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On Photogrammetric Distortion

Compensation for model deformations is afforded by electrical analogue computers both for planimetry and elevation, allowing one to generate mutually independent correction surfaces in both directions.

Abstract: *The term "distortion" is precisely defined as the symmetrical dislocation of image points between the mathematical model of perspective projection and the physical image formation model. On this physical distortion, which is always rotation-symmetrical, is superposed a summation of unavoidable tolerances in the manufacture of optical systems, for which the term "deformation" is introduced. Further considerations deal with external influences on photogrammetric photography and lead to the conclusion that at the present state of knowledge no definite causes can be established for image point dislocations in photographic image formation. Based on the assumption that model deformations are caused by imperfections of the photograph's geometry, it is advantageous to compensate those imperfections by correction surfaces generated analytically or by analogue instrumental techniques from discrepancies at known control points. Conventional solutions using optical compensation plates or reseau cameras are criticized.*

Introduction

With the advance of photogrammetric methods with respect to measuring accuracy, systematic model deformations are claiming closer attention. In relevant publications, frequent use is made of the word *distortion* as a component of various terms such as *asymmetrical distortion, tangential distortion,* etc. In photogrammetric instrument engineering, however, the term *photogrammetric distortion* in a narrower sense describes certain properties of an optical system used in photogrammetry, so that from the instrument design aspect it is no longer desirable to ascribe the term to a sum of ill-defined residual systematic errors in the photogrammetric measuring system. Some deeper exploration seems to be imperative and should help to find possibilities of eliminating systematic errors in the restitution process.

Distortion in Optical Imaging Systems

Normally, image formation by an optical-photographical system is, to a first approximation, based on the mathematical-geometrical model of perspective projection. In any physical implementation of optical imaging by glass elements of (mostly) spherical curvature, deviations from this model will occur due to the laws of geometric optics. Such deviations are defined as *distortion*.

In a photolens, the mathematical projection center is replaced by the centers of the entrance and exit pupils, which are the vertices of an object-side and an image-side cone of rays. If in a mathematical perspective projection we know the spacing c (Figure 1) of the projection center O from an image plane and the angle of incidence τ_1 of an imaging ray emerging from the object point P relative to the projection axis, then the image distance s' to the image point P' from the point of

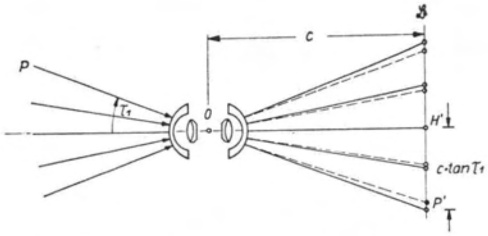

FIG. 1. Definition of distortion.

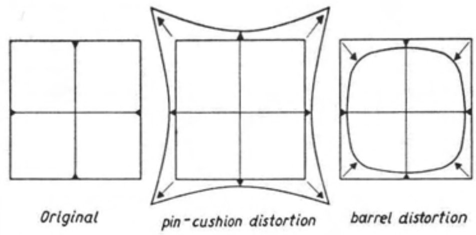

FIG. 2. Typical distortion patterns.

intersection of the projection axis with the image plane is found by the simple equation

$$s' = c \cdot tan\ \tau_1. \quad (1)$$

In the described imaging model of mathematical perspective projection, c is a constant applicable to any angle of incidence.

In computing an optical imaging system it will be found that constant c in Equation 1 has to be replaced with

$$c = F\ (\tau_1). \quad (2)$$

If c increases with increasing angles of incidence, all image points will be displaced outwards (away from the principal point H') to an extent depending on the angle of incidence. Any straight line not intersecting the optic axis will necessarily appear in the image plane as a curve deflected towards the principal point. If, in the opposite sense, c decreases with increasing field angles, the images of analogous object straight lines should be deflected in the opposite way. The image of a square located symmetrically to the optic axis will in the first instance have the shape of a pin-cushion, in the second situation, that of a barrel (Figure 2). This phenomenon can be explained also by assuming a magnification varying with the ray's angle of incidence relative to the optic axis. Barrel distortion will occur at a collecting lens having its aperture diaphragm in front of it (Figure 3). Refraction of the principal rays incident in the marginal zones, i.e., at a large angle from the optic axis, is proportionally greater, which locally reduces the magnification. Similarly, pincushion distortion can be explained by a limiting aperture behind the lens (Figure 3). For axial-parallel rays extending from a test object, again those incident near the lens rim are refracted by a greater angle and consequently cannot pass the aperture. The aperture diaphragm is reached only by a cone of rays slightly diverging in the object space. Magnification thus increases as object points recede from the optic axis.

What has been said so far about magnification applies to image formation in the optic axis. The distortion of a photolens can be represented in a rectangular coordinate system, where distortion values (i.e., deviations of image point locations from those of mathematical perspective projection) are plotted vs. the associated angles of interference relative to the optic axis (Figure 4). The

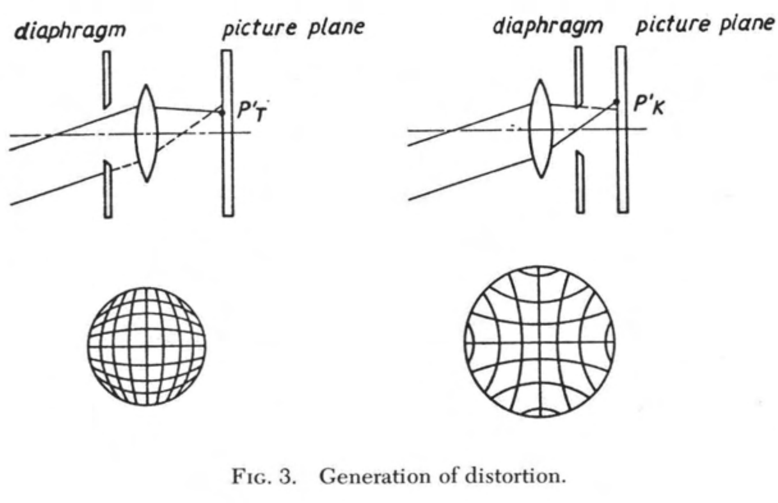

FIG. 3. Generation of distortion.

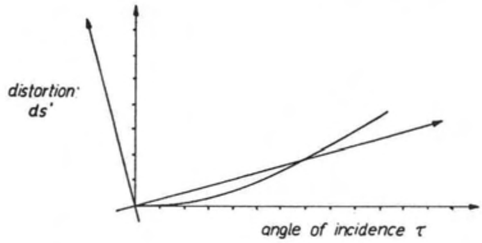

FIG. 4. Absolute distortion and photogrammetric distortion.

abscissa, representing angles, will necessarily be tangent to the curve in the coordinate origin, because by definition the distortion

$$ds' = s' - c \cdot tan\ \tau_1 \qquad (3)$$

s' denoting the distance measured between the principal point and the respective image point. Distortion in that sense may be called *absolute distortion*, the imaging constant c corresponding to the Gaussian focal distance (along the optic axis).

Photogrammetric Distortion

For the purpose of measurement where geometrical dimensions are taken from the photograph, the concept of *absolute distortion* is impractical. Reference to a mean optical magnification, e.g., to the effect of demanding that distortion maxima and minima are equal throughout the image field, is a more useful approach. This implies that the imaging constant c no longer applies to the Gaussian focal distance (in the optic axis), but to a suitably chosen angle of incidence τ_1 of a principal imaging ray relative to the optic axis. Resulting from an imaging constant c thus defined, the statement $ds' = O$ is true both for the optic axis and for the angle of incidence τ_1. The distortion referred to such a mean optical magnification can be obtained with sufficient accuracy from the distortion diagram described by a corresponding rotation of the coordinate system.

Demands placed on a photogrammetric camera lens system for the least possible distortion mostly imply the concept that in this application the object-side and image-side cones of rays should be congruent within the accepted tolerance. This constraint, however, is unnecessary. Let us, generally, denote the angles of incidence of a principal ray relative to the optic axis by τ and their corresponding angles in the image space by τ'. If we put the condition

$$\frac{tan\ \tau'}{tan\ \tau} = k = constant \qquad (4)$$

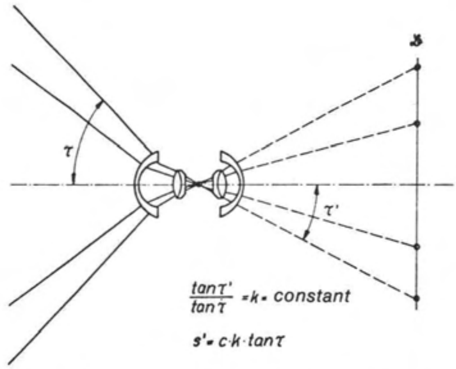

FIG. 5. Distortion-free superwide-angle lens.

the image distance from the principal point H' in the image plane to a particular image point will become

$$s' = c \cdot k \cdot tan\ \tau'. \qquad (5)$$

It is evident that congruent cones of rays in the object and image spaces constitute a special case only for the general relationship where $k = 1$. The absence of the necessity to have both cones of rays congruent is very significant for extremely wide-angle photogrammetric photolenses (Figure 5). In this instance, too, the optical designer is enabled to provide that the imaging rays strike the photographic emulsion at the smallest possible angle to the direction of incidence.

Distortions as defined in the preceding paragraphs can be determined only for an agreed wavelength of the light used for imaging (Figure 6). It may differ appreciably in other wavelength ranges (e.g., in the frequently-used near-infrared), depending on the lens design.

Deformation of the Distorted Model by Manufacturing Errors

Proceeding from the mathematical-geometrical model of perspective projection, we have so far considered those deviations only that occur in the physical and optical realization of such projection representing

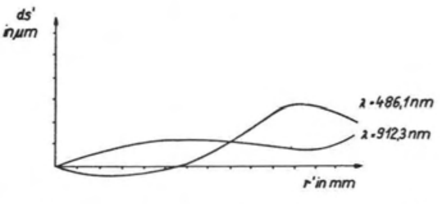

FIG. 6. Distortion as a function of the wavelength.

systematic errors. These considerations are based on the assumption that it might be technically possible to manufacture an optical system exactly equal to the physical design concept. This assumption actually is wrong. Although optical instrument manufacturers are able to keep inaccuracies within tolerances as narrow as a wavelength of light, the accumulation of such inaccuracies in a multi-component optical system will result in appreciable deformations of the imaging model. Practical experiments with a photolens will therefore reveal a superposition of the different influences involved. Attempts at determining the effective imaging function for a certain photogrammetric camera lens in one azimuth only from the optic axis will fail. If the test is repeated in several azimuths, results may contradict each other in a relatively high degree, especially for modern high-performance photogrammetric lenses having low absolute distortion (Figure 7).

Asymmetric Deformation

Investigating the image of a grid of equal geometrical spacing arranged symmetrically about the optic axis will generally reveal dislocations of the image points, to which may be ascribed a radial and a tangential component referred to the image center. As an effect of the radial component, imaging rays incident at equal angles to the optic axis are associated with different distances in the image plane (measured from the intersection point of the optic axis to those of the respective imaging rays) depending on the azimuth. In effect the radial component corresponds to an accidental tilt of the camera in taking the photograph of the test object. Consequently, it should be possible to compensate radial dislocation by intentionally tilting the taking axis toward the object (or the projection camera of a plotting machine in the course of relative orientation). Asymmetric deformations of the distortion model in the photograph are caused by decentrations in the optical system and tilting of its elements (Figure 8). Geometric-optical computation based on the assumption of such asymmetric-point dislocation discloses the existence of tangential components in addition to radial ones.

As a consequence, straight object lines may appear as curves in the image plane even if they pass the image center (Figure 9). Direct compensation of this tangential deformation effect in restitution is possible only if analytical techniques are employed. To account for both dislocation components, American publications have presented a simple-model concept based on the fact that an ideal lens with a weak prism (Figure 10) put in front of it can dislocate image points in a similar manner. This *thin prism theory* permits the derivation of a mathematical relationship between the radial and tangential components, and in accordance with practical experiments on optical systems it was found that the tangential share should be smaller than the compensable radial distortion. Investigations conducted by G. Würtz (G.D.R.) on modern, sophisticated high-performance lenses, however, did not generally confirm the thin prism concept, which was supported mainly by F. E. Washer (USA).

Affine Deformation

From the effective image point dislocations it is possible to isolate affinity errors in addition to asymmetries. Affinity errors make equal distances in the object appear con-

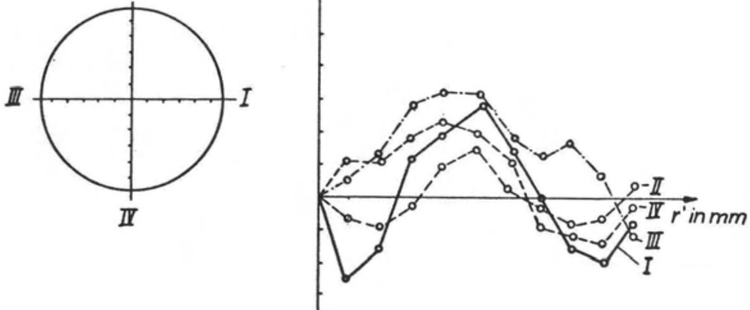

Fig. 7. Deformation of radial distortion in different azimuth directions.

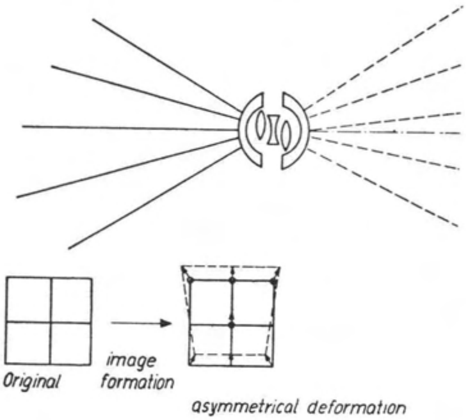

FIG. 8. Generation of asymmetrical deformation.

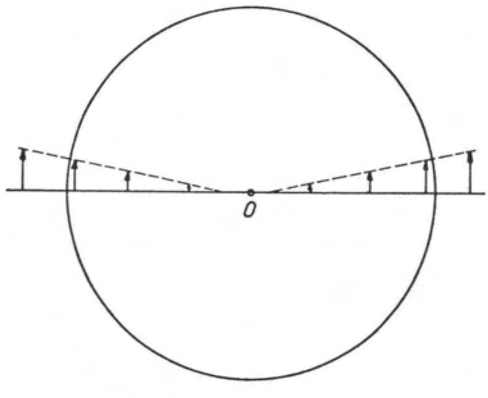

FIG. 9. Tangential components of asymmetrical deformations.

tracted or expanded (Figure 11) in the image depending on their azimuth position, so that a circle, e.g., would form an elliptical image. This error is possibly due to deviations of optical surfaces from the true spherical shape caused by imprecise polishing or by warping under the influence of pressure exerted by parts of the lens mounting. Deformations of that kind do not always act in a rotation-symmetrical way, so that they might in turn influence asymmetry as well. Exact compensation of affine deformations can be accomplished only if analytical restitution methods are employed.

In this connection it should be brought to attention that, theoretically, the elimination of deformations in the image plane by means of optical aspherical compensation plates used on the plotting machine is possible only as far as these deformations are situated symmetrically around the photo center. Such compensation plates, in turn, are likely to be afflicted by similar engineering deficiencies imparting new deformations. That is why optical compensation plates frequently fail to yield the expected gain in plotting accuracy for photographs taken through modern, high-performance lenses.

CAMERA GEOMETRY

The effective imaging function of an individual photogrammetric camera is described by the camera geometry. In addition to the properties of the lens system, the camera geometry comprises the influences of the camera body on image formation. These include the orientation and flatness of the focal-plane frame, the flatness and stability of the vacuum back in film cameras and the operation of the vacuum mechanism (Figure 12). Air survey cameras are frequently exposed to extreme temperatures. Fundamental studies of the temperature gradient behavior in air survey cameras and its influence on the calibrated focal length have been reported by U. Zeth (G.D.R.).

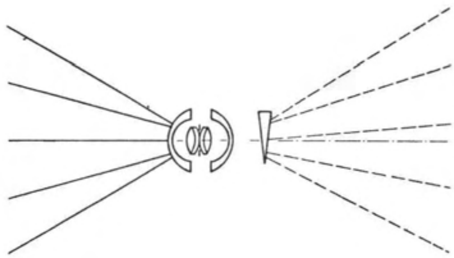

FIG. 10. Theory of the prism effect.

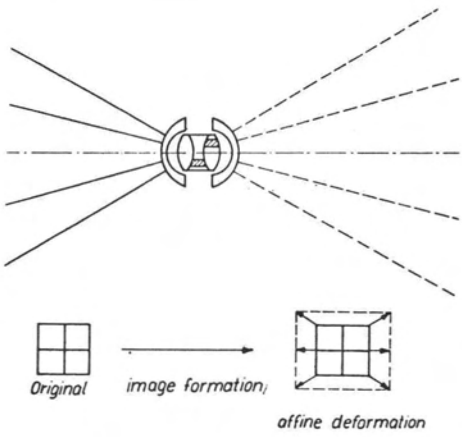

FIG. 11. Generation of affine deformation.

External Influences on Photogrammetric Photography

In aerial photogrammetry where photographs are taken from high altitudes and through wide-angle lenses, refraction effects and earth curvature (Figure 13) produce point dislocation in the image plane that may give rise to appreciable systematic errors in plotting. Perspective projection may further be impaired by the use of a sealing glass port in the airplane bottom. Controlled experimentation has proved that familiar models of deformation affecting the taking cone of rays are insufficient to describe the actual conditions. This suggests that the individual physical properties of air currents disturbed by the aircraft significantly influence the photographic geometry (Figure 14). Further impairments of the photograph may result from instabilities of the emulsion base, especially where film is used.

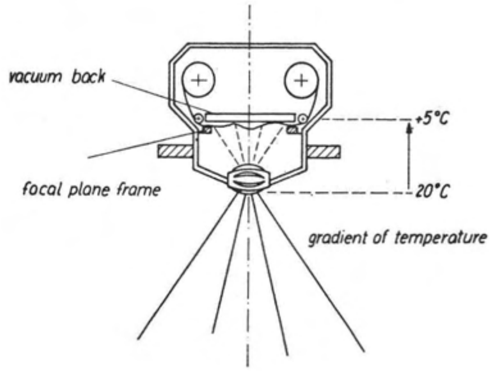

Fig. 12. Camera geometry.

Quasi Distortion in Analogue Plotters

Most analogue plotting machines reconstruct the two taking cones of rays so as to generate an optical model. If this is effected by means of a projection lens, i.e., by optical projection, it is self-evident that this process involves deformations analogous to those occurring in taking the photograph (Figure 15). Even where this reconstruction is implemented by mechanical projection, using either spatial guide rods or two-dimensional systems of straightedges for two projection planes, error patterns of a quite similar nature occur. Possible causes are deformations of space rods (frequently due to their own weight) and unavoidable residual errors in the space-rod gimbal joints.

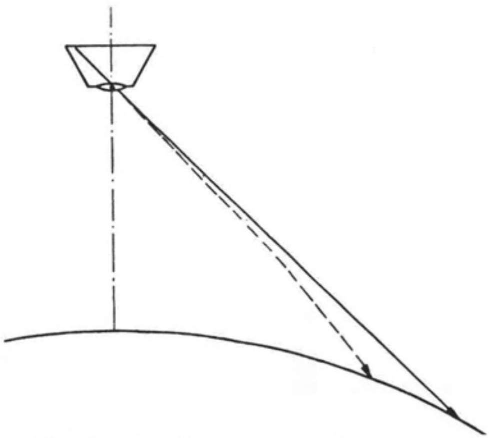

Fig. 13. Earth curvature and refraction.

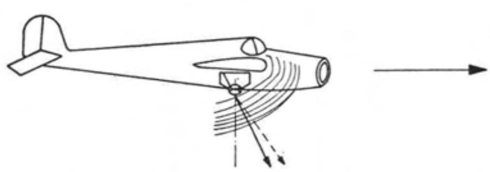

Fig. 14. Deflection of the image beam near the aircraft.

The Compensation of Systematic Model Deformations in Plotting

Attempts at correcting systematic imaging errors by instrumental means have so far made use of two devices. One is an optical correction plate inserted into the projection beam of the plotting machine as an optically refractive element to compensate for any residual *distortions* and for effects of earth curvature and atmospheric refraction. The other is a calibrated focal-plane grid (reseau) photographed along with the picture, the known reseau cross positions serving to improve the picture geometry.

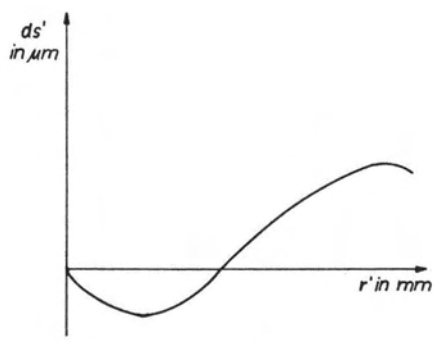

Fig. 15. Quasi-distortion in analogue plotters.

Optical Compensating Plates

In the light of the above discussion, optical compensating plates (Figure 16) are capable of correcting image point dislocations

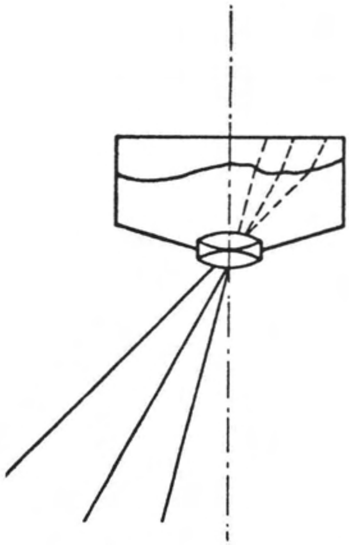

FIG. 16. Optical compensation plates.

situated symmetrically about the center, whereas they will not help to eliminate tangential and affine dislocations. Because radial deformation components in the modern, high-performance photogrammetric camera lenses are extremely small and, on the other hand, because compensating plates may in turn cause deformations on account of unavoidable manufacturing tolerances, the use of such plates will frequently fail to improve plotting results. Attempts to compensate symmetrical dislocation components resulting from earth curvature and atmospheric refraction are complicated by the necessity to employ different plates for changing flying altitudes.

RESEAUS

The idea behind a reseau image superposed on the picture (Figure 17) during exposure is to derive corrective interpolations from the known (calibrated) nominal positions of the reseau crosses, and from their actual coordinates measured in the photograph. The measurement of the reseau points on a comparator means considerable extra work. Such a method will be restricted to analytical restitution techniques. Comparing the results of practical applications of standard-type and reseau cameras, the so-called *Oberschwaben* test revealed no significant difference. This should be interpreted as a positive establishment of the fact that errors of interior camera geometry (which can only be corrected by a reseau) after correction are of secondary importance

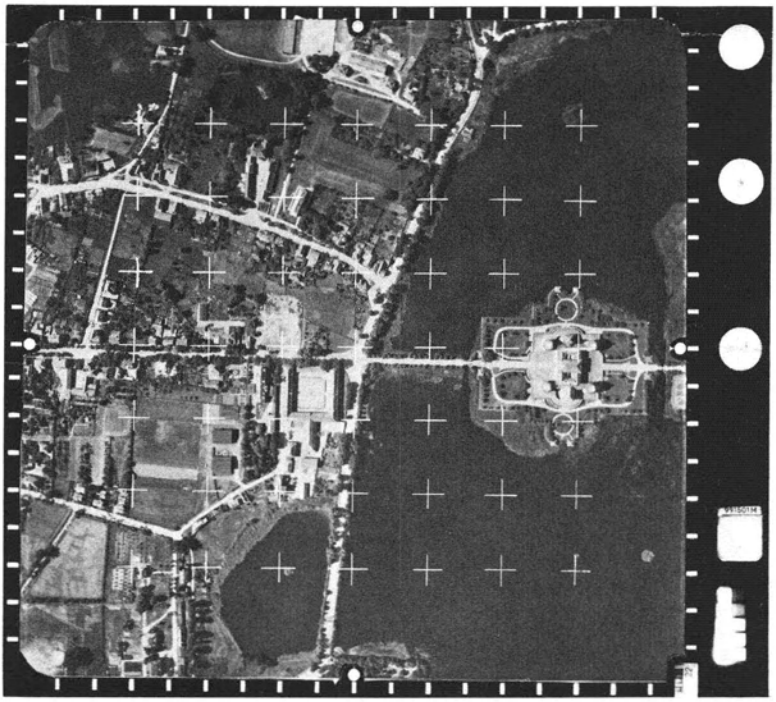

FIG. 17. Photograph with superimposed reseau.

compared to external influences on photogrammetric photography.

Correction of Model Deformations

If we have to acknowledge that the effective systematic deformation of the optical model formed by relative and absolute orientation is, after all, due to a number of causes still resisting evaluation, it seems appropriate to develop a correction model having corresponding degrees of freedom, the parameters of which can be determined from errors at known control points. Nothing in the analytical-photogrammetric restitution process would impede the implementation of that principle, and some initial promising approaches have been made to eliminate systematic error causes in aerial triangulation nets in that way.

To eliminate model deformations using precision analogue plotters, the Jena instrument designers have also been following that path. Compensation in the plotting equipment of optical model deformations is being effected by an electrical analogue computer (Figure 18) both for planimetry and elevation. The device has degrees of freedom that allow one to generate mutually independent cylindrical correction surfaces with x and y as their axes and being either concave or convex. The combination of both x and y corrections permits one to generate ellipsoidal surfaces of any desired orientation (Figure 19). Apart from the possibility of entering the operating parameters determining the degrees of freedom (which requires the causes of the model deformation to be known quantitatively), it is more important that the correction surface can be generated empirically by the elimination of discrepancies at control points of known coordinates. Practical experience gathered so far has clearly confirmed the utility of the device and its highly welcome universality, which is lacking in optical compensation plates.

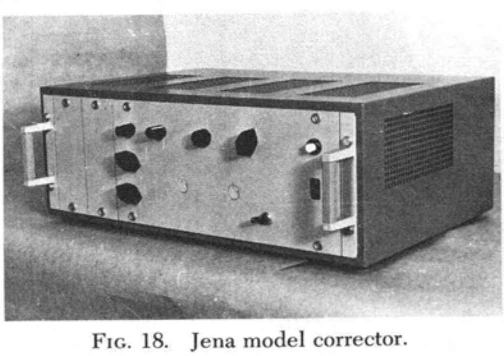

Fig. 18. Jena model corrector.

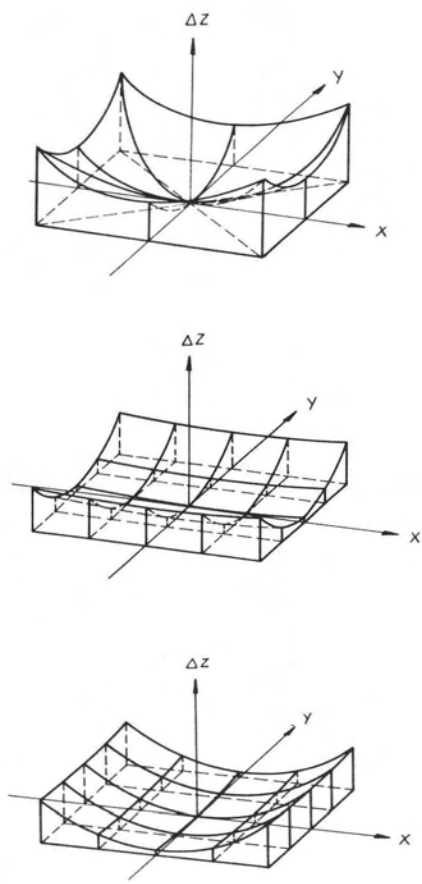

Fig. 19. Correction surfaces.

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Engineering Reports

Business—Equipment—Literature

Gordon R. Heath

Wild Announces PEM-1 Exposure Control System for RC10 Camera

Wild Heerbrugg Ltd. has introduced an automatic exposure control system which can be used without modification for all of the interchangeable

lens cones of the RC10 Universal Camera. The PEM-1 Exposure Meter provides continuous automatic control of the exposure time for optimum uniformity in exposures throughout the flight strips. For additional information contact Wild Heerbrugg Instruments, Inc., 465 Smith Street, Farmingdale, New York 11735.

An Australian Camera That Can Scale a Mountain

Precision rescaling with ease and speed for finished work by automatic processing is the idea behind the vertical graphic reproduction camera. The Precision Rescaler camera, designed by Artiscope Manufacturing Co. Pty. Ltd., enables a greater range for larger areas, and it supplies higher enlargements and reduction ratios. Constant demand for scale change, whether it be private companies searching for minerals; exploration, the laying of cables or pipelines; mapping; or the rezoning of cities make this camera a vital contribution. Cunningham & Walsh, Inc., 1888 Centry Park East, Los Angeles, Calif. 90067 is handling distribution of this new camera in this country.

Envrionmental Mapping of Powder River Basin in Wyoming Planned

A $50,000 contract has been awarded to the University of Wyoming Geology Department by the U. S. Geological Survey, Department of the Interior, to map current land use, vegetation, and surface features of about 25,000 square miles in the Powder River Basin of northeastern Wyoming as part of an environmental overview study of the basin. The one-year project is aimed at producing a series of three maps at scales of 1:500,000 (one inch equals nearly 8 miles) for use in regional land use planning, resources management, and environmental protection in the Powder River Basin. The basin contains some of the largest reserves of energy resources in the Nation, including coal, oil, gas and uranium.

Perkin-Elmer Releases Improved Microdensitometer

The microdensitometer produced by the Boller & Chivens Division of Perkin-Elmer forms the heart of a high speed data acquisition system which is completely interfaced to a committed computer. Hardware systems have been standardized, but the specific selection and arrangements are sufficiently flexible to allow for tailoring to specific applications. Efficient general purpose software system programs are furnished at no cost; special programs geared to individual data analysis or augmentation requirements may be furnished at nominal extra cost. You may contact them at 916 Meridian Ave., S. Pasadena, Cal. 91030.

International Light Has New UV Photometer

The new IL440 photometer is a low cost ($349.00), accurate, ultraviolet light measuring instrument capable of UV measurements from 300 to 500nm with probe A and 300 to 400nm with probe B. The sensitivity is variable in seven ranges from 100 microwatts/cm^2 full scale to 100 milliwatts/cm^2 full scale. Both negative and positive photo-resist exposure levels can be monitored. A small probe head conveniently fits into most mask aligners and exposure system image planes. A new six page