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Analytical On-Line Systems in Close-Range Photogrammetry

Generalized analytical concepts, applicable to on-line close-range photogrammetry and man-machine interaction, are presented.

INTRODUCTION

A SIGNIFICANT part of the credit for the remarkable achievements of close-range photogrammetry in the last decade should undoubtedly be given to the development of analytical methods. Quite naturally, the general character of analytical solutions can well cope with the great variety of conditions encountered in close-range photogrammetry, microtriangulations, photography with given non-metric systems with atypical image geometry, and an increasing need for performing some additional digital analysis of data. Most of the analytical methods are applied in an *off-line* mode; this means that the phases of data acquisition and processing are physically and timewise separated and the photogrammetric reconstruction is based

ABSTRACT: *There is a distinct trend in close-range photogrammetry to employ on-line digital systems, such as the analytical plotter, for solutions which otherwise may not even be feasible. It is shown that the basic characteristics of these systems are well suited to cope with the variety of conditions and objectives typical for close-range applications. After the introduction of basic concepts analytical relations are discussed, as needed for the definition of image geometry, for the model reconstruction phase, and for the detailed photogrammetric compilation. The analytical structure of these procedures is presented in a general way which allows for the most universal application. Finally, additional auxiliary functions of the system and its high potential for an efficient man-machine interaction are analyzed.*

whereas the uniformity of solutions offered by standard analog systems is not too compatible with the real needs in this field. Other factors which contributed towards this trend are the requirements for an extremely high accuracy in some applications, the use of setups involving multiple photographs and on a discrete point representation of the object. However, in many projects the element of continuity in data sampling, in processing, and in the presentation of results is predominant so that the *on-line* principle of simultaneous data collection and processing becomes desirable.

With reference to a detailed analysis of computational (or analytical) photogrammetric systems given by Jaksic (1974) or Makarovic (1974), only a few basic concepts and terms are reviewed here. In general, the existing methods of photogrammetric processing can be classified as *analog*, *digital*, and *hybrid*. The analog systems represented by conventional photogrammetric plotters, i.e., by special purpose analog computers, are inherently on-line processors providing an immediate feedback from the model to the image spaces and a continuity of processing. Conventional plotters interfaced with digital computers form hybrid systems which combine the basic features of the analog and digital processing, as shown by Makarovic (1970) and Dorrer (1972). An equivalent on-line link in digital systems is provided by a computer interface to a measuring device simpler than a stereoplotter, usually a stereocomparator.

If the interface allows information to flow only in one direction, i.e., from the photograph to the computer, the on-line link is characterized as an *open-loop* system. The collected information can be instantly processed, but the operation and its control is in no way affected by the outcome of processing. Consequently, the function of this system is somewhat limited. An example of an open-loop on-line system is the proposal of the Image-Space plotter conceptually described by Forrest (1971). The first commercial product in this category appeared recently when C. Zeiss Oberkochen introduced the Stereocord G2 based on the well known Stereotop design supported by an electronic desk calculator and a small plotting table.

A *closed-loop* on-line system has a computer-monitored positioning of images, so enabling a full, rigorous, and universal control of the operations on the photogrammetric model. The system is capable of working in real-time when the system response delay is negligibly small (<20 ms), or in near-real-time when more complex computations are necessary and are inserted as off-line interruptions ranging from seconds to minutes, which can still be acceptable. This category includes large and highly automated systems, such as the UNAMACE or AS11 Analytical Plotter, as well as human operated less complicated systems like the NRC Analytical Plotter (Jaksic, 1974) and the Digital Stereocartograph (Inghilleri, 1972). The Gestalt Photo Mapper (Hobrough, 1971) which represents a special purpose automated system can also be classified as a closed-loop on-line setup.

Undoubtedly, the closed-loop on-line systems are most versatile in the photogrammetric use. The function of the system can be modified or extended by changes in the programming software. The mode of operation can be made simple or sophisticated depending on the user's choice. It allows full freedom of applying diverse mathematical formulations for the image and model geometries. In recognition of this potential the present paper attempts to discuss the basics of this type of processing in the field of close-range photogrammetry. The scope is limited to a general treatment of the analytical aspects. Related technical details concerning the design of individual components, the structure of the system, or any review of existing practical applications are not considered.

General Considerations

On-line analytical reconstructions in close-range photogrammetry are characterized by the same features which, in general, make close-range applications so different from procedures used in cartographic production. A high efficiency of the latter is achieved by securing uniform conditions and by adhering to standard solutions whereas close-range projects cover a broad scope of individual approaches. These reflect a great variety of diverse conditions and consequently make any unification or generalization very difficult or even impossible. However, at least some degree of uniformity can be established in close-range solutions based on the analytical principle, especially in their on-line versions. This is achieved by a suitable formulation of the system functions and by a development of versatile programs which can be readily modified by the operator at the time of their execution.

Operation Control in Photogrammetric Systems

Figure 1 illustrates how various photogrammetric systems are controlled by an operator or a computer, to perform their basic functions. To allow a broader comparison the flow charts are presented not only for on-line analytical systems, but also for analog and off-line analytical systems. The boxes in the Figure represent individual system components: photograph, analog model, graphical model or map, digital model, etc. In closed-loop on-line analytical systems the primary input from X and Y handwheels and Z foot disk represents operations on a physically non-existent, imaginary model which is digitally formed in the computer memory as an equivalent to the analog model in the analog systems. Let us call it a *virtual* model. The source of control is denoted by a double circle and control outputs by single circles properly positioned with respect to relevant boxes. Individual directed lines show the control flow. The control is described for system functions in both the orientation and final operation phases.

The practical difference between off-line and on-line systems is obvious from the way in which the operation control can be effected. In closed-loop on-line systems the control originates from the model space with an instantly enforced proper positioning of photographs, thus fully eliminating any stereoscopic disparities in the observed three-dimensional optical model. In off-line systems, however, the control is confined only to the two-dimensional space of photographs. Because of the missing feedback from the model to the photo space, the observed stereoscopic model must be steadily adjusted by manual corrections and the data cannot be collected in sequences related to the three-dimensional coordinate system of the model.

BASIC CHARACTERISTICS OF ON-LINE ANALYTICAL SYSTEMS

From the operational point of view there are three basic phases in the process of an on-line analytical operation:

- definition of the image geometry,
- reconstruction of the photogrammetric model, and
- detailed photogrammetric compilation of the model.

The first two phases are preparatory and actually proceed in an off-line mode, whereas the final phase is a typical realtime operation fully dependent on the use of the computer feedback control.

In defining the image geometry one actually chooses from existing models by determining the type of general conditions, and further specifies the characteristics of the image by its interior orientation and distortion parameters. Obviously, the image geometry essentially affects the process in the following operational phases. The reconstruction of a photogrammetric model results in a good description of relations between the images and the object by means of the parameters of exterior orientation. The reconstruction phase includes a one-step collection of measured data and usually an iterative solution of the parameters. The compilation phase represents an operation appearing practically continuous even though it is simulated by a fast repetitive cycle of digital computations. These proceed in a stream of densely spaced discrete data points defined by the operator's control of the floating mark in the observed optical model. In general, this computation is based on transformations between the model and image spaces with the use of all previously derived parameters of interior and exterior orientation. Included in the computations are corrections for any image distortions. The output from the photogrammetric compilation can be presented in an analog form on a plotting table or on a CRT screen, and in a digital form by coordinate readouts and printed listings. If the output is stored in the computer memory, numerous possibilities are available for further editing and additional processing of data, including their display in the form of computer-generated graphics. The physical control of the compilation process can be arranged or programmed in many different ways ranging from manual through mixed to fully automated computer control. In the mixed mode partial control drive is generated by the computer and additional necessary changes or adjustments are then continuously introduced by the operator.

The main advantage of the on-line analytical processing is that it allows for an immediate and useful man-machine interaction

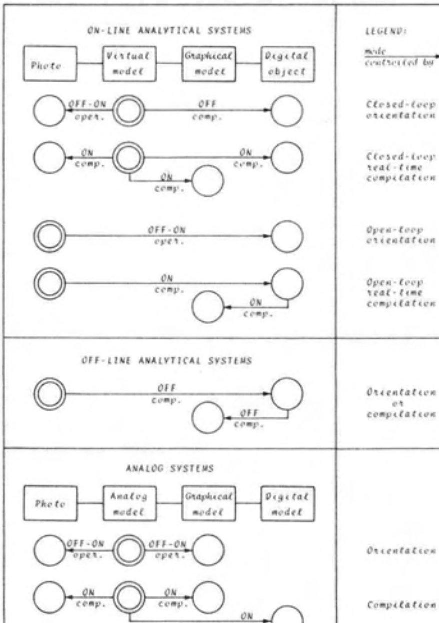

FIG. 1. Control flow in photogrammetric systems.

in practically all phases of the process. This fact increases the universality of the system and makes it flexible for a wide range of applications. The immediate interaction capability is, of course, a feature completely missing in off-line analytical procedures and, although present in analog systems, its potential there is rather limited. Most of the modifications and changes in the on-line system's function, as decided on by the operator at the time of execution, are built into the programming software supplied by the manufacturer of the system, and can be potentially expanded or further developed by users. Obviously, the key component of the system is the computer, and its performance limits the function of the system. Fortunately, contemporary minicomputers, such as the PDP series 11, are powerful and fast enough to handle practically everything needed in close-range photogrammetry.

The full potential of an analytical on-line system can be exploited only if the closed-loop design is used. The open-loop version has limitations both in the scope of its functions and in the lower accuracy because of lack of feedback.

TYPICAL FEATURES OF ANALYTICAL CLOSE RANGE PHOTOGRAMMETRY

One of the main characteristics of close-range photogrammetry is the impossibility of preserving standard conditions in the data acquisition phase. For example, the range of photo scales is substantially different in such applications as scanning electron microscopy and terrestrial photogrammetry. Also, the use of photogrammetric cameras is, for various reasons, not entirely universal. In some instances, the practical aspects prevail and a preference is shown for ordinary non-metric cameras which may be more readily available, more versatile, less bulky, or easier to operate. Fast moving objects call for photography with movie cameras or with other high frequency systems. Finally, some photogrammetric evaluations must simply rely on given photoimaging systems, such as scanning electron microscopes, X-ray machines, ophthalmologic instruments, line scanners, etc. As a rule, these cannot be modified for metric use or replaced by metric cameras.

This variety must be reflected in the way in which the image geometry is defined for the on-line analytical processing. In most applications the image is considered to be a central projection, with some systematic deviations from the concept due to lens distortion and film deformation. For non-metric cameras these parameters are either unknown or unstable and must be derived with the use of self-calibrating procedures (Kölbl, 1972) or in an on-the-job calibration (Faig, 1975). In some instances, the central perspective may not be an adequate imaging model because the distortion may exceed certain reasonable limits, e.g., in the fish-eye or anamorphotic lens design. Sometimes, the perspective bundle becomes very narrow or is represented even better by a parallel beam of imaging rays (Kratky, 1975a and b). Evidently, the character of images should be reflected by various modified projection equations.

Another generalization is required if the imaging geometry becomes time dependent. In contrast to a simple *frozen* model of an instant exposure, one is confronted with the dynamics of sequential imaging typical for TV cameras, scanning electron microscopes, and other systems employing line-scanning principle. All these geometries, atypical in photogrammetry, can be handled in on-line analytical systems, provided that the calibration of the dynamic projection is feasible.

Although metric cameras should be applied wherever possible, one cannot always guarantee their arrangement in a regular setup which yields a normal or nearly normal photogrammetric case. The pictures to be photogrammetrically treated may be taken individually, in pairs, or in larger groups depending on the form of the object. It is also quite typical for close-range photogrammetry that a set of pictures produces a complete view of an enclosed three-dimensional object from directions around the full circle. With the use of mirrors all partial images can be contained in a single stereopair. In this instance, and very often also in otherwise standard stereopairs, the area of interest for the photogrammetric processing covers only a minor part of the overlapping photographs. The configuration of control and intersection points used for the model reconstruction is then not too suitable. Consequently, a standard solution turns out to be unstable or inaccurate and may ultimately even fail. To ensure a reliable photogrammetric reconstruction one has to use some additional information on the exterior orientation of cameras. These auxiliary data can then be used in the form of constraints to the solution when applied and enforced with properly assigned weights. In on-line analytical systems the operator can enter the constraints at execution time, immediately check their effect, and possibly make modifications in a rerun. In a similar way the operator can decide on data rejections or remeasurements with the full advantage of comparing the digital results with the observed optical stereomodel.

Analytical Formulations

General Concepts

As a basic rule for formulations and programming in on-line analytical systems one can state that simple and fast computations are preferable. However, this applies mainly to the real-time operations where the time aspects are most critical. Depending on the type of the computer used, this programming may be necessary at the lower level language, such as the assembler, while the other programs seem to be handled well with higher level languages like FORTRAN, with the convenience of their easier reading and potential modifications by the user.

The variety of conditions in close-range photogrammetry requires that an on-line analytical system be supported by a set of program modules for formulations fitting all possible image geometries. The modules should provide ready-to-use solutions just by selecting the suitable type. For the reconstruction phase a more universal and flexible formulation is needed to accommodate different conditions, in the process of the operator-computer interaction.

In general, two different groups of variables characterize the image-object relationship in on-line analytical systems:

- inner geometry $[\boldsymbol{x}', \boldsymbol{d}]$
- outer geometry $[\boldsymbol{X}, \boldsymbol{g}]$

Here, vector $\boldsymbol{x}' = (x', y')^T$ represents the image coordinates, $\boldsymbol{d}$ is a vector of distortion parameters typical of the imaging system, $\boldsymbol{X} = (X,Y,Z)^T$ is a vector of object coordinates, and $\boldsymbol{g}$ is a vector of parameters of exterior orientation. In a symbol form one can write for the imaging process

$$\boldsymbol{X} \xrightarrow{\boldsymbol{g},\, \boldsymbol{d}} \boldsymbol{x}' \tag{1}$$

and for the reconstruction process

$$(\boldsymbol{x}', \boldsymbol{x}'', \ldots, \boldsymbol{d}) \xleftrightarrow{\boldsymbol{g}} \boldsymbol{X} \tag{2}$$

After deriving the unknowns $\boldsymbol{g}$ with the use of suitable control points and with a previous knowledge of $\boldsymbol{d}$, one can start the routine intersection of individual model points $\boldsymbol{X}$ from stereoobservations $\boldsymbol{x}', \boldsymbol{x}''$)

$$(\boldsymbol{x}', \boldsymbol{x}'') \xrightarrow{\boldsymbol{d},\boldsymbol{g}} \boldsymbol{X} \tag{3}$$

Some or even all of the parameters $\boldsymbol{d}$ can be considered as unknown and determined together with $\boldsymbol{g}$ during phase (2); however, in most instances parameters $\boldsymbol{d}$ are predetermined in separate calibrations.

Image Geometry

The imaging process performs a suitable conversion of data from a three-dimensional object space into a two-dimensional image space. This conversion is always achieved by the use of physical means which represent a real projecting system. This is true for both basic modes of instantaneous or sequential imaging. Depending on the position of the effective projection center, one can categorize the projection as central or parallel (Kratky, 1975b). The imaging rays are considered as straight lines which can be broken in the effective projection centers, or in a more general way as curved lines substituting for suitable trajectories or flux lines of a physical field, e.g., in electron microscopy.

To unify the various possibilities one can always base the photogrammetric reconstruction on scaled projections using straight lines derived from corrected image coordinates. Whenever applicable the corrections should also include the effect of the original curvilinear projection. This approach yields

$$\boldsymbol{x}' + \boldsymbol{c} = \frac{1}{\mu} \bar{\boldsymbol{P}}^T \Delta \boldsymbol{X} \tag{4}$$

where $\boldsymbol{c}$ are corrections for image distortions, μ is scale factor which is variable in central projections and constant in parallel projections, and $\Delta\boldsymbol{X} = (\Delta X, \Delta Y, \Delta Z)^T$ are object coordinates reduced with respect to a suitable reference point. Projection matrix $\bar{\boldsymbol{P}}(3,2)$ represents the orientation of the projection bundle or beam in the object coordinate system. In the central projection, $\bar{\boldsymbol{P}}$ forms the upper part of the exterior rotation matrix $\boldsymbol{P}$ for the photograph, and the reference point for $\Delta\boldsymbol{X}$ is given by the projection center (Jaksic, 1967). In the parallel projection matrix $\bar{\boldsymbol{P}}$ is a product of a matrix $\boldsymbol{O}$ for oblique parallel projection and of the rotation matrix $\boldsymbol{P}$

$$\bar{\boldsymbol{P}}^T = \boldsymbol{O}\, \boldsymbol{P}^T$$

where

$$\boldsymbol{O} = \begin{bmatrix} 1 & 0 & \xi \\ 0 & 1 & \eta \end{bmatrix} \tag{5}$$

and (ξ, η) are parameters of the oblique parallel projection (Kratky, 1975b). Only the left part of Equation 4 is important for the first phase of on-line operations as a preparation of the model reconstruction.

The image distortion is ususally described by a linear transformation

$$\boldsymbol{c} = \mathrm{F}(\boldsymbol{x}', \boldsymbol{d}) = \boldsymbol{D}_x{}'\boldsymbol{d} \tag{6}$$

where matrix $\boldsymbol{D}_{x'}$ is a suitable function of the point position $\boldsymbol{x}'$ in the image. Depending on the type of imagery, matrix $\boldsymbol{D}_{x'}$ assumes different forms to express the effect of affine, radial, tangential, spiral, or other distortions. If possible the distortion can be regarded as radial and then computed as a function of the radial distance. Distortion parameters $\boldsymbol{d}$ are usually known from separate calibrations and available for on-line operations as constants to be used in Equation 6. Another possible way of applying corrections is based on an interpolation from a computer-stored lookup table.

MODEL RECONSTRUCTION

This chapter deals with the geometry represented by the central projection. Typical modifications for parallel projection can be obtained from derivations given by Kratky (1975b).

Basic relations. In any off-line computation the photogrammetric model can be reconstructed by matching corresponding sets of photo coordinates $\boldsymbol{x}', \boldsymbol{x}''$ with control coordinates $\boldsymbol{X}$ as expressed in Equation 2, using a suitable mathematical model for the expected relationship. In on-line analytical systems the same relationship is expressed in a slightly different form. In accordance with Figure 1 the communication between photo and object coordinates is mediated by virtual model coordinates $\boldsymbol{x} = (x,y,z)^T$

$$\begin{tikzcd} (\boldsymbol{x}',\boldsymbol{x}'',\boldsymbol{d}) \arrow[r, leftrightarrow, "g_1"'] \arrow[rr, leftrightarrow, bend left, "g"] & \boldsymbol{x} \arrow[r, leftrightarrow, "g_2"'] \arrow[d] & \boldsymbol{X} \\ & \tilde{\boldsymbol{x}} & \end{tikzcd} \quad . \tag{2a}$$

The coordinate system of the virtual model becomes a master for the remaining systems, which now also include the graphical output $\tilde{\boldsymbol{x}}$.

Before the feedback link $(\boldsymbol{x} \rightarrow \boldsymbol{x}')$ is established photo-coordinates $\boldsymbol{x}', \boldsymbol{x}''$ are measured in on-line systems in the same way as in off-line systems. Derived parameters $\boldsymbol{g}$ can then be used in the on-line mode with an arbitrary decomposition $\boldsymbol{g}^T = (\boldsymbol{g}_1^T\, \boldsymbol{g}_2^T)$ where $\boldsymbol{g}_1$ represents the return in the feedback link $(\boldsymbol{x} \rightarrow \boldsymbol{x}')$.

In general, the model coordinate system can be defined in any arbitrary manner with respect to the object, but it is advantageous to assume equal photo and model scales $M = 1/m$ so that the flying altitude is equal to the negative principal distance f. Then it holds

$$\Delta \boldsymbol{X} = m\Delta \boldsymbol{x} \tag{7}$$

where $\Delta \boldsymbol{x} = \boldsymbol{x} - \boldsymbol{x}_c$, and the projection centers in a stereomodel can be assigned special values, e.g., $\boldsymbol{x}_{c_1} = 0$ and $\boldsymbol{x}_{c_2}^T = (b\ 0\ 0)$ where b is the photogrammetric base in the photo scale.

The computation of parameters $\boldsymbol{g}$ is then based on the relation $\boldsymbol{x} \leftrightarrow \boldsymbol{x}'$ rather than on $\boldsymbol{X} \leftrightarrow \boldsymbol{x}'$. For that purpose $\boldsymbol{X}$ is converted into $\boldsymbol{x}$ in accordance with Equation 7. This is done with the use of suitable estimates of coordinates for the first projection center $\boldsymbol{X}_{c_1} = \boldsymbol{C}$ and with the use of $\boldsymbol{x}_{c_1} = 0$, $\Delta \boldsymbol{x} = \boldsymbol{x}$ so that

$$\boldsymbol{x} = (\boldsymbol{X} - \boldsymbol{C})/m \quad . \tag{8}$$

Here, vector $\boldsymbol{C}$ is derived from an arbitrary given single pair of $\boldsymbol{X}$ and $\boldsymbol{x}$ according to $\boldsymbol{C} = \boldsymbol{X} - m\boldsymbol{x}$. A successful reconstruction of the model ultimately yields the rotation matrices $\boldsymbol{P}$ and the vectors $\boldsymbol{x}_c$ for both images.

Now the original Equation 4 is modified, by substitution of Equation 7, into

$$\boldsymbol{x}' + \boldsymbol{c} = \lambda \bar{\boldsymbol{P}}^T \Delta \boldsymbol{x} \tag{9}$$

where

$$\lambda = \frac{m}{\mu} \quad \text{and} \quad \mu = -\frac{1}{f}\boldsymbol{p}^T \Delta \boldsymbol{x} \quad . \tag{9a}$$

The value of λ is always very close to unity. Here, matrix $\bar{\boldsymbol{P}}$ and vector $\boldsymbol{p}$ are defined by column partitioning of the rotation matrix $\boldsymbol{P}$

$$\boldsymbol{P} = [\bar{\boldsymbol{P}}\ \boldsymbol{p}] \quad . \tag{10}$$

Thus, the working equations of an on-line analytical system which is physically driven via the virtual model, can be given by

$$\text{FULL RETURN} \qquad \boxed{\begin{aligned} \boldsymbol{x}' &= \lambda \bar{\boldsymbol{P}}^T \Delta \boldsymbol{x} - \boldsymbol{c} \ , \\ \boldsymbol{X} &= \boldsymbol{C} + m\boldsymbol{x} \ , \\ \tilde{\boldsymbol{x}} &= v\boldsymbol{x} \end{aligned}} \tag{11}$$

where v is an arbitrary scale factor for generating a graphical plot. The first formula in Equation 11 represents the transformations for both images in a stereopair. In this system, the exterior orientation is fully returned to the $(\boldsymbol{x} \rightarrow \boldsymbol{x}')$ link except for the photo scale factor m which is used in the $(\boldsymbol{x} \rightarrow \boldsymbol{X})$ computation. In operations, the floating mark is driven in the directions of the object coordinate system.

In photogrammetric compilations, it is always necessary to establish a horizontal x,y plane, but in some instances it may be inconvenient to fit the control drive with the object X,Y axes especially if the photogrammetric base is azimuthally rotated. Rotation matrices $\boldsymbol{P}$ can then be factorized as $\boldsymbol{P} = \boldsymbol{K}\boldsymbol{T}$ where any arbitrary rotation $\boldsymbol{K}$ around the Z axis defines a new orientation $\boldsymbol{T} = \boldsymbol{K}^T\boldsymbol{P}$ to be returned to the photo feedback $(\boldsymbol{x} \rightarrow \boldsymbol{x}')$ while $\boldsymbol{K}$ is applied in the $(\boldsymbol{x} \rightarrow \boldsymbol{X})$ conversion

$$x' \underset{T}{\overset{P}{\rightleftarrows}} x \underset{K}{\rightleftarrows} X \quad .$$

Using a truncation of $\boldsymbol{T}$ equivalent to Equation 10 the new operational formulas are

PARTIAL RETURN

$$\boxed{\begin{aligned} x' &= \lambda \overline{T}^T \Delta x - c = \lambda \overline{P}^T K \Delta x - c \quad . \\ X &= C + mKx \quad , \\ \bar{x} &= vx \quad . \end{aligned}} \qquad (12)$$

Finally, if one desires, the whole orientation can follow the classical routine of the relative-absolute orientation. The measurements are arranged in two steps yielding intermediate model coordinates $\bar{\boldsymbol{x}}$ after the relative orientation and final coordinates $\boldsymbol{x}$ after the absolute orientation. With the notation of $\boldsymbol{R}$ for the matrix of relative orientation, $\boldsymbol{A}$ for the matrix of absolute orientation, $\boldsymbol{L}$ for the matrix to level the model, and $\boldsymbol{K}$ for the matrix of azimuthal rotation, one can write

$$P = AR \quad , \quad A = KL \quad , \quad P = KLR$$

and eventually arrive at the following formulas:

(a) after solving five parameters for $\boldsymbol{R}_1$, $\boldsymbol{R}_2$

$$\text{RELATIVE} \quad \boxed{\begin{aligned} x' &= \lambda \overline{R}^T \Delta \bar{x} - c \quad , \\ X &= mA\bar{x} + dX \quad , \end{aligned}} \qquad (13a)$$

(b) after solving seven parameters for m, $\boldsymbol{A}$, $d\boldsymbol{X}$

$$\text{ABSOLUTE} \quad \boxed{\begin{aligned} x' &= \lambda (L\overline{R})^T \Delta x - c \quad , \\ X &= mKx + dX \quad , \\ \bar{x} &= vx \quad . \end{aligned}} \qquad (13b)$$

Mathematical considerations. Mathematical formulations suitable for applications in close range photogrammetry are discussed in detail by Wong (1975) in his invited paper to the XIIIth ISP Congress. For this reason, the analysis here is limited to some special aspects typical of on-line systems.

In general, the basic equation for perspective bundles is the well known collinearity condition which can be modified as an affinity condition to fit the relations in projections with parallel beams. Both conditions can be further extended to include the unknown parameters of inner orientation and of image distortions. For example a more general formulation of the collinearity equation (Abdel-Aziz and Karara, 1971; Jahn, 1975) included the elements of interior orientation extended by two additional parameters to compensate for a general affine distortion of a photograph.

An on-line analytical system can handle a multiple orientation of images although the final processing can obviously proceed only in stereopairs sequentially formed from appropriate combinations of images. The model reconstruction is based on a simultaneous micro-block adjustment using suitable models for the image geometries. This arrangement has a great self-calibrating potential. The procedure is applicable to multiple stereoviews based on the use of mirrors, or to several smaller sized pictures as long as they can be simultaneously accommodated in the photocarriers of the analytical on-line instrument.

An exclusive use of a single collinearity or affinity condition leads to a uniform formulation of the equation system. One can avoid using an additional coplanarity condition for intersections of conjugated rays, which is inadequate for a multiple orientation anyway. In this instance the unknown coordinates of intersected points are sequentially eliminated from the solution in a point-by-point procedure well known in photogrammetric bundle adjustments.

In the formulation of the least squares adjustment it is advantageous to consider the initial linearized system of condition equations with corrections $\boldsymbol{v}$, unknown parameters $\boldsymbol{g}$, and condition residuals $\boldsymbol{u}$

$$Av = Bg + u = 0 \qquad (14)$$

as an equivalent to a system of parametric equations based on quasi-observations $\boldsymbol{u}$ and associated weights $\boldsymbol{P}$

$$Bg + u = w \quad , \quad P = (AA^T)^{-1} \quad . \qquad (15)$$

Weighting of the original observations is neglected here. If required, their variance-covariance matrix $\boldsymbol{Q}$ is introduced and then the expression for $\boldsymbol{P}$ in Equation 15 is modified into $(AQA^T)^{-1}$.

Assuming a suitable partitioning $\boldsymbol{B} = [\boldsymbol{B}_o \; \boldsymbol{B}_x]$ and $\boldsymbol{g}^T = (\boldsymbol{g}_o^T \; \boldsymbol{g}_x^T)$, the elimination of model coordinates $\boldsymbol{g}_x$ leads to an equivalent system of quasi-observations $\boldsymbol{u}$ which are newly correlated through a weight matrix $\boldsymbol{P}_o$. This system contains only orientation parameters $\boldsymbol{g}_o$

$$\begin{aligned} B_o g_o + u &= \overline{w} \quad , \\ P_o &= P - PB_x(B_x^T P B_x)^{-1} B_x^T P \quad . \end{aligned} \qquad (16)$$

These equations are used to contribute sequentially towards the normal equations

$$B_o^T P_o B_o g_o + B_o^T P_o u = 0 \quad .$$

In this scheme the given control coordinates can be weighted with the estimations of $\boldsymbol{P}_x$ and regarded as additional constraints in the same form of quasi-observation $\boldsymbol{u}$ as in Equation 16, but with modified weight matrices

$$\boldsymbol{P}_o = \boldsymbol{P} - \boldsymbol{P}\boldsymbol{B}_x(\boldsymbol{P}_x + \boldsymbol{B}_x^T\boldsymbol{P}\boldsymbol{B}_x)^{-1}\boldsymbol{B}_x^T\boldsymbol{P} \quad . \qquad (16a)$$

An absolute enforcement of this coordinate constraint is feasible by increasing the diagonal values in $\boldsymbol{P}_x$ to infinity, which will change the weight matrix $\boldsymbol{P}_o$ into $\boldsymbol{P}$.

Any other auxiliary information concerning the orientation parameters $\boldsymbol{g}_o$ should be added to the system again in the form of quasi-observations $\boldsymbol{u}$ which, this time, are associated with a different matrix $\boldsymbol{B}_c$ and with a weight matrix $\boldsymbol{P}_c$. Otherwise, a full uniformity of the formation of the equations is preserved.

The solution of normal equations is repeated while regularly updating vector $\boldsymbol{g}_o$ and quasi-observations $\boldsymbol{u}$ in each iteration. Upon applying $\boldsymbol{P}_o$ from Equation 16 the unknown vector $\boldsymbol{g}_x$ is rigorously eliminated in each step and is ultimately computed only after the completion of the iterative process by

$$\boldsymbol{g}_x = -(\boldsymbol{B}_x^T\boldsymbol{P}\boldsymbol{B}_x)^{-1}\boldsymbol{B}_x^T\boldsymbol{P}\boldsymbol{u} \quad . \qquad (17)$$

Although possible, there is no need to update the model coordinates after each iteration.

DETAILED COMPILATION

The first two operational phases in a standard on-line analytical process are concerned with the derivation of a valid analytical model for image geometry and, by using it, with the analytical reconstruction of the model for object geometry. In both phases the on-line function proper is prepared and checked, but the essence of what is done is practically identical with the function of off-line analytical procedures. Only after the models of both image and object geometries are derived are they ready to be used in a process of detailed photogrammetric compilation which represents a typical real-time operation with all the characteristics of an on-line process. The control of this operation is characterized by Equation 2a or in a modified form by

$$\boldsymbol{x} \left\{ \begin{array}{l} \boldsymbol{d}, g_1 \rightarrow (\boldsymbol{x}', \boldsymbol{x}'') \\ g_2 \longrightarrow \boldsymbol{X} \\ v \longrightarrow \bar{\boldsymbol{x}} \end{array} \right. \quad .$$

With the computer performing all the three involved computations as given by Equations 11 to 13 in a high frequency cycle, the operator retains the dynamic control of the system through $\boldsymbol{x}$ and, with a perfect illusion of continuity, receives his feedback from stereoobservations of computer positioned image details for $\boldsymbol{x}'$ and $\boldsymbol{x}''$. For the real-time operation the link $(\boldsymbol{x} \rightarrow \boldsymbol{x}')$ is most critical because it includes the image correction $\boldsymbol{c} = \boldsymbol{Dd}$ and the computation is repeated for $\boldsymbol{x}'$ and $\boldsymbol{x}''$. The second link $(\boldsymbol{x} \rightarrow \boldsymbol{X})$ must be implemented with a frequency dependent on the type of output. If $\boldsymbol{X}$ is to be digitized and stored continuously, the required density of data should determine the computation frequency. When $\boldsymbol{X}$ is to be displayed or recorded only occasionally, for discrete points, the frequency can be lowered. Depending on the user's need and sophistication, the $\boldsymbol{X}$-digitization may also include additional operations, such as an interpolation of residuals remaining in control points. The graphical output following the conversion $(\boldsymbol{x} \rightarrow \bar{\boldsymbol{x}})$ is relatively less demanding as far as the computation frequency is concerned and, in general, it can be different for a plotting table or for a CRT-display.

A few comments are in order on continuous digitizations. Some of the operations proceed under full control of the operator, some can be computer-assisted, and some may be fully controlled by the computer. The latter is possible if the system includes an automatic correlator, but even in this instance the operator usually monitors the action and is ready to step in. This is typical for the Gestalt Photo Mapper (Hobrough 1971). The operator terminates the automatic correlation process in individual orthophoto patches and can modify the derived height level if necessary. A reversed situation arises when the computer assists the operator by assuming control in a part of the operation. This is useful in profiling when the computer provides the basic constant drive in any desired direction or in a more general pattern and the operator controls the elevation. In parallel profiling the computer can duplicate the preceding profile, leaving the operator with relatively small adjustments.

Additional Functions

AUXILIARY OPERATIONS

An on-line analytical system can also be used in further data processing and editing, mostly in a better way than a pure off-line system (Masry 1973). Some of these functions are performed directly during the real-time operation, some of them in periodically inserted off-line time periods. One should mention a reduction of the data volume which is useful for any continuous digitization ultimately suffering from a high redundancy of data. A more sophisticated editing can smooth out the data stream, filter out the noise, provide some refinement with respect to already existing data, and do all necessary checks important for the acceptance of data. Immediately after the editing action is completed the modified results can be rerun for visual inspection and otherwise graphically displayed. The operator monitors the process.

A series of additional operations are very compatible with the function of an on-line analytical system. Auxiliary transformations can be applied to change the coordinate system for the control of the floating mark motion in the virtual model. This is extremely useful in architecture (Jachimski, 1974) where multiple facades can be plotted from a single model upon guiding the floating mark in generally oriented planes fitted to individual walls. Computations of different geometric parameters, such as angles, distances, areas, volumes, differential changes, deformations, etc. represent a simple use of digitized data immediately after their acquisition. A more sophisticated data analysis can be selectively combined with an on-line process to enable surface fitting, interpolations, and statistical evaluations important for further operations.

Probably one of the most useful auxiliary functions is provided by graphical displays built as a part of the on-line system. A plotting table is elementary for any modified graphical presentation of available data, but an interactive CRT-display is much faster and more powerful. With its use one can produce all types of projections, perspectives, slope maps, contour renditions, and other thematic plots which can be immediately modified by the operator to provide the most useful presentation. Through its use additional data can be obtained from the on-line function. Only after the operator is satisfied, is the latest data stored or line-drawn in the final format on the plotting table.

MAN-MACHINE INTERACTION

As already mentioned, a moderately powerful minicomputer, such as one of the PDP-11 family, is capable of carrying and performing a series of basic and auxiliary photogrammetric programs stored together with the computer system monitor on a magnetic disk. The individual tasks and solutions can be organized into program modules and libraries tied by conversational programs into a universal photogrammetric software package. Its flexibility depends on built-in communication channels which determine the level of the potential man-machine interaction.

Here, we attempt to show only some of the general possibilities and options which can be used by the system. Basically, some form of communication with the computer is indispensable to modify the current operation at any level of the system performance, e.g., to terminate the action, to accept new information, to transfer control to any other function, or to continue the previous one. The system invites information or specific actions and the operator follows accordingly. Essential conversational patterns frequently used can also be prestored and the communication speeded up just by specifying the code of a particular combination of actions to be invoked.

When starting a task a selection of conditions is made out of existing options included in the photogrammetric software. In the phase of formulating the image geometry it must be stated what type of projection is required, which are the known parameters, and which parameters are to be determined on the job. Further information should also be supplied on where to allocate the parameters, if they are prestored, and how to use them. In the phase of the model reconstruction the conditions and required constraints for the solution are specified first. Then the measurements are performed and resulting data recorded in a sequence controlled by the operator or by the system from prestored instructions. Weights for observations may be entered and control support specified independently for each data point. The routine can be interrupted, measurements rejected or repeated, and all previous settings checked with the use of an automatic computer-controlled drive. The computation is monitored by means of auxiliary printouts to show progress. Ultimately, a complete listing can be printed out or only the final accuracy assessment displayed. Computations can also be repeated from an edited data set or with added or corrected information until the reconstruction is considered satisfactory. The real-time operations necessary for detailed photogrammetric compilation are started after specifying its conditions, the type of computer control, and other requests concerning the storage, display, or editing of data.

CONCLUSIONS

Analytical on-line systems are becoming more available. At the present a few new systems are developed, which are primarily oriented towards an ordinary user and not towards exclusive, huge mapping agencies as in the past. The cost and operational complexity are expected to be moderate and the reliability high.

In general, the functional range of on-line systems is greater than that of any other photogrammetric system, and the expansion of functions is mostly a matter of software development. The users can actually build their own processing systems tailored to their needs. This fact combined with the versatility of on-line analytical systems make them extremely attractive for close range applications, especially in biomedicine and engineering. One can expect a significant future development in these fields.

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Articles for Next Month

A. Biache, Jr., P. I., Photogrammetry, and Public Policy.
William R. Harris, Remote Imagery Support for International Organizations.
Ronald F. Stowe, Diplomatic and Legal Aspects of Remote Sensing.
J. Robert Porter, Jr., Remote Sensing Issues and Policies—Industry's View.
Amrom H. Katz, A Retrospective on Earth-Resource Surveys: Arguments about Technology, Analysis, Politics, and Bureaucracy.
Ida R. Hoos, Utilization of Remote Sensing Data—The Sociological Perspective.
Arthur T. Anderson and *Jane Schubert*, ERTS-1 Data Applied to Strip Mining.
Percy P. Batlivala and *Fawwaz T. Ulaby*, Radar Look Direction and Row Crops.
Dr. C. P. Lo, Photographic Analysis of Water Quality Changes.
Harold E. Lockwood and *Lincoln Perry*, Shutter/Aperture Settings for Aerial Photography.
Robert B. McEwen, *William J. Kosco*, and *Virginia Carter*, Coastal Wetland Mapping.