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Autocorrelation Functions in Stereoscopy

For sharp targets with annuli greater than 0.25 mrad, pointing precisions are constant with respect to AUTOPROD, a new parameter.

INTRODUCTION

THE PRECISIONS with which the visual system performs monocular, binocular and stereoscopic pointing tasks leads to some interesting postulations on how the different tasks may be performed. Pointing precisions of these tasks have been compared (Trinder, 1972) for different qualities of circular targets using a circular measuring mark, and comments have been made on the possible visual processes involved in these tasks.

ABSTRACT: *Autocorrelation functions, which are computed in automatic plotters during heighting measurements, are derived for circular targets for which steroscopic pointing precisions for a human observer are known. A parameter, AUTOPROD, is derived from the percentage decrease in the auto-correlation function multiplied by the target diameter. A simple relationship has been derived between AUTOPROD and pointing precision which agrees with findings of physiologists. The investigation indicates that pointing precision basically depends on the* quality and length *of the target border.*

In automatic correlation equipment which measures terrain heights from two photographs, it is common to evaluate the convolution of luminance intensities received by the electronic scanners of the two photographs. The location of the electronic scanners at which the convolution is a maximum for a certain selected sized area on the photograph, provides the information for height determination by the equipment of that area.

With such processes in mind it was considered that an investigation of properties of the function after convolution of the objects viewed in stereoscopic pointing should prove fruitful. The observations presented (Trinder, 1972) were used for such a test. The rate of change in the convolution function from the central value if the floating mark was displaced by the standard deviation of the pointing observations was then investigated and some significant conclusions have been derived. These conclusions give some further information on the visual processes in stereoscopic vision.

CONVOLUTIONS AND AUTOCORRELATION FUNCTION

Convolution of two functions $g(x)$ and $f(u)$ is given by:

$$h(x) = \int_{-\infty}^{+\infty} f(u)\, g(x-u)\, du\ .$$

The general process of convolution of two functions can be described as one in which one function is displaced with respect to the other, and the common area enclosed by the two functions evaluated. If these two functions are identical, the process produces an Auto-correlation function. This is often called *self convolution*.

Convolution of functions of the intensity profiles of circular targets used for this research will produce autocorrelation functions because the two observed targets were identical. The best way to perform these computations (and this was the procedure adopted in this study) is to transform the function into the frequency domain by Fourier Transforms, square the function in that domain (because convolutions in the frequency domain are determined from the product of the two functions), and then to transform back into the spatial domain. No normalization was necessary as percentage differences only were computed.

In automatic instruments, convolutions of the luminance intensities distributions received by electronic scanners of two corresponding photograph points are the basis for the determination of correct relative positions of electronic scanners, and hence correct terrain elevations (Bertram, 1965). The distributions of the intensities represent the functions to be convolved. Bertram demonstrates that the spot scanners used in automatic equipment may each be located in certain time-dependent positions relative to the corresponding image points on the two photographs, as they rapidly scan the two photographs. Time delays may be introduced into the motion of the scanners. This effectively displaces one distribution of intensities with respect to another. As the time delays are varied, the value of the convolution function between the two distributions can be determined. At its maximum the scanners are in their correct relative positions for height determination.

In stereoscopic heighting, the task may be considered similarly. The two images viewed are the object points. Two measuring marks must be brought into coincidence with the stereoscopically observed object points. Though the measuring marks are not identical to the flying spot scanners, they may be likened to them inasmuch as they are the means by which the visual system locates one object relative to the other. If the correct heighting position is determined, the respective measuring marks visually coincide with the corresponding objects. Computation of the autocorrelation function for different displacements between the two identical images should therefore produce a similar function to that used by automatic instruments for heighting.

For sharp images the autocorrelation function is triangular in cross-section, and it is clear that small displacements of the measuring marks produce a significant change in the autocorrelation function. However, as the targets become blurred, this change is not so sharp, and hence logically heighting precision should suffer. This is the aspect which has been investigated in this paper. The significant factor of the autocorrelation function affecting pointing precision was expected to be the rate at which the autocorrelation curve changed. Because the standard deviation is a measure of pointing precision, the percentage decrease from the maximum value of the autocorrelation function was computed if the two targets viewed were separated by a displacement equal to the standard deviation.

Observations were made (Trinder, 1972) on sharp and blurred targets whose luminance profiles were known. Autocorrelation functions on these profiles were derived using the computer program prepared by Trinder 1973 and the percentage drop in the autocorrelation function was computed. The percentage drop in autocorrelation function (Column 4), together with this value multiplied by the corresponding target diameter—from here on called AUTOPROD (Column 5)—are shown in Table 1 and plotted in Figure 1. Characteristics of the target, grade of blur of the density profile, size (subjective size in the case of blurred targets) and the pointing precision in each case are also shown in Table 1. This table is discussed in the following section.

Discussion

Sharp Targets

The table of percentage decrease in autocorrelation function for sharp targets with annuli greater than 0.25 mrad indicates a strictly linear relationship with target diameter. The *product* of percentage decrease in autocorrelation function and target diameter, AUTOPROD, however leads to a constant which is consistent with the pointing precisions obtained with such targets (Column 6, Table 1.) Computations were made only for circular targets up to 5 mrad in diameter because it was impossible to obtain sampling points in the computation sufficiently close for larger targets. Extrapolation of values in Column 4 for the larger targets is valid because the behavior of the autocorrelation function for sharp targets is entirely predictable.

This constancy of AUTOPROD, which has no parallel in monocular observations, may be compared with conclusions of Andersen and Weymouth (1923) which still appear to be the most suitable theories on stereoscopic vision. They stated that stereoscopic observations depend on the so-called *local signs* of individual receptor cones in the retina. Each cone has its own *local sign*, an assumption only justifiable if such cones have individual

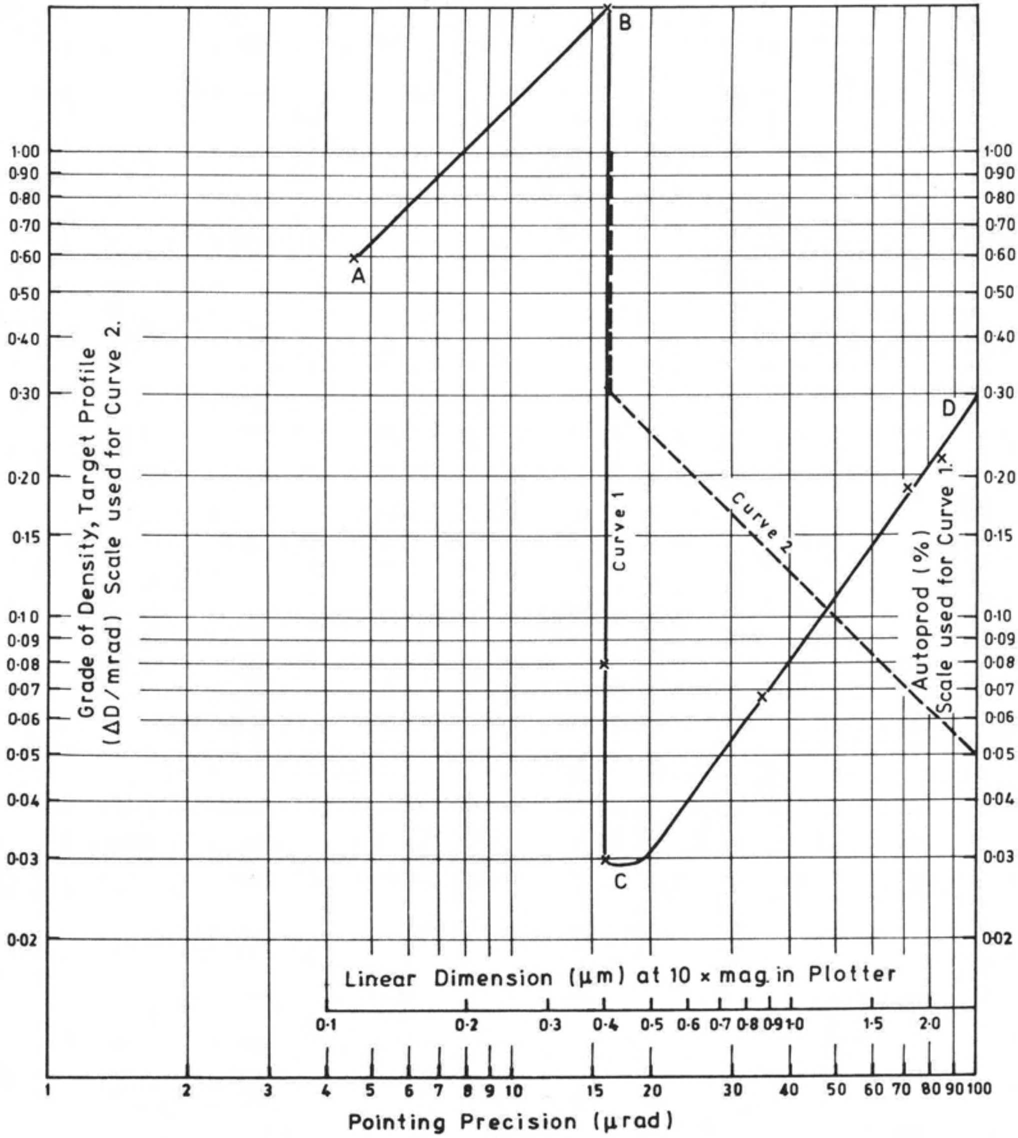

FIG.1. The relationship between AUTOPROD (on right hand abscissa scale) and pointing precision for sharp and blurred targets is given in Curve 1. The relationship between grade of the density profile (left hand abscissa scale) and pointing precision for the same targets is given in Curve 2.

connections through the visual system. The local signs of these cones will be related to those of other cones similarly stimulated. By virtue of the micro-nystagmus, or involuntary eye movements of the visual system, a still larger number of cones will be stimulated by the same object. The relative stimulation of the many cones will then produce a *mean local sign* which is used by the visual system in pointing.

Andersen and Weymouth stated that the larger the object the higher the accuracy of object location. The values in Column 4 of Table 1 therefore seem to be consistent with this statement. That is, the actual percentage decrease in autocorrelation function required for pointing decreases as the target diameter increases. If multiplied by the target diameter and therefore the circumference, a constant value is obtained for all target diameters, consistent with constant pointing precisions. It is noticeable in Table 1 that it is the circumference or length of target perimeter that leads to a constant

TABLE I.

1 Target Size mrad*	2 Grade of Density Profile ΔD/mrad	3 Size Annulus mrad	4 % Diff in Autocorr. fn	5 AUTOPROD	6 Pointing Precision (mrad)	7 Revised Values of AUTOPROD
Sharp Targets						
1.1	–	0.05	0.53	0.58	4.5	0.018
1.2	–	0.1	0.81	0.97	7.5	0.028
1.4	–	0.2	1.11	1.56	12.0	0.042
1.5	–	0.25	1.39	2.08	16	0.056
2.0	–	0.5	1.04	2.08	16	0.050
3	–	1.0	0.69	2.08	16	0.04
4	–	1.5	0.52	2.08	16	0.04
5	–	2.0	0.42	2.08	16	0.038
11	–	5.0	0.19	2.08	16	0.035
21	–	10.0	0.10	2.08	16	0.035
Blurred Targets						
5.0	>1.0	2.0	0.016	0.080	16	0.030
4.2	0.43	1.6	0.007	0.030	16	0.034
5.2	0.43	2.1	0.006	0.030	16	0.030
5.3	0.18	2.2	0.009	0.046	29	0.055
4.3	0.16	1.7	0.017	0.071	35	0.077
4.3	0.12	1.7	0.022	0.092	44	0.115
6.0	0.071	2.5	0.031	0.19	71	0.20
5.0	0.059	2.0	0.045	0.22	85	0.24
4.6	0.044	1.8	0.103	0.475	110	0.48

* 0.3 mrad = 1 min. of arc = 7.5 μm linear dimension when viewed in a plotter under an optical magnification of 10×.

AUTOPROD rather than the area. This factor is a variation on Andersen and Weymouth's conclusions, and it seems that the receptors stimulated by the perimeter only, or edge of the target, are used by the visual system.

The importance of edges or contours of objects in vision has been well established. Fry (1947) found that long straight borders of objects were more significant than saw-tooth or wavy edges. Lamar, *et al.*, (1947, p545) proposed that for targets larger than 0.6 mrad the critical region of the target is the ribbon inside its perimeter approximately 0.3 mrad wide. O'Connor (1967) believes that contour perception and associated mechanisms in the visual system involved in producing Mach bands are of great importance in acuity results. The work embodied in this paper further reinforces the importance of target borders in visual observations, and particularly stereoscopic observations.

For sharp targets with annuli less than 0.25 mrad the relationship between AUTOPROD and pointing precision follows a decreasing linear relationship with decreasing annulus size. This section is not easily explained but an attempt will be made in the following section. It should be pointed out, however, that these targets are smaller than those which normally occur in photogrammetric practice, and therefore do not require much discussion.

BLURRED TARGETS

AUTOPROD was also computed for blurred targets with annuli (the ribbon between the measuring mark and target edges) as shown in Table 1, in which the subjective target size was used. AUTOPROD is plotted against pointing precision in Figure 1, Curve 1, for which the right-hand abscissa scale has been used. Curve 2 is that derived by Trinder (1972) which relates pointing precision to grade of the density profile, marked on the left-hand abscissa scale. The sharp change in direction at point *C*, Curve 1, corresponds with the point of discontinuity in Curve 2. Section *B* to *C*, curve 1, corresponds with the vertical section of Curve 2. Sharp target characteristics therefore are found in the line *A* to *B*, curve 1, and towards infinity on the top of Curve 2.

Whereas a simple relationship exists between AUTOPROD and pointing precision for sharp targets, there does not seem to be a simple relationship for blurred targets. Indeed, along section *B*-*C*, Curve 1 (i.e., where the grade of target blur is greater than 0.3 ΔD/mrad) pointing precision is constant and, therefore, is independent of AUTOPROD. It is interesting that AUTOPROD drops to as low as 0.03 before pointing precisions deteriorate, yet a value of 2.08 is required for sharp targets. It seems that the constant precision of 16 μrad is the highest that can be attained by the visual system for a significant range of target blur and target sizes, despite the relatively large values of AUTOPROD for the sharp targets.

For increasing blur, with grade of the density profile less than 0.3 ΔD/mrad (the point of discontinuity in Curve 2), the curve takes a change in direction indicating the increased difficulty in stereoscopic pointing to very blurred targets. Clearly the curve would approach infinity as pointing becomes increasingly difficult. Over this section of the curve, the greater the rate of change in autocorrelation function the lower the pointing precision. This factor is in accordance with expectations and demonstrates that AUTOPROD is related to pointing precisions for significantly blurred targets. As most targets on aerial photographs have a density profile of 0.1 ΔD/mrad or less, section *B-C* applies to most photographic images.

Further thought on the significance of section *B-C*, Curve 1, Figure 1, led to consideration of influences of the blurring effects of the visual system on the quality of the image *actually seen by the observer's visual system* as pointing is performed. This subject has been discussed at length by Hempenius (1969) and Trinder (1971) for monocular pointing using the Point Spread Function of the optics of the eye and the visual system. Hempenius adopted a three-dimensional Gaussian function with $\sigma = 150$ μrad, for the Point Spread Function of the eye, whereas Trinder used several different functions in attempting to include also the effects of inhibition of the visual system. It was pointed out at that time, however, (Trinder, 1971) that the essential assumption of linearity in the behavior of the visual processes is not strictly valid, and therefore the method must be used with caution. In spite of this, both authors were able to derive from the method conclusions which explained aspects of monocular pointing otherwise not understood. Although the numerical values derived may suffer from the inaccuracies in the method, the general conclusions derived seem to be well founded. With such factors in mind, a similar technique seems appropriate in this paper.

Because the blurring effects of the eye will cause a reduction in the quality of the images seen by the observer, all targets should be convolved by the Point Spread Function of the eye and new AUTOPROD values computed. (The convolution computes the characteris-

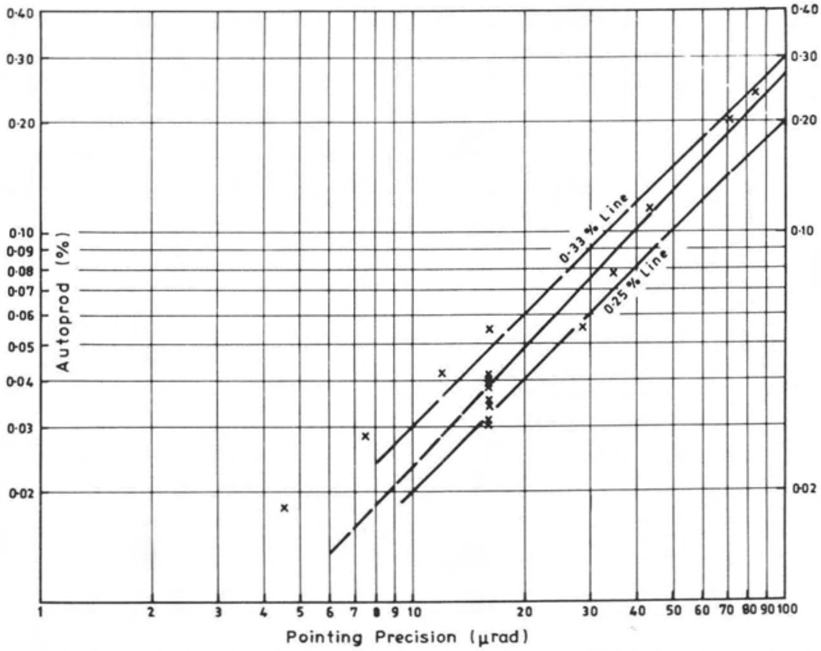

FIG. 2. The relationship between the revised values of AUTOPROD (Column 7, Table 1) and pointing precision.

tics of the target image on the retina). The new AUTOPROD values will then represent those available to the visual system for the pointing process. A three-dimensional Gaussian Point Spread Function with σ equal to 200 μrad was assumed. The effect of the blurring will be most noticeable on the AUTOPROD values of the sharp targets, and become less noticeable for the more blurred targets. The values of AUTOPROD after the convolution are given in Column 7, Table 1, and plotted in Figure 2.

In Figure 2 the relationship between AUTOPROD and precision is completely transformed. The section *B-C* in Figure 1 is now plotted in almost a single point between 0.03 and 0.04, and a simple linear relationship is shown. Further, the section *A-B* in Figure 1 is shifted almost on an extension of the interpolated straight line. Indeed, considering the complexities of the visual system for such small targets, and previous comments regarding imperfections in this approach, it is not difficult to visualize this section for small targets falling on the dashed line. The use of a Point Spread Function with σ equal to 300 μrad would achieve this. Therefore it is not unreasonable to state that the interpolated line closely approximates the complete relationship between pointing precision and AUTOPROD for circular targets.

Linear relationships on logarithmic scales such as in Figure 2 are well known in psychophysics (Stevens, 1962). From Figure 2 it is possible to state the simple relationship between pointing precision and AUTOPROD as follows:

AUTOPROD = 0.25% pointing precision (μrad) .

It may therefore be concluded that AUTOPROD of the image of the target on the retina of the eye is related to Pointing Precision by a simple relationship consistent with theories in psychophysics.

Conclusions

- For sharp targets with annuli greater than 0.25 mrad, pointing precisions are constant with AUTOPROD and agreement is found with conclusions of Andersen and Weymouth (1923). It may be concluded that the length of a target circumference is of prime importance in stereoscopic pointing. This factor agrees with findings of scientists who have investigated visual processes.
- A clearer picture of the relationship between AUTOPROD and pointing precision is gained if AUTOPROD is computed for the image presented by the eye on the retina of the visual system. Though there are some approximations in the approach used, it is valid to state that a simple relationship exists between AUTOPROD and pointing precision even for the very small targets. This relationship is consistent with general theories in psychophysics.
- Overall, the greater AUTOPROD, the lower the pointing precision. Since AUTOPROD depends on size and quality of a target, pointing precision depends on length and quality of its border.
- The results presented demonstrate the application of autocorrelation functions with one set of data. There are many other sets of data which should be investigated to find conclusively whether autocorrelation or convolution functions can be applied to the different sets of data viewed on aerial photography. It is considered, however, that results in this paper demonstrate an important agreement with physiological data.

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