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Three-Dimensional Transformations of Independent Models*

A simultaneous three-dimensional linear conformal adjustment of independent models is analogous to the bundle adjustment.

ABSTRACT: *A method for the simultaneous three-dimensional linear conformal adjustment of the independent models in a block has been developed and programmed. The method is shown to be analogous to the bundle type of adjustment. In fact, both methods are encompassed in one computer program. The ISP simulated test block is used to test the accuracy of the method. The results show that this method is of the same order of accuracy as the bundle adjustment.*

INTRODUCTION

SIMULTANEOUS block adjustment, as referred to in this paper, is the solution for all the unknown parameters for the block at one time, using all the photogrammetric observations and ground coordinate observations for the block. The unknown parameters are (1) the orientation parameters for the basic photogrammetric measurement units, e.g., photos or models, (2) any added parameters that are the same for all the units, e.g., symmetric lens distortion, and (3) the uncontrolled ground coordinates. The photogrammetric observations are the photo coordinates in the case of fully analytical photogrammetry, or the model coordinates in the case of semi-analytical photogrammetry. Photo coordinates are the comparator measurements corrected for all known systematic distortions; likewise, the model coordinates are the stereoplotter measurements corrected for all known systematic distortions. In practice, stereoplotter measurements usually are used directly as model coordinates.

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The simultaneous adjustment of photos or bundle adjustment has been used in fully analytical block triangulation for over ten years. Until recently, the closest thing to a simultaneous adjustment for semi-analytical aerotriangulation was the "half-photo" method in which the stereoplotter measurements are resected to create coordinates in the two "half-photos" that made up the model. These "half-photos" are then subjected to a bundle adjustment.

At the XIIth Congress of the International Society of Photogrammetry in 1972, Prof. Ackermann, et al., of Stuttgart University presented a paper dealing with the simultaneous adjustment of independent model observations[1]. Two methods were discussed. The first involved the simultaneous three-dimensional similarity or linear conformal transformations of all the independent models in a block. In the second, the block adjustment was accomplished by iterative sequential horizontal and vertical adjustments. It is this second method that was actually programmed at Stuttgart, designated as PAT-M 43, and used in the 1972 ISP Working Group and also for results reported by Ackermann in 1974[2].

In order to investigate the potential benefits of a block adjustment by three-dimensional linear conformal transformations of independent models, hereafter referred to as 3-D independent models, a method based on the linearized projective equations was developed and programmed. Tests were performed by using the simulated test block of the ISP Commission III 1968-1972 Working Group. Testing with simulated data is especially suited for determining potential accuracy and for comparing different approaches to the same problem. True capabilities can only be determined using actual field data. No results using field data are reported in this paper.

Mathematical Basis

The reasons for approaching the problem with the projective equations are that the physical situation is easy to visualize (see Figure 1), and that the collinearity equations, which are the basis for the bundle adjustment, may be derived from these equations.

The Projective Equations

The projective equations relate the object space system to the image space system as

$$\begin{bmatrix} x - dx \\ y - dy \\ z - dz \end{bmatrix}_{ij} = s_i M_i \begin{bmatrix} X_j - X_i^c \\ Y_j - Y_i^c \\ Z_j - Z_i^c \end{bmatrix} \qquad (1)$$

where

x_{ij}, y_{ij}, z_{ij} are the image space coordinates of the j-th ground point in the i-th image space with reference to an assumed origin;

$dx_{ij}, dy_{ij}, dz_{ij}$ represent the distortions to the ij-th image due to any added parameters;

s_i is the scale of the i-th image space;

M_i is the orthogonal rotation matrix for the i-th image space;

$$M_i = \begin{bmatrix} A & B & C \\ A' & B' & C' \\ D & E & F \end{bmatrix}_i$$

$$= \begin{bmatrix} c\phi c\kappa & c\omega s\kappa + s\omega s\phi c\kappa & s\omega s\kappa - c\omega s\phi c\kappa \\ -c\phi s\kappa & c\omega c\kappa - s\omega s\phi s\kappa & s\omega c\kappa + c\omega s\phi s\kappa \\ s\phi & -s\omega c\phi & c\omega c\phi \end{bmatrix}_i$$

X_j, Y_j, Z_j are the object space coordinates of the j-th ground point;

X_i^c, Y_i^c, Z_i^c are the object space coordinates of some discrete point in the i-th image space.

Equations (1) are linearized by Taylor's series expansion about the measured quantities (x_{ij}, y_{ij}, z_{ij}), and the initial approximations for the unknown parameters $(s_i, \omega_i, \phi_i, \kappa_i, X_i^c, Y_i^c, Z_i^c, X_j, Y_j, Z_j)$ and any added parameters.

The Collinearity Equations

The collinearity equations used in the bundle adjustment may be derived from the projective equations (1). Consider the photo coordinate system as the image space system (see Figure 2). The term $(-f_i)$ is substituted for the term $(z - dz)_{ij}$. This represents that the focal length, f_i, is constant for the i-th photo. This third equation is divided into the equations for $(x - dx)_{ij}$ and $(y - dy)_{ij}$ resulting in the collinearity equations,

$$(x - dx)_{ij} = -f_i \frac{(X_j - X_i^c)A_i + (Y_j - Y_i^c)B_i + (Z_j - Z_i^c)C_i}{(X_j - X_i^c)D_i + (Y_j - Y_i^c)E_i + (Z_j - Z_i^c)F_i}$$

$$(y - dy)_{ij} = -f_i \frac{(X_j - X_i^c)A_i' + (Y_j - Y_i^c)B_i' + (Z_j - Z_i^c)C_i'}{(X_j - X_i^c)D_i + (Y_j - Y_i^c)E_i + (Z_j - Z_i^c)F_i} \qquad (2)$$

Equations (2) are linearized by Taylor's series expansion about the measured quan-

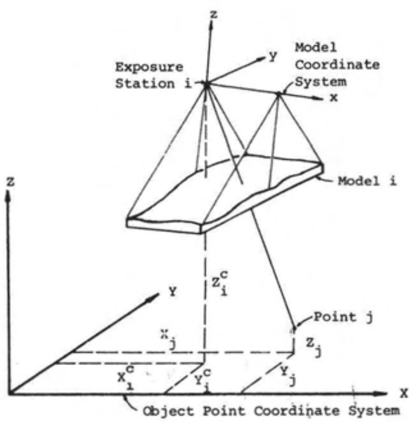

FIG. 1. Projective equations.

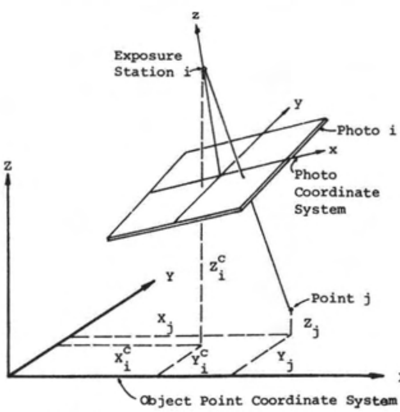

FIG. 2. Collinearity equations.

tities (x_{ij}, y_{ij}), and the initial approximations for the unknown parameters $(\omega_i, \phi_i, \kappa_i, X_i^c, Y_i^c, Z_i^c, X_j, Y_j, Z_j)$ and any added parameters.

The Linearized Observation Equations

The linearized form of either equations (1) or (2) may be expressed in matrix notation as

$$\boldsymbol{v}_{ij} + \dot{\boldsymbol{B}}_{ij}\dot{\Delta}_i + \bar{\boldsymbol{B}}_{ij}\bar{\Delta} + \tilde{\boldsymbol{B}}_{ij}\tilde{\Delta}_j + \boldsymbol{F}_{ij} = 0 \quad (3)$$

where

- $\underset{a,1}{\boldsymbol{v}_{ij}}$ is the vector of residuals for the photogrammetric observations;
- $\underset{a,b}{\dot{\boldsymbol{B}}_{ij}}$ is the matrix of partial derivatives for the orientation parameters of the i-th image space;
- $\underset{a,c}{\bar{\boldsymbol{B}}_{ij}}$ is the matrix of linear coefficients for the added parameters;
- $\underset{a,3}{\tilde{\boldsymbol{B}}_{ij}}$ is the matrix of partial derivatives for the object space coordinates of the j-th ground point;
- $\underset{b,1}{\dot{\Delta}_i}$ is the vector of corrections for the orientation parameters of the i-th image space;
- $\underset{c,1}{\bar{\Delta}}$ is the vector of added parameters;
- $\underset{3,1}{\tilde{\Delta}_j}$ is the vector of corrections for the object space coordinates of the j-th ground point;
- $\underset{a,1}{\boldsymbol{F}_{ij}}$ is the discrepancy vector for ij-th photogrammetric observation;
- a is the number of observation equations per photogrammetric observation (3 for independent model, 2 for bundle);
- b is the number of orientation parameters per image space (7 for independent model, 6 for bundle); and
- c is the number of added parameters.

Equations 1, 2, and 3 have been presented here mainly to demonstrate the similarities between the independent model adjustment and the bundle adjustment. The projective Equations (1) are the basis for the simultaneous three-dimensional linear conformal adjustment of independent models. The collinearity Equations (2) are the basis for the simultaneous adjustment of photos or bundle adjustment. The linearized observation Equations (3) apply to either the projective or the collinearity equations. Analytically, there is no difference between the two methods of adjustment after the linearized coefficients have been formed.

The reduced normal equations are formed and solved, and the object point coordinates are adjusted in exactly the same manner for either method.

The Computer Program

Both the 3-D independent model adjustment and the bundle adjustment are incorporated in one computer program called ALBANY. This stands for the "*a*djustment of *l*arge *b*locks with *any* number of photos, points and images, using *any* photogrammetric measuring instrument, and on *any* computer." The program will adjust a block of virtually unlimited size in 120K bytes on an IBM 360 computer, or 100k octal words on a CDC 6600. Random access files are used to store all the data not in immediate use by the computational routines. Initial approximations for the orientation parameters are input and, from these, the initial approximations for the ground coordinates are computed by the program. It takes one to three iterations for a solution, depending on the accuracy of the initial approximations. Between one and two (1-2) system seconds per photogrammetric unit per iteration is required on a CDC 6600.

The ALBANY program began in 1972 at the University of California, Berkeley, where the author wrote the computer program TURTLE, which performs simultaneous adjustment of photos and camera calibration using up to 25 photos. This program later became the basic production program for analytic aerotriangulation for the California Department of Transportation (CALTRANS). The author also developed a program for CALTRANS which performs a bundle adjustment on a block of virtually an unlimited number of photos. It is to this latter program that the author, as an independent researcher, added the 3-D independent model capability to produce the ALBANY program.

Tests With Simulated Data

In order to determine the potential accuracy of the method of 3-D independent models, and to compare it with others, the ISP simulated test block was used.

ISP Test Block

As used by the ISP Commission III 1968-1972 Working Group, this block consists of 5 strips of 20 photos each, with a nominal overlap of 60 per cent and a nominal sidelap of 20 per cent. The flight height above mean terrain is 11,000 meters, and terrain relief over the block is 1000 meters. The camera focal length is 152 millimeters, resulting in an average photo scale of 1:66,000.

In the final summary of the ISP Commission III Working Group on Analytical Block Adjustment[3], the test results of the 12 participating organizations are presented. Each organization adjusted the block with its own computer programs, but all used the same control configurations. In this way, the individual programs and methods could be compared.

The three regular control configurations were used, designated as A, B and C, and a modified version of A, designated as A*. Only one set of simulated image coordinates was used in this study. These contained random normal deviates having a standard error of 6 micrometers plus residual systematic deviates. The corresponding test cases are designated as 2A, 2B and 2C (see Figure 3).

Prior to the simultaneous adjustment, the simulated photo coordinates were subjected to two photo relative orientation, using the collinearity equations, in order to obtain the simulated independent model coordinates.

3-D INDEPENDENT MODEL TEST RESULTS

Table 1 summarizes the results of some of the participants of the ISP Commission III Working Group. In addition, entries 13 and 14 are included. The participants are listed below.

1. National Research Council of Canada, Mr. G. H. Schut. These results are the best of the sequential type adjustment.
8. Stuttgart University, using the PAT-M 43 program.
9. Helsinki University of Technology, using a bundle adjustment.
11. D.B.A. Systems, Inc., using a bundle adjustment.
12. National Ocean Survey (NOS) NOAA, using a bundle adjustment.
13. California Department of Transportation, using a bundle adjustment.
14. 3-D independent model adjustment facility of the ALBANY program.

In Table 2, all the other results are compared to the 3-D independent model results. The values in the table are the percentages that the RMSE is reduced (−) or increased (+) by the use of 3-D independent models. A minus sign (−) means that the 3-D independent model results are better.

The method of 3-D independent models shows an average improvement both horizontally and vertically of 30 per cent over the sequential adjustment of participant 1. The maximum improvement occurs with the sparse control configuration. Comparison with the PAT-M 43 program shows average horizontal

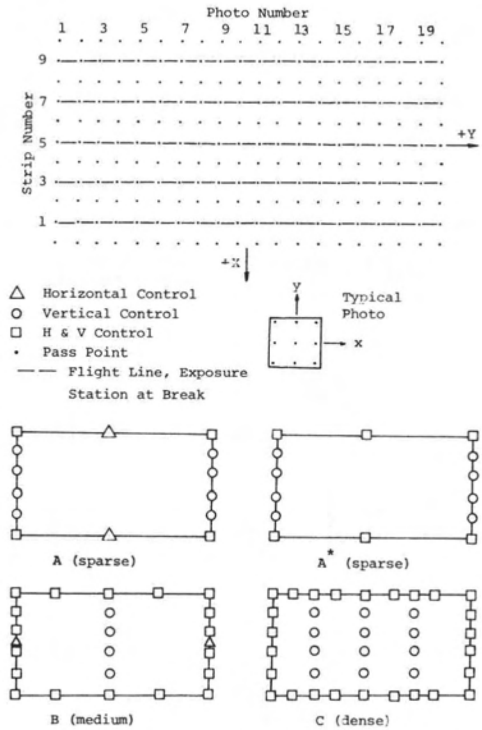

FIG. 3. ISP block and control configurations.

TABLE 1. HORIZONTAL AND VERTICAL RMSE DISCREPANCIES OF CHECK POINTS (IN METERS).

Participant	2A (sparse)		2B (medium)		2C (dense)	
	m_{XY}	m_Z	m_{XY}	m_Z	m_{XY}	m_Z
1	3.5*	3.5*	1.5	2.4	1.3	1.4
8	2.27	5.20	1.92	2.45	1.33	1.66
9	2.34	2.39	1.31	2.03	0.69	1.32
11	2.21	3.18	2.12	2.25	0.68	0.89
12	1.45	3.19	1.41	1.86	0.70	0.90
13	1.72	2.80	1.60	2.10	0.66	0.95
Avg. Bundle	1.93	2.89	1.61	2.06	0.68	1.01
14	1.94	3.64	1.28	1.95	1.02	1.11
14	1.95*	2.08*				

* Using modified control configuration A.
1—Sequential
8—PAT-M 43
9-13—Bundle
14—3-D Independent Model

TABLE 2. PER CENT CHANGE BY USING 3-D INDEPENDENT MODEL AS COMPARED TO OTHER METHODS.

Participant	2A (sparse)		2B (medium)		2C (dense)	
	m_{XY}	m_Z	m_{XY}	m_Z	m_{XY}	m_Z
1	−44*	−41*	−15	−19	−22	−21
8	−15	−30	−33	−20	−23	−33
9	−13	+52	−02	−04	+48	−16
11	−12	+14	−40	−13	+50	+25
12	+34	+15	−09	+05	+46	+23
13	+12	+30	−20	−07	+55	+17
Avg. Bundle	00	−27	−20	−06	+30	+09

and vertical improvements of 24 and 26 per cent respectively. Similar comparisons with the bundle adjustments show mixed results. Depending on the participant compared, discrepancies are increased by as much as 17 per cent, or decreased by as much as 55 per cent. Considering only the average bundle adjustment, 3-D independent models shows horizontal improvement for medium control, vertical deterioration for sparse control, and horizontal deterioration for dense control.

CONCLUSIONS

The simultaneous three-dimensional linear conformal transformations of independent models yields results which were improved when compared with (a) the sequential type adjustment, (b) the simultaneous independent model method in which the horizontal and vertical adjustments are sequential, and (c) some of the bundle adjustments.

The 3-D independent model method promises to be the best analytical solution for blocks of independent models, and also for blocks containing both independent model measurements from a stereoplotter and photo measurements from a comparator. In this latter instance, the photo coordinates would first be subjected to image coordinate refinement and two-photo relative orientation. Then these "independent models" would be combined with the stereoplotter measurements to form the block.

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