B. SHMUTTER
Israel Institute of Technology
Haifa, Israel

Connecting Adjacent Models

The distribution of residual errors in the connection of adjacent models in a strip is discussed.

STRIP TRIANGULATION with independent models is used widely in photogrammetric practice. One of the problems inherent in various triangulation procedures is the connection of two consecutive models in the strip to each other. Whatever procedure is employed to perform the connection, it is usually based on an adjustment which utilizes redundant observations, these being the observed coordinates of points common to the two adjacent models. Because of the redundant observations, the adjustment yields residual errors, so that after the connection of the models has been performed, each point common to both models is associated with two sets of strip coordinates: one set that was determined while observing coordinates in the previous model, and another that resulted from the transformation of the observed coordinates of the "new" model. The differences between the values of these two sets of coordinates are the residual errors. Since each point should be determined by a unique set of strip coordinates, the question arises of how to handle the residuals in order to assign a single set of coordinates to each point. An analysis of this problem is presented.

ABSTRACT: *The paper discusses the problem of distributing residual errors that remain after connecting two consecutive models to each other in a strip. The strip coordinates of points located in the common zone of adjacent models are determined uniquely.*

It is assumed that the connection of a new model in the strip to the previous one is carried out by a coordinate transformation, the transformation elements being derived from the equation:

$$\boldsymbol{X} = X_o + \lambda \boldsymbol{a}\boldsymbol{x} \qquad (1)$$

in which:

$\boldsymbol{X}$ = coordinates of a point observed in the previous model,
$\boldsymbol{x}$ = coordinates of the same point observed in the new model,
X_o = a translation,
λ = a scale factor, and
$\boldsymbol{a}$ = an orthogonal rotation matrix whose elements are computed from three angles.

Each common point (including the projection centers) contributes one equation of type (1), i.e., three equations in coordinates, for the solution of the unknowns. Since Equation 1 is non-linear with respect to the sought scale factor and rotation angles, the unknowns have to be solved for by an iteration process which makes use of linear expressions obtained from expanding the above equations around an initial solution. Secondly, in order to fulfill Equation 1, all the measured quantities X and x have to be corrected. Hence, the demand for a unique determination of the coordinates of the common points, while solving the unknown elements for the connection procedure, leads to an adjustment which involves condition equations with unknowns

$$X_o + \lambda \boldsymbol{a}\,(\boldsymbol{x}+\boldsymbol{v}_n) - (\boldsymbol{X}+\boldsymbol{v}_p) = 0 \quad . \qquad (2)$$

The subscript n denotes the new model, and p – the previous one.

After linearizing Equation 2 the following expression is obtained:

$$\bar{\lambda}\bar{\boldsymbol{a}}\boldsymbol{v}_n - \boldsymbol{v}_p + \boldsymbol{A}\boldsymbol{Y} + (\bar{X}_o + \bar{\lambda}\bar{\boldsymbol{a}}\boldsymbol{x} - \boldsymbol{X}) = 0 \qquad (3)$$

where A is a coefficient matrix and Y is a vector which represents the sought corrections to the elements assumed in the initial solution (denoted in the equation by a bar).

Each common point yields three equations for solving the unknowns and the entire system of the condition equations has the form

$$BV + AX + W = 0 . \tag{4}$$

The V are the corrections to the observed coordinates and W the discrepancies (the bracketed term is Equation 3).

If the correlations between the observed coordinates are disregarded, which is common practice in many triangulation procedures, and the observations are assumed equally weighted, the condition Equation 4 provides the following solutions:

$$\begin{aligned} V &= B^T K \\ K &= -(BB^T)^{-1}(AY+W) \\ Y &= -(A^T(BB^T)^{-1}A)^{-1}A^T(BB^T)^{-1}W \end{aligned} \tag{5}$$

The matrix B has a quasi-diagonal form and can be presented by

$$B = \begin{bmatrix} B_1 & 0 & . & . & 0 \\ 0 & B_2 & . & . & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ 0 & 0 & . & . & B_n \end{bmatrix} . \tag{6}$$

Each matrix B_i is associated with the corrections to one common point. All matrices B_i are equal to each other and have the following form:

$$B_i = [\bar{\lambda}\bar{a} \quad -I]. \tag{7}$$

Since $\bar{a}$ is an orthogonal matrix and I a unit matrix, we have

$$\begin{aligned} BB^T &= (\lambda^2 + 1)\, I \\ (BB^T)^{-1} &= \frac{1}{\lambda^2 + 1} I. \end{aligned} \tag{8}$$

It follows therefore that the correlates for computing corrections to the observed coordinates are given by

$$K = -\frac{1}{\lambda^2+1}\, I\,(AY + W) \tag{9}$$

and the unknown corrections to the orientation elements are solved from

$$Y = -(A^T A)^{-1} A^T W . \tag{10}$$

According to Equation 10 the solution of the required quantities Y is identical with that which would be obtained if the problem were stated in terms of usual observation equations.

The solution of the orientation elements is based on an iteration process; for each step the matrix A and the discrepancies W are computed anew utilizing the corrections Y found in the previous step. After the last iteration step (convergence of the process) the corrections Y will be very small, practically equal to zero. Thus, the correlates K may be computed from

$$K = -\frac{1}{\lambda^2 + 1} W \tag{11}$$

where the W represent the final discrepancies between the coordinates and are defined as follows:

$$W = X_{(transformed)} - X. \tag{12}$$

The corrections to be added to the observed coordinates are given by

$$V = B^T K = -\frac{1}{\lambda^2+1} B^T W . \tag{13}$$

Taking into account the form of the matrix B, the required correction to the observed coordinates are computed as follows: The corrections for the coordinates of the previous model equal

$$v_p = \frac{1}{1+\lambda^2} W \tag{14}$$

and, for the corrections of the new model, one has

$$v_n = -\frac{\lambda}{1+\lambda^2} a^T W . \tag{15}$$

The corrections v_n ought to be added to the observed coordinates of the new model, and the corrected coordinates inserted into the transformation equations. Instead, regarding the fact that the transformation is linear with respect to the coordinates, the values v_n can be transformed separately, and their transformed values then added to the transformed coordinates which were obtained during the solution of the orientation and utilized for the determination of the discrepancies W.

$$v_{n\,(transformed)} = \lambda a v_n = -\frac{\lambda^2}{\lambda^2+1} W . \tag{16}$$

Equation 16 holds because a is an orthogonal matrix, and aa^T is a unit matrix.

Now we arrive at the following conclusions: The discrepancies between the coor-

dinates which remain after performing the connection between two models are distributed according to the ratios stated by Equation 14 and 16.

Usually the scale variations between two consecutive models are small and the value of λ is very close to unity. Hence,

$$\frac{1}{1+\lambda^2} \simeq \frac{\lambda^2}{1+\lambda^2} \simeq \frac{1}{2}$$

It follows, therefore, that when the scale alterations between consecutive models are small, the common practice of averaging the two sets of coordinates for each point is fully justified.

The author believes that the justification for averaging the coordinates as presented above will satisfy all who have intuitively applied that rule, even when they had doubts about its theoretical basis.

References

1. B. Shmutter, Triangulation with independent models, *Photogrammetric Engineering*, June 1969.

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