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Bayesian Decision Theory and Remote Sensing

Bayesian decision theory may be employed to determine the net benefits of remote sensing data obtained from small scale imagery versus that obtained from more accurate but more costly large scale imagery.

Introduction

Water resources planning in urban areas commonly employs mathematical models that simulate the hydrology of the watershed. A number of these models are so designed that remote sensing, i.e., multispectral, can provide some of the required input data.

A recent investigation of the Anacostia River Basin (Ealy, Ragan, and McCuen (1975)) in the Maryland suburbs of Washington, D.C., showed that the landcover and percent imperviousness needed in urban hydrologic modeling could be obtained by computer aided analysis of LANDSAT data. Overall, the results were good but errors were noted. However, the investigation produced the information in four man-days. A previous investigation (Ragan and Rebuck (1974)) based on human interpretation of 1:4800 scale aerial photographs gave more accurate results but required 94 man-days.

A decision maker, forced to choose between data accuracy and the cost of collection and processing, should attempt to evaluate the trade-off in dollars and cents. A criterion for evaluation is economic efficiency. The determination of the most economically efficient data source requires the evaluation of the net benefits of the information provided. Net benefits is the difference between benefits and costs. The costs are deterministic and procedures such as those developed by Cheeseman (1973) can be used for estimation. The benefits are difficult to quantify.

Economic analysis using Bayesian Decision Theory allows the potential user of remotely sensed information to determine the expected value of the information before he purchases it. In essence, the technique converts statistical information of source performance into monetary terms.

Abstract: *Remote sensing can provide valuable information for decision-making in a wide variety of problems. The inability of the data producer to express the capabilities of remote sensing in monetary terms inhibits its use as an information source. Bayesian Decision Theory can provide monetary information for evaluating the worth of data, in some cases. An outline of Bayesian Decision Theory for finite-discrete problems is presented. An example problem in land use identification for water resources planning is discussed.*

Remotely Sensed Information as an Economic Commodity

Marschak (1968) presents an instructive way of viewing the decision-making process. Decision making is a production process that uses inputs of information and man-hours in a model that produces the decision variable.

Microeconomic analysis of production processes requires that profit be maximized. Profit maximization requires the determination of the optimal input combination. Profit

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and input units must be measurable, and knowledge of the value of the input is assumed to be perfect.

Conventional analysis fails to provide the answers in evaluating remote sensing information for a number of reasons. First, applications of remote sensing are commonly associated with public projects. The measurement of profits becomes difficult, but not impossible, due to the occurrence of nonmonetary benefits. Techniques such as benefit/cost analysis are used for estimation.

A second reason for the failure of conventional economic analysis is that a general measure of information hasn't been developed. Therefore it isn't possible to employ techniques that optimize the input mixture.

Finally, the information provided by a source will not be perfect. The potential user cannot know the value of the information without possessing and applying it. Therefore, if he must pay for information knowing that it is imperfect, he will be reluctant to do so. On the other hand, if he is allowed to examine the information before purchase, he will then possess the knowledge it contains and will have no reason to buy it. Bohm (1973) identifies the conflict as the central problem of information economics.

The problems discussed are those which currently prevent the full exploitation of remote sensing techniques as information sources. Rather than give up and rely on potential users to gamble on remote sensing, a second-best solution to the problem can be attempted. One approach is to use Bayesian Decision Theory.

Economic Analysis of Information Sources Using Bayesian Decision Theory

The discussion of Bayesian Decision Theory presented here focuses on the type of problem in which there is a closed set of discrete values for the information variable, such as land use classification. The presentation is by necessity cursory. The interested reader is referred to the works of Raiffa (1968) and Schlaifer (1959) for a more comprehensive treatment of the subject.

A decision problem can be defined as the choice of an action to optimize a criterion. The criterion will be called the payoff and is identical to the net benefits of a project. If the decision maker chooses a nonoptimal action, he suffers an opportunity loss. An opportunity loss is defined as the difference between the payoffs of the chosen nonoptimal action and the optimal action (OL_{ij}; the opportunity loss of choosing action j when action i is optimal).

Most decisions are made based upon the knowledge of one or more parameters. The parameters identify the optimal action. Without knowledge of the parameter values, the decision maker must employ a decision rule such as minimax to choose an action. Decision rules such as minimax can lead to illogical actions.

Prior Analysis

The decision maker usually has some information concerning the parameter values. The information may be based on his own experience, observation, or other sources. The acceptance of the principles of Bayesian Decision Theory allows the decision maker to translate the information into subjective prior probabilities for the parameter values. The only restriction placed on the prior probabilities is that they sum to one over the set of n values for the parameter, S_j.

$$\sum_{j=1}^{n} P(S_j) = 1 \tag{1}$$

The expected opportunity loss of any action, a_i, is the sum of the opportunity losses of choosing a_i weighted by their respective prior probabilities. The optimal action is that which minimizes the sum,

$$R_o = \min_i EOL(a_i) = \min_i \sum_{j=1}^{n} OL_{ij} \cdot P(S_j) \tag{2}$$

The variable R_o is the cost of uncertainty or the expected value of perfect information. A logical decision maker should be willing to spend up to R_o to obtain the true value of S_j since on the average he will lose this amount due to uncertainty.

Posterior Analysis

The decision maker may be able to obtain additional information and must know if it will be of any value to him. An information source can be represented by an information structure. Marshack and Radner (1972) define an information structure as a function which identifies a signal $(y_k;\ k=1, l)$ with a subset of parameter values. To completely define a source a performance index also is needed. The index is the probability of correct and incorrect parameter identification based on past performance.

In remote sensing the signal might be a classification and the parameter values the true classes. The information sources, $IS(M)$, may reduce the cost of uncertainty by providing additional data on the parameter values. The additional information is the conditional probability of a particular signal given a true parameter value ($P[y_k|S_j]$), the performance index discussed previously. The complete set of the conditional error probabilities can be presented in matrix form. In remote sensing the set is commonly called the confusion matrix.

Given a particular signal y_k the decision maker can apply Bayes' Theorem for the revision of prior probabilities to obtain the posterior probability of each variable of the parameter

$$P(S_j \mid y_k) = \frac{P(S_j) \cdot P(y_k \mid S_j)}{\sum_{j=1}^{n} P(S_j) \cdot P(y_k \mid S_j)} \quad (3)$$

The posterior probability, $P(S_j|y_k)$, represents the probability of the parameter value after the signal has occurred. The denominator of the right hand side of Equation 3 is the marginal probability of the signal occurring.

After calculating the posterior probability for each class, the decision maker can use the values in the same manner as he used the priors to calculate the expected opportunity loss. The minimum value is selected from the set of expected opportunity losses and identifies the optimal action given the signal y_k.

$$\begin{aligned} R_1(y_k) &= \min_i EOL\,(a_i \mid y_k) \\ &= \min_i \sum_{j=1}^{n} OL_{ij} \cdot P(S_j \mid y_k) \end{aligned} \quad (4)$$

The variable $R_1(y_k)$ is the value of perfect information after the additional information provided by the signal has been considered. The value of the information, $V(IS(M),y_k)$, provided by the signal is the reduction in the cost of uncertainty and is calculated by subtracting the results of Equation 4 from Equation 3.

$$V(IS(M),y_k) = R_o - R_1(y_k) \quad (5)$$

Posterior analysis typifies the problem of considering information as an economic commodity. The decision maker sacrificed resources to obtain the signal y_k but was unable to judge the worth of his effort until after he knew the signal. If the cost of the information was greater than the value as determined by Equation 5, he made a wrong choice.

PREPOSTERIOR ANALYSIS

If the procedure outlined for posterior analysis was performed for each possible signal the decision maker would know the optimal action and expected opportunity loss for each signal. The marginal probability of each signal was defined as

$$P(y_k) = \sum_{j=1}^{n} P(S_j) \cdot P(y_k \mid S_j). \quad (6)$$

Using the marginal probability and expected opportunity loss for a signal, an expected value of the expected opportunity loss can be calculated. The value is obtained by summing the multiples of the results of Equation 6 and Equation 5 over the set of signals.

$$R_2(IS(M)) = \sum_{k=1}^{l} P(y_k) \cdot \min_i EOL(a_i \mid y_k) \quad (7)$$

The value of the information source is the reduction in the cost of uncertainty and is calculated by using Equation 8.

$$V(IS(M)) = R_o - R_2(IS(M)) \quad (8)$$

If the value of the information exceeds the costs there is a net gain to the decision maker. The net gain is defined as

$$G(IS(M)) = V(IS(M)) - C(IS(M)) \quad (9)$$

If a number of alternative sources exist the net gain for each should be computed and the source that maximizes Equation 9 chosen.

CONCLUSION

In remote sensing a number of alternative techniques can usually perform the same task. However, these techniques will differ in accuracy and cost. The approach presented employs Bayesian Decision Theory to develop a rational method to evaluate the trade-off in cost and accuracy.

The approach is problem specific. In some situations the development of the economic data; i.e., the opportunity loss matrix, may create its own collection problems. However, the data requirements for Bayesian analysis are complementary with those of current water resources investigations and cost benefit studies.

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Appendix

Hypothetical Example Problem

An agency is charged with reducing flood damages in a particular region. The only run off control to be considered is a reservoir of varying capacity. The required capacity is determined by a watershed model that requires land use, area, and rainfall as inputs. Existing policy determines the specific rainfall event that is used for design.

The states of nature, S_j, are the land uses, classified in order of increasing magnitude of runoff. The actions, a_i, are the reservoir sizes.

The benefit provided by a reservoir is based upon flood reduction and a probabilistic function. For any land use the watershed model calculates the change in the flood regime caused by a particular reservoir. The benefit is the reduction in future flood damages and the cost includes construction, operation, and maintenance.

A particular reservoir is optimal for the land use if it maximizes net benefits, i.e., benefits minus costs. If a nonoptimal action is taken then the maximum net benefits are not obtained and an opportunity loss is incurred. An opportunity loss can occur if, due to data inaccuracy, the land use is incorrectly identified. The opportunity loss matrix for this example problem is presented in Table 1. It is noted that underdesign will result in larger losses than overdesign.

The agency has at its disposal a land use prediction model that uses population and geographic data. The model is not very reliable but does provide a first pass estimate of the land use class. The model is applied with a prior probability vector for the classes of land use is developed. The results are presented in Table 2.

The agency can also purchase remotely sensed land-use information. Two sources are available. These might be LANDSAT and aerial photographs or two different methods of LANDSAT analysis. Both sources have performed land-use classification previously and based upon their experience have developed error matrices which are the best indication of future performance they can provide to the agency. The cost of the two sources are $C(1)$ and $C(2)$. The confusion or error matrices are presented in Tables 3 and 4.

The agency's first step in analysis is the determination of the expected opportunity

Table 1. Opportunity Loss Matrix.

State S_j / Action a_i	1	2	3	4	5	6
1	0	10	25	45	70	100
2	5	0	15	35	60	90
3	10	5	0	20	45	75
4	15	10	5	0	25	55
5	20	15	10	5	0	30
6	25	20	15	10	5	0

Table 2. Land Use Prior Probability Vector.

State S_j	1	2	3	4	5	6
$P(S_j)$	0.02	0.15	0.40	0.25	0.10	0.08

Table 3. Confusion Matrix for Information Source IS (1).

State S_j / Signal y_k	1	2	3	4	5	6
1	0.70	0.10	0.00	0.00	0.00	0.00
2	0.15	0.60	0.05	0.00	0.05	0.00
3	0.05	0.20	0.60	0.10	0.05	0.00
4	0.05	0.10	0.30	0.55	0.05	0.05
5	0.05	0.00	0.05	0.30	0.70	0.15
6	0.00	0.00	0.00	0.05	0.15	0.80

Table 4. Confusion Matrix for Information Source IS(2).

State S_j / Signal y_k	1	2	3	4	5	6
1	0.75	0.05	0.00	0.00	0.00	0.00
2	0.15	0.85	0.10	0.00	0.00	0.00
3	0.05	0.10	0.80	0.05	0.00	0.00
4	0.05	0.00	0.10	0.75	0.10	0.05
5	0.00	0.00	0.00	0.15	0.80	0.05
6	0.00	0.00	0.00	0.05	0.10	0.90

loss with only the prior information. Equation 2 is used and the results are presented in Table 5.

The optimal action is to choose land use a_5, the value of R_o, the value of perfect information, is 10.3.

The next step of the analysis is to perform the preposterior procedure for each of the alternative information sources. The optimal actions and their associated expected opportunity losses are presented in Tables 6a and 6b for the two sources.

Equation 7 is applied to the results in order to obtain the expected value of the expected opportunity loss for each structure. The results are presented in Table 7.

The value of the information source is determined by applying Equation 8. The results are presented in Table 8.

The results presented in Table 8 indicate that both sources can reduce the cost of uncertainty. The chosen source will be that which yields the highest positive net gain as determined by Equation 9. The final decision rule for the agency is presented in Table 9.

It was noted previously that the Bayesian approach is problem specific and generalizations should not be based on one study. Consider the decision rule presented in Table 9 and the results of the Anacostia River Study. If the man-days were valued at one hundred dollars each, then the difference in the costs of the two sources would be $9,000. Next consider the difference of 2.23 in value of the two information sources in Table 9. If the difference in value was $223,000, then the cost difference would be insignificant. At the other extreme, if the difference in value was $2230, the cost difference would dominate the decision.

TABLE 5. EXPECTED OPPORTUNITY LOSSES WITH PRIOR INFORMATION.

a_i	1	2	3	4	5	6
EOL	37.75	28.05	16.45	10.70	10.30	12.50

TABLE 6. EXPECTED OPPORTUNITY LOSSES OF THE OPTIMAL ACTION GIVEN THE SIGNAL.

	(1) $IS(1)$			(b) $IS(2)$	
y_k	a_i	$R_1(y_k)$	y_k	a_i	$R_1(y_k)$
1	4	8.92	1	3	7.32
2	3	4.92	2	2	3.59
3	4	4.27	3	3	1.60
4	4	5.53	4	4	3.07
5	5	6.02	5	5	3.23
6	6	2.19	6	6	1.68

TABLE 7. EXPECTED VALUES OF THE EXPECTED OPPORTUNITY LOSS.

IS (M)	1	2
$R_2(IS(M))$	5.13	2.90

TABLE 8. VALUE OF THE INFORMATION SOURCES.

IS(M)	1	2
V(IS(M))	5.17	7.40

TABLE 9. DECISION RULE FOR CHOOSING THE INFORMATION SOURCE.

	IF	Choose *IS(M)*
	$C(2) - C(1) > 2.23$	
and	$C(1) \leqslant 5.17$	$M = 1$
	$C(2) - C(1) \leqslant 2.23$	
and	$C(2) \leqslant 7.40$	$M = 2$
	$C(1) > 5.17$ and $C(2) \geqslant 7.40$	Neither

Articles for Next Month

Ralph N. Baker, LANDSAT Data: A New Perspective for Geology.
Timothy C. Bidwell, College and University Sources of Remote Sensing Information.
D. J. Brooks, LANDSAT Measures of Water Clarity.
David W. Gaucher, James D. Turinetti, and *Kenneth R. Piech,* Special Color Analysis of Runway Conditions.
Robert D. Lawrence and *James H. Herzog,* Geology and Forestry Classification from ERTS-1 Digital Data.
Janet E. Nichol, Photomorphic Mapping for Land-Use Planning.
F. L. Scarpace, R. P. Madding, and *T. Green III,* Scanning Thermal Plumes.
C. Vigneron, MATRA Type 910 Automatic Third Camera.