_1 \\ y_1 \\ -n_1 \end{pmatrix} + \tfrac{1}{2}\boldsymbol{T}_2 \begin{pmatrix} x_2 \\ y_2 \\ -n_2 \end{pmatrix} \qquad (26)$$

The inaccuracy in the intersection shows up as a y-parallax

$$p_y = n_1\mu' - n_2\mu'' - b_y \,. \qquad (27)$$

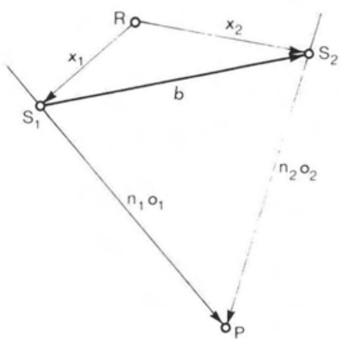

FIGURE 16

COMPUTATION OF ORIENTATION PARAMETERS

The computation of the orientation parameters can be arranged in one of several possible ways depending on the number of control points available to support the solution. Full orientation of two parallel beams includes 12 unknown parameters to be determined. As in bundle solutions, one needs at least seven absolute control values to obtain a non-singular normal matrix. These control values are usually not available except if one is applying the solution to artificial models as suggested for calibration and testing purposes. The other extreme in computing the model is characterized by a complete neglect of absolute information if a non-scaled, non-oriented model is formed in the process of a relative orientation. This type of solution includes only five unknowns whereas seven parameters must be predetermined and kept fixed. Such an uncontrolled model can be additionally subjected to a partial absolute orientation if required. One can decide on deriving a scaled model, a leveled model, or both. The last four alternatives are applicable to real situations provided that the limited control support is obtained from suitable estimations. Table 3 illustrates possible parameter arrangements and specifies conditions for the solutions mentioned.

The analytical formulation can be done by a proper modification of the collinearity equations, eliminating the unknown object coordinates and fixed parameters from the solution to fit the particular reconstruction situation. Another feasible solution maintains the general form of the collinearity equations; the unknown object coordinates are eliminated in the course of the numerical solution and corrections for the fixed parameters are omitted from the equations.

To demonstrate the performance of the model formation from parallel projections, an

TABLE 3. DIFFERENT TYPES OF SOLUTIONS FOR RECONSTRUCTION OF MODELS FROM PARALLEL PROJECTION.

	Solution parameters												Control support	
	Left						Right							
	dx′	dy′	dm	dα	dβ	dγ	dx′	dy′	dm	dα	dβ	dγ	#	type
Full orientation (FO)	X	X	X	X	X	X	X	X	X	X	X	X	7	XYZ/XYZ/Z
Relative orient. (RO)	0	0	0	0	0	0	0	X	X	X	X	X	0	—
Relative orient. (RO)	0	0	0	X	0	0	0	0	X	X	X	X	0	—
RO + leveling (L)	0	0	0	X	X	X	0	X	X	X	X	X	3	Z/Z/Z
RO + scaling (S)	0	0	X	0	0	0	0	X	X	X	X	X	1	D
RO + L + S	0	0	X	X	X	X	0	X	X	X	X	X	4	Z/Z/Z/D

Legend: X—to be derived
0—to be fixed in advance
D—distance

example is presented here of an experiment arranged with full control. The calibration block as described in the previous section, but without the simulated eye pupil, was photographed with a Zeiss photo slit lamp from a distance of about 80 mm. The maximum distance between the control targets was approximately 5 mm and their vertical range 1 mm. Although the image was formed virtually from perspective bundles with the centers close to the objective lenses, the narrow angle of the bundles caused an unsatisfactory convergence and resultant failure of the iterative bundle solution, which included also the determination of interior orientation parameters. However, the computation with the use of parallel beams was successful after four iterations. Figure 17 shows the computer listing. Arbitrarily selected coordinates of the reference point are printed before the parameters of interior orientation. The scale factor m defines that the images are 3.6 times larger than the object. The print-out of the rotation matrices shows a convergence angle of approximately 11°. For each control point participating in the computation, its object coordinates X, Y, Z and associated residual errors of fitting with the rays are printed out in millimeters. The residual image parallax and least squares corrections to measured coordinates are expressed in micrometers. The intersected additional point, number 33, is listed by its computed coordinates and with no error assessment possible in the object space. All the discrepancies, as well as the resulting standard error of unit weight are typical of the relatively low quality of the image definition. The final part of the print-out shows the uncertainty of the solution in terms of estimated standard errors for the computed parameters: for the image shift dx', dy' in millimeters, for the scale factor in no metric units, and for the rotation parameters $d\alpha$, $d\beta$, $d\gamma$ in radians.

USE OF ANALYTICAL PLOTTER

The occurrence of perspective as well as affine bundles, geometric properties of which are so dependent on conditions of imaging, inevitably limits the practical use of analog photogrammetric instruments. On the other hand, a digital approach which is capable of providing solutions fitting the geometric peculiarities of ophthalmologic systems, suffers from limitations imposed by the mode of operation on discrete points. Thus, only the combination of digital and analog features as materialized in the design of the analytical plotter makes it possible to cope with virtually all metric and practical problems of the ophthalmic stereophotogrammetry.

The basic stereoresection of images can be performed off-line in a form, the complexity of which suits the computer supporting the analytical plotter. A general description of applicable solutions was presented in the previous sections. The on-line computation used for the computer-controlled positioning of the measuring mark in the plotter must also be modified for the use of parallel projections. Formulas for this real time operation with perspective bundles represent a projection of a common point from the object space onto an image plane by reduction, transformation and variable scaling of coordinates, as described, e.g., by Jaksic[8]. Projection Equations 22 give the equivalent formulation

MAR. 07 1974

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* SLITLAMP ZEISS * DEC 70 * A-BLOCK * 3X MAGN. * FRAME # 5 ******* *

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MCDEL FRCM PARALLEL PROJECTIONS 5 - 5

ORIENTATION PARAMETERS

	X	Y	Z
REF	85.00	47.00	100.00
	DX	DY	M
5	-0.147	0.506	0.28003
5	-0.094	0.459	0.27804
LEFT	0.99603069	0.04562040	-0.07643116
	-0.04391566	0.99875069	0.02383928
	0.07742321	-0.02038813	0.99678987
RIGHT	0.99013811	0.07331097	0.11938190
	-0.07494247	0.99714506	0.00922899
	-0.11836439	-0.01808474	0.99280554

PT	X	Y	Z	DX	DY	DZ	PY	VXL	VYL	VXR	VYR
11	83.58	48.43	101.00	-0.01	0.00	0.03	-7	12	-10	32	-17
12	87.03	48.47	101.00	0.01	-0.00	0.04	22	-62	5	-33	26
13	87.07	45.02	101.00	-0.00	0.00	-0.04	18	21	-14	-11	4
14	83.62	45.00	101.00	-0.01	0.00	-0.01	-40	23	13	19	-27
21	84.14	47.79	100.00	-0.01	-0.01	0.01	-11	28	31	34	21
22	86.36	47.73	100.00	-0.01	-0.00	-0.02	-48	24	29	16	-19
23	86.33	45.54	100.00	-0.00	0.01	0.06	46	-9	-37	29	9
24	84.09	45.53	100.00	0.02	-0.00	-0.07	0	-35	-7	-85	-7
33	85.25	46.65	100.04				19	-1	-10	1	10

STANDARD UNIT ERROR 35.3 MICRCNS

STANDARD ERRORS OF UNKNOWNS

0.017	0.017	0.00171	0.00170	0.00686	0.00672
0.018	0.017	0.00173	0.00174	0.00669	0.00656

FIGURE 17

for parallel beams of rays. Parallel projection formulas are simpler because the scaling factor is constant for all rays and a single reference point R replaces the projection centers in the reduction phase of the computation.

CONCLUSIONS

The main peculiarity of the ophthalmologic stereophotography is its geometric instability. Relatively small variations in the mutual position of the eye and of the instrument may result in significant changes in the position of the effective projection centers. For this reason, analytical photogrammetric methods are much more suitable and reliable in ophthalmologic applications than analog solutions.

Perspective bundles of rays, rigorously reconstructed from ophthalmologic images, are usually very narrow and in some instances the projection centers even vanish to infinity. Arising difficulties can be overcome using parallel beams of rays instead of perspective bundles for some analytical photogrammetric reconstructions. Under certain conditions such an approach often proves useful and more efficient also in substituting conventional bundle procedures in situations where the equation system becomes ill-conditioned and the convergence of its iterative solution is not satisfactory.

REFERENCES

1. Allen, L. et al, "Instant Positive Photographs and Stereograms of Ocular Fundus Fluorescence". *Arch. Ophthal.*, 75:192, 1966.

2. Bynke, H. G., Krakau, C.E.T., "An Improved Stereophotographic Method for Clinical Measurements of Optic Disc Protrusion". *Acta Ophthal.*, 38:115, 1960.

3. Crock, G., Parel, J. M., "Stereophotogrammetry of Fluorescein Angiographs in Ocular Biometrics". *Med. J. Austr.*, 2:586, 1969.

4. Crock, G., "Stereotechnology in Medicine". *Trans. Ophthalmol. Soc. U. K.*, 90:577, 1970.

5. Havelka, G., *Geometrical Optics* (in Czech), CSAV, Prague 1955.

6. Holm, O., Krakau, C.E.T., "A Photographic Method for Measuring the Volume of Papillary Excavations". *Annals of Ophthalmology*, 1969-1970, pp. 327-332.

7. Holm, O., et al., "Volume of the Optic Disc Cup". *Am. J. Ophthal.*, 73:876, 1972.

8. Jaksic, Z., "Solution of Aerial Triangulation Problems Using the N. R. C. Analytical Plotter". *Photogrammetria*, 22:59-71, 1967.

9. Jenkins, F.A., White, H.E., *Fundamentals of Optics*, McGraw-Hill, New York 1957.

10. Jönsas, C. H., *Stereophotogrammetric Techniques for Measurements of the Eye Ground*, Munksgaard, Copenhagen 1972.

11. Kottler, M.S., et al., "Digital Photogrammetry of the Optic Nervehead".*Investigative Ophthalmology*, 13:116, 1974.

12. Roos, W., "Üuber die Lage der Projektionzentren bei einem Objektiv und ihre Bedeutung in der Bilmessung". *Bildmessung und Luftbildwesen*, 1940, pp. 88-96.

13. Saheb, N.E., Drance, S.M., Nelson, A., "The Use of Photogrammetry in Evaluating the Cup of the Optic Nervehead for a Study in Chronic Simple Glaucoma". *Can. J. Ophthal.*, 7:466, 1970.

14. Schirmer, K.E., Kratky, V., "Photogrammetry of the Optic Disc". *Can. J. Ophthal.*, 8:78, 1973.

15. Schirmer, K.E., "Instamatic Photogrammetry". *Can. J. Ophthal.*, 9:81, 1974.

FORUM

College Needs Used Instruments

Ferris State College (Big Rapids, Michigan) recently initiated a four-year curriculum leading to the degree of Bachelor of Science in Surveying. Included in this curriculum are two required courses in photogrammetry which I teach.

Ferris is a unique college. Student participation in each mapping technique is emphasized in addition to the traditional classroom treatment of the theory underlying the technique. In the photogrammetry class, each student learns how to perform relative and absolute orientation on our Wild A-8 Autograph. This philosophy of "learning by doing" has proven so successful that we now have over 140 students enrolled in the surveying program which only a few years ago had 60 students.

As a result of this growth, we need photogrammetric equipment. Specifically, we need another plotter, preferably a universal one, with which to teach orientation and bridging techniques. Unfortunately our department budget cannot support the purchase of such an instrument. We are therefore looking to the photogrammetric profession for assistance. It is our hope that the American Society of Photogrammetry can advise us of a potential donor. I feel that this need is sufficiently important that I must write to ASP personally.

I am very excited about the surveying program at Ferris.We serve an educational need by teaching students how to perform practical jobs that cannot be learned other places. There is an excellent chance that we can put unwanted equipment to good use at this college. Your assistance in locating such equipment would be very welcome.

— *Prof. Jens Otto Rick*

Prof. Rick:

I hope that some of our readers will come to your aid.

You may be unaware how this problem is being solved by the International Training Center for Aerial Survey and Earth Science (I.T.C.) where 300 new students are being trained each year. The school uses dozens of special economical training instruments on which the students learn the elements of relative and absolute orientation before practicing on a first-order instrument. Inasmuch as I am not familiar with the source nor cost of these instruments, I believe that you can obtain this information from Prof. A. J. van der Weele, Director, I.T.C., P. O. Box 6, Enschede, the Netherlands.

— *Editor*

(Continued on page 89)