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The Use of Non-Metric Cameras in Monitoring High Speed Processes

Formulations for interior and exterior orientation of non-metric cameras, and their application to high speed processes, are presented.

Introduction

Despite the fact that, in recent years, a wide selection of close-range photogrammetric cameras has become available, it is expected that ordinary, non-metric, cameras will continue to be used in photogrammetric applications. These cameras generally offer a wide focusing range, often combined with a choice of different types of lenses. Some of them, including movie cameras, have electrical film transport and exposure control to provide a rapid sequence of exposures and remotely controlled operation.

A drawback of non-metric cameras is that lens distortion generally exceeds the acceptable limits and that, as the result of dimensional instabilities and improper film flattening at the time of exposure, the interior orientation is poorly defined. As a result, it is usually not possible to calibrate these cameras with sufficient precision.

These problems can, to a certain degree, be overcome by providing control points from which the elements of interior and exterior orientation can be determined for each individual photograph, and image coordinates can be corrected for effects of lack of film flatness, lens distortion, etc. This, of course, requires special precautions and time consuming preparations prior to the photography. It would therefore not be practical to use non-metric cameras in routine, close-range applications such as architectural photogrammetry and the recording of traffic accidents in which many different sites and camera stations are involved. There are, however, examples, particularly in research and development work, where the above control points can be provided. Since it is not always possible to purchase or to build special photogrammetric cameras, non-metric, "off-the-shelf" cameras often are used in this type of application.

Other applications extract information from photographs taken with non-metric cameras for other purposes. In these cases it is usually not possible to establish control points prior to photography, so that the necessary information on interior and exterior orientation has to be collected after the photographs have been taken.

This paper reviews various projects concerned with monitoring high speed processes, carried out by the National Research Council of Canada, in which non-metric cameras were used. The operations needed to provide the control data for establishing the elements of interior orientation during the experiment are described, and the accuracy of the photogrammetrically compiled information is evaluated.

Interior and Exterior Orientation

REMARKS REGARDING THE CAMERA GEOMETRY

If metric data are to be derived from a photograph, sufficient additional information is required to reconstruct the object bundle which formed the photograph.

Let us assume for a moment a distortion-free lens. In this case the photograph can be put into the object bundle, in position $\bar{i}$, in such a way that the projection center O, the image point $\bar{p}$, and the actual point P are located on a straight line. This condition—the collinearity condition—holds true for all points photographed with this lens (see Figure 1).

In order to reconstruct the object bundle from the photograph it is necessary to define the projection center 0 with reference to the image.

First, it is stipulated that the projection center is located on that object ray which is perpendicular to the image plane. This ray will be referred to as the chief ray, and its intersection with the image plane as the principal point of autocollimation. In practice, the image plane i is located on the image side of the projection center parallel to the plane i; the principal point of autocollimation is then defined as the point of intersection of the chief ray and the image plane i. This ray is, prior to entering the lens, perpendicular to the image plane i. After passing through a lens, which introduces a certain amount of distortion, the chief ray will, in all likelihood, no longer be perpendicular to the image plane i. The advantage of the above definition for the principal point is that it can be determined by autocollimation. In practice, however, the point of best symmetry is used to define the projection center with respect to the image plane. For the point of best symmetry, the radial lens distortion is, to the best approximation, rotationally symmetrical, giving certain advantages in the correction procedures.

ABSTRACT: *This paper discusses the use of non-metric cameras in photogrammetric applications. The derivation of the elements of interior orientation, using control points in the photographs, is formulated providing the possibility for "self-calibration" of non-metric cameras. Various projects concerned with monitoring high speed processes using non-metric cameras, which were carried out at the National Research Council of Canada, are described. The accuracy of the obtained information is evaluated.*

RESUMÉ: *Cet exposé étudie l'usage des chambres non-métriques en photogrammétrie. La dérivation des éléments d'orientation intérieure se servant de points de contrôle dans les prises de vues est formulée donnant la possibilité d' "auto-calibrer" les chambres non-métriques.*

Plusieurs projets, éxécutés au Conseil National de Recherches, touchant au contrôle de procédés à grande vitesse et utilisant des chambres non-métriques sont décrits. La précision des résultats est évaluée.

ZUSAMMENFASSUNG: *Die Verwendung nicht-metrischer Kameras für photogrammetrische Zwecke ist Gegenstand dieses Berichtes. Die Herleitung der Elemente der inneren und der äußeren Orientierung mithilfe von im Bild gegebenen Kontrollpunkten wird besprochen.*

Verschiedene, am National Research Council of Canada durchgeführte Auswertungen von Aufnahmen sich schnell bewegender Objekte werden erläutert und Hinweise auf die erzielte Genauigkeit gegeben.

In addition to the principal point, the distance between projection center and the image plane is needed. Most amateur cameras will have the same image scale in all directions. Some professional movie camera lenses, on the other hand, have a non-uniform image scale to accommodate screen format requirements. These lenses are called anamorphic lenses. The

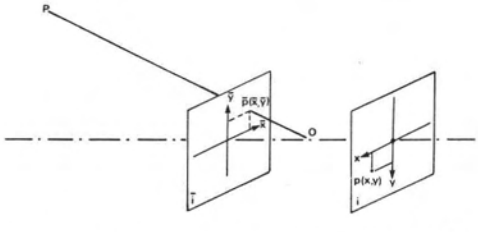

FIG. 1. Photogrammetric central projection.

lenses with a uniform scale factor will be referred to as amorphic lenses. Photographs produced with amorphic lenses require a single projection distance c for the reconstruction of the bundle, while anamorphic lenses require two projection distances c_x and c_y in planes perpendicular to each other.

Non-metric cameras are usually not equipped with suitable fiducial marks defining the image coordinate system. Hence, the direction of both major and minor axis (x and y) of an anamorphic lens is, in general, unknown.

PHOTOGRAMMETRIC CENTRAL PROJECTION

Let us assume an origin O and a plane $\bar{i}$ (Figure 1), O being the origin of a Cartesian spatial coordinate system X, Y, Z and $\bar{i}$, with the coordinates $\bar{x}, \bar{y}$, being a plane parallel to the plane X, Y. Each point $\bar{p}$ of the plane $\bar{i}$ can be uniquely defined by the ray $O\bar{p}$ which in turn is defined by any point P located on it, as long as P is not identical with O. The spatial coordinates X, Y, Z of point P can therefore be used to determine the location of $\bar{p}$ in $\bar{i}$ as long as Z is not zero. In addition to P, ∞ points P_i with X_i, Y_i, Z_i are also located on the ray OP. These points can all be derived from P by multiplying X, Y, Z with a suitable scale factor S_i,

$$\begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix} = S_i \, . \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

A rotation of a plane around the origin of the coordinate systems can be described by a system of homogeneous linear equations. The coefficients of such a system form an orthogonal matrix with the coefficients a_{ij}. If the scale factor S is also introduced, one can write these equations as

$$\begin{bmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} .S. \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

In practically all photogrammetric applications it is impossible to arrange for the center of the object bundle O to be located in the origin of the spatial coordinate system X, Y, Z. Hence, the location of O with the coordinates X_o, Y_o, Z_o must be introduced into the formulae

$$S \cdot \begin{bmatrix} X - X_o \\ Y - Y_o \\ Z - Z_o \end{bmatrix} = \begin{bmatrix} S & 0 & 0 & -X_o \\ 0 & S & 0 & -Y_o \\ 0 & 0 & S & -Z_o \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ S \end{bmatrix}$$

resulting in the pseudo-homogeneous coordinates (X, Y, Z, S).

Introducing the shift of the origin into the projection equation, one can now re-write the transformation equation as

$$\begin{bmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} S & 0 & 0 & -X_o \\ 0 & S & 0 & -Y_o \\ 0 & 0 & S & -Z_o \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ S \end{bmatrix} .$$

It may be necessary in certain photogrammetric operations to consider the origin $\bar{o}(\bar{x}_o, \bar{y}_o)$ of the image coordinates as unknown:

$$\begin{bmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\bar{x}_o \\ 0 & 1 & -\bar{y}_o \\ 0 & 0 & \bar{z} \end{bmatrix}^{-1} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} S & 0 & 0 & -X_o \\ 0 & S & 0 & -Y_o \\ 0 & 0 & S & -Z_o \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ S \end{bmatrix} .$$

Anamorphic lenses can be considered to have two projection distances: one applies for all measurements to be taken in the direction of film transport in the camera, the other for all measurements taken perpendicular to this direction. These two projection distances are here introduced indirectly as the scale factors c_x and c_y. Hence

$$\begin{bmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\bar{x}_o \\ 0 & 1 & -\bar{y}_o \\ 0 & 0 & \bar{z} \end{bmatrix} \begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} S & 0 & 0 & -X_o \\ 0 & S & 0 & -Y_o \\ 0 & 0 & S & -Z_o \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ S \end{bmatrix} .$$

Non-photogrammetric cameras are not equipped with reference marks defining the image coordinate system. It is therefore not possible to orient the photograph in the measuring instrument. If an amorphic lens is used, a rotation between the image coordinate system and the amorphic coordinate system of the measuring instrument will be included into the orthogonal matrix **A**. However, because of the scale differences in an anamorphic photograph, it is also necessary to define for these photographs the angle of rotation α between the anamorphic photograph and the amorphic measuring system. Hence

$$\begin{bmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\bar{x}_o \\ 0 & 1 & -\bar{y}_o \\ 0 & 0 & \bar{z} \end{bmatrix}^{-1} \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\cdot \begin{bmatrix} S & 0 & 0 & -X_o \\ 0 & S & 0 & -Y_o \\ 0 & 0 & S & -Z_o \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ S \end{bmatrix}$$

If $\bar{z} = 1$, this equation contains the following twelve unknowns: the coordinates of the origin of the image coordinate system $\bar{x}_o$ and $\bar{y}_o$, the rotation α between the anamorphic and amorphic coordinate systems, the anamorphic scale factors c_x and c_y, the rotational elements a_{12}, a_{13}, and a_{23} (for κ, ϕ and ω respectively), the scale factor S, and the coordinates of the position of the projection center X_o, Y_o and Z_o. The matrices in the above equation can all be multiplied. The resulting matrix

$$\begin{bmatrix} 1 & 0 & -\bar{x}_0 \\ 0 & 1 & -\bar{y}_0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\cdot \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} S & 0 & 0 & -X_o \\ 0 & S & 0 & -Y_o \\ 0 & 0 & S & -Z_o \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \end{bmatrix}$$

can then, after division of all elements by $b_{34}.S$, i.e., $c_{ij} = \frac{b_{ij}}{b_{34}\,S}$, be used to form the projection equations

$$x = \frac{c_{11}X + c_{12}Y + c_{13}Z + c_{14}}{c_{31}X + c_{32}Y + c_{33}Z + 1}$$

$$y = \frac{c_{21}X + c_{22}Y + c_{23}Z + c_{24}}{c_{31}X + c_{32}Y + c_{33}Z + 1}$$

which no longer contain the scale factor S as unknown.

Linearization of the two projection equations leads to normal equations permitting the determination of the remaining eleven unknowns. The solution of these equations, however, will only yield acceptable results if a sufficient number of object points with suitable distribution is available. It is always advisable with the use of non-metric cameras, first to derive the correlation matrix, and then to decide on which of the eleven possible parameters should actually be used.

If an amorphic lens was used, for example, α and κ cannot be distinguished and $c_x = c_y$. Often, a strong correlation may also exist between x_o and ϕ (or a_{13}) and/or between y_o and ω (or a_{23}).

Once the object bundle has been re-established and oriented, it is possible to search along the object ray for that point which corresponds to the object point, by enforcing a minimum distance constraint between ray and object point. Jahn[1] replaced the rotation α by a shear factor e. This has not been introduced here because there is no element in the optical projection suggesting the use of such a factor.

DETERMINATION OF THE LENS DISTORTION

If sufficient redundant object points are available, the inclusion of parameters into the projection equation for the lens distortion could be considered. An amorphic lens will have a certain design distortion which is rotationally symmetrical. The assembled lens will have additional lens distortion caused by minor imperfections in the production of each lens element as well as in the centering of all the elements. This latter distortion, which is commonly referred to as decentering distortion, has both radial and tangential components.

It is assumed for the following presentation that the lens distortion of an anamorphic lens is proportional to the coordinates themselves.

The rotationally symmetrical lens distortion for an anamorphic lens can then be expressed as

$$d\bar{x}_1 = \bar{x}\,(d_1r^2 + d_2(r^2)^2 + d_3(r^2)^4 + \ldots)$$
$$d\bar{y}_1 = \bar{y}\,(d_1r^2 + d_2(r^2)^2 + d_3(r^2)^4 + \ldots)$$

with $r^2 = c_x^2\,\bar{x}^2 + c_y^2\,\bar{y}^2$ and $\bar{x}$ and $\bar{y}$ referenced to the defined center of the photograph. The decentering distortion can be expressed as

$$d\bar{x}_2 = \{e_1(3c_x^2\,\bar{x}^2 + c_y^2\,\bar{y}^{\,2}) + e_2 \cdot 2c_x\,\bar{x}\,c_y\,\bar{y}\}\{1 + e_3r^2 + e_4(r^2)^2 + \ldots\}$$

$$d\bar{y}_2 = \{e_1 \cdot 2c_x\bar{x}\,c_y\bar{y} + e_2 \cdot (c_x^2\bar{x}^2 + 3c_y^2\bar{y}^{\,2})\}\{1 + e_3r^2 + e_4(r^2)^2 + \ldots\}$$

with r^2, $\bar{x}$, and $\bar{y}$ as before.

An attempt to calibrate non-metric cameras on camera calibration equipment designed for mapping cameras could also be considered. There are, however, several factors which speak against such a calibration:

- Non-metric cameras often lack a well defined image frame permitting the definition of the principal point of autocollimation;

- Non-metric cameras can be focused, thus the lens distortion often changes with a change in the focusing distance; and
- Non-metric cameras often have rather small lens angles, hence only a few collimators of a standard calibrator may be imaged.

It should be noted here that the lens distortion has not been corrected in any of the applications to be discussed later in this report.

DETERMINATION OF THE IMAGE DEFORMATION

Image deformation consists of dimensional changes in the actual photographic record occurring between exposure and the measurement, and of image displacements resulting from improper placement of the photographic material (film or glass) into the image plane.

Since non-metric cameras are not equipped with fiducial marks, dimensional changes of the original photographic record cannot be determined and therefore remain uncorrected. An overall scale change will result in a projection distance change. A differential scale change is unlikely if modern films with a polyester base are used. Older films with a less stable base, for example acetate cellulose, often displayed not only differential scale changes but also shear between the two coordinate axes. If such films were used, Jahn's[1] partial matrix

$$\begin{bmatrix} C_x & -eC_y & 0 \\ 0 & C_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

could be included as well.

Improper placement of the photographic surface into the image plane must be considered a more serious error source because non-metric cameras using film are hardly ever equipped with a satisfactory film flattening mechanism. The resulting positional displacements are directed in radial directions and, therefore, interpretable as lens distortion. Since the improper film flattening may result in rather irregular radial distortion, the determination of the lens distortion by introducing additional parameters becomes somewhat questionable for lenses with larger lens angles.

It should be noted here that the attempted placement of the photographic emulsion into the image plane will produce a different result for each frame. If the possibility of a change in focus is also considered, it must be concluded that the interior orientation must be redetermined for each new photograph.

Examples for Single Camera Solutions

DETERMINATION OF TWO-DIMENSIONAL INFORMATION

In the photogrammetric determination of small distances or displacement vectors, systematic image errors caused by inaccuracies in the camera's interior orientation, including the film flatness, have little effect, and satisfactory results can therefore be expected from non-metric cameras. This is particularly true when the information is derived from a single photograph, which, in deformation measurements, is possible by multiple exposures on the same photographic emulsion.

An example of such an application is the determination of the mode shapes of a thin wing, vibrating at various frequencies between 50 and 500 cps[3]. The size of the wing was approximately 60 cm by 30 cm. In deformation measurements of thin objects, photogrammetry is particularly suitable since it does not interfere with the actual process, which is usually the case with mechanical or electrical pick-up systems.

Reference points were provided on the wing by means of a regular grid consisting of narrow grooves (0.2 mm thick) which were filled with a yellow dye. These points offered a good photographic contrast against the wing surface which was painted matte black. After the wing was forced to oscillate, using a jet exciter, it was photographically recorded using a one-second exposure. The camera was oriented with its optical axis nearly perpendicular to the oscillation direction at a distance of 1.5 m from the wing, slightly above the plane of the wing. The image plane was parallel to one of the directions of the reference grid defined by the chordline of the wing, so that the photographic scale (approximately 1:10) was constant for each grid line. The scale factor was independently determined for the various reference points from the distances between adjacent grid points along these grid lines and corresponding values measured in the photograph.

In a harmonic oscillation the points of interest are the endpoints of the motions, which on the photograph were clearly defined as the result of the reversal of the direction of motion (Figure 2).

The amplitude was calculated from the coordinate differences of the endpoints of each line exposed by the vibrating reference points. A simple transformation formula was used, taking the locally determined scale factor and the inclination of the camera axis with respect to the wing into consideration. As a result of the short distances used for deriving both the amplitude and the scale factor from the photo coordinates, lens distortions did not have to be taken into consideration. An approximate value for the principal distance, derived from the nominal value of the focal length which was marked on the lens (158 mm), was used in the computation.

An analysis of the accuracy of the results obtained indicated that errors in the determined amplitude generally did not exceed 0.2 mm corresponding to 2 to 4 per cent of the maximum deflections in the various mode shapes.

DETERMINATION OF THREE-DIMENSIONAL INFORMATION

Controlled experiment. A single camera approach was selected to determine the path and the orientation of a vehicle in testing various types of highway barriers[2]. The accuracy requirements in this project combined with travelling speeds of the vehicle of up to 90 km-per-hour would require extremely precise synchronization (0.001 sec.) of the photographic exposures if two cameras were used. It was therefore decided to base the photogrammetric solution on a single camera and, as the result of the required time interval of the photographs, a 16 mm movie camera was selected. Such a camera is generally not designed for precise photogrammetric operations and special precautions had to be taken to assure an accurate reconstruction of the bundle of projection rays. A control net was established for this purpose which included the precise location of the camera station, in addition to a system of targets behind the scene of the experiment. This net provided the necessary means for accurate reconstruction of the bundle of projecting rays without the need for information on the interior orientation of the camera.

In order to determine the position and the orientation of the vehicle from a single photograph, it was necessary to attach a system of targets to the car, the x, y, and z coordinate of which had to be measured accurately. Figure 3 shows enlarged movie frames representing various stages of the experiment. The targets attached to the vehicle and the reference targets in the background can easily be identified in the photographs.

The camera used in this project was a Millikan 16 mm movie camera with a focal length of 80 mm and a speed of 129 frames per second. The camera was located at 86 m from the system of reference targets behind the scene of the experiment. Its optical axis was perpendicular to the plane defined by these reference targets. The image coordinates of the car targets were measured together with some targets of the background control target system. In practice, only a total of six or seven background targets, located near the car target images, were measured for

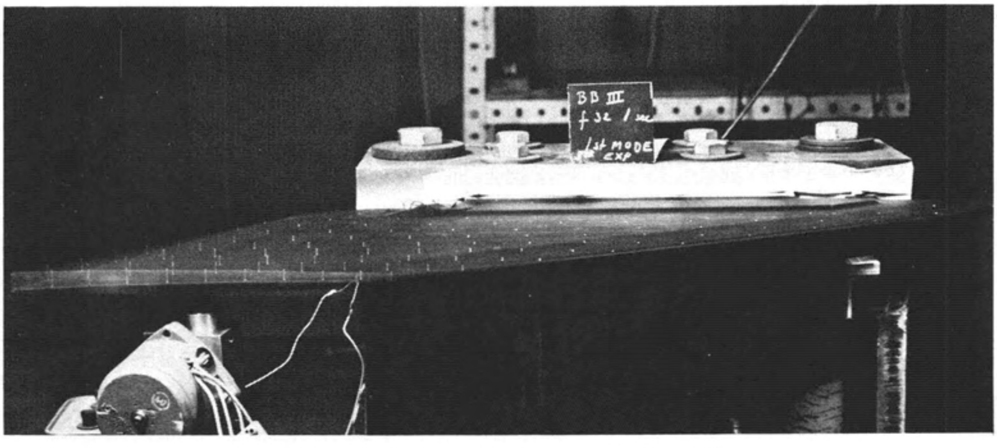

FIG. 2. 1 sec. exposure of wing oscillating at 65.3 cps.

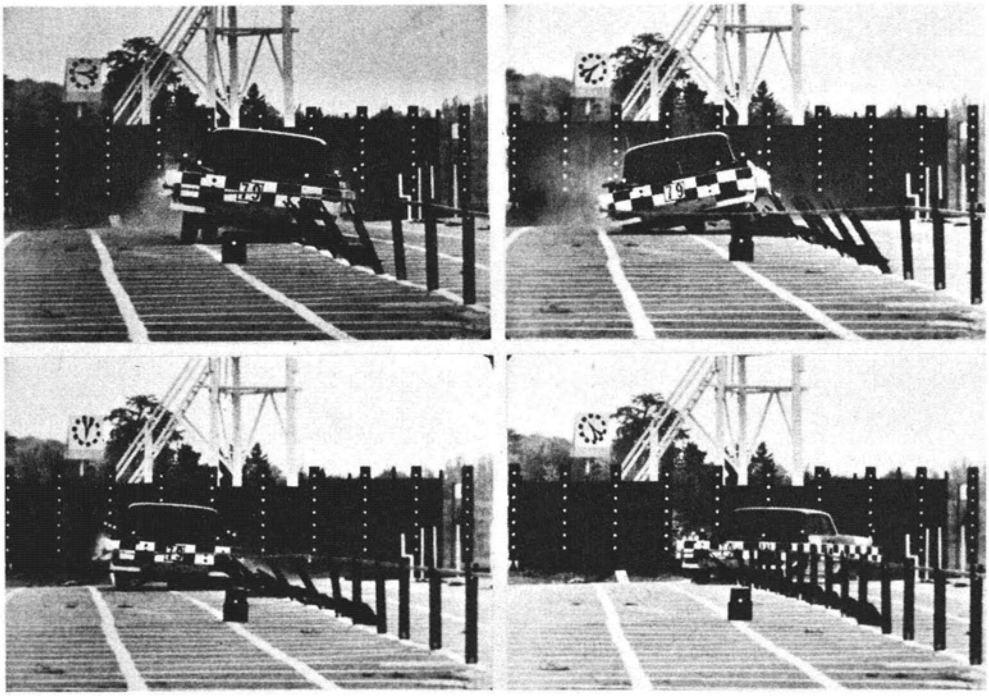

FIG. 3. Enlarged movie frames representing various stages of the experiment.

each frame. The measurements were performed on an NRC monocomparator using a copy of the original 16 mm movie film.

Various tests were made to determine the accuracy of the obtained information. The following standard errors were found:

for the car position:	$m_x = m_y = 1$ cm
	$m_z = 7$ cm
for the car orientation:	$m_\phi = m_\kappa = 2.4'$ (minutes of arc)
	$m_\omega = 3.9'$ (minutes of arc)

The x and y coordinates are parallel to the image plane and the z coordinate is parallel to the optical axis of the camera. The angles κ, ϕ, and ω represent the yaw, pitch, and roll of the vehicle, respectively.

Uncontrolled experiment. The Photogrammetric Research Section has repeatedly become involved in the evaluation of uncontrolled non-metric photography in connection with aircraft accidents. In these cases single frames or movies were provided and either the absolute aircraft position(s), or changes in the aircraft position from frame to frame, were asked for.

In each case only a single camera of unknown make had been used. Therefore, the interior orientation was unknown. In addition, the camera position was only approximately known. Since a crashing aircraft approaches the surface of the earth, a camera following its path may eventually show the aircraft surrounded by topographical features suitable to derive, by spatial resection, both interior and exterior orientation. This, however, necessitates first the determination of coordinates for suitably located topographical features, usually within a local net (Figure 4).

Once interior and exterior orientation have been determined for each photograph, from control point coordinates and measured photograph coordinates for these points, it is possible not only to project the object bundle back into space but also to intersect the object bundle with suitable planes. One such plane, perpendicular to the chosen direction of the local coordinate grid and approximately 1 km away from the determined camera station, was used to compute projected coordinates for the aircraft. These coordinates $\overline{X}, \overline{Z}$ were then also derived from the given airplane dimensions (Figure 5):

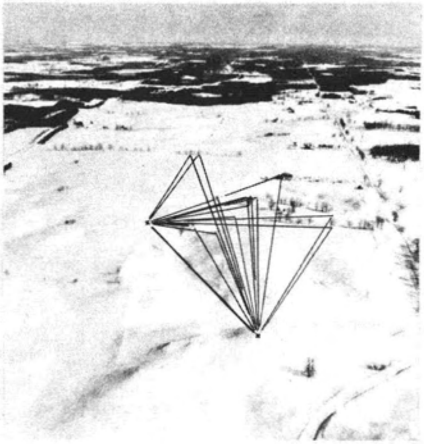

FIG. 4. An oblique photograph of the accident scene showing the local net used to determine reference points for the determination of interior and exterior orientations. The camera taking the movie was located in the vicinity of the station in the foreground.

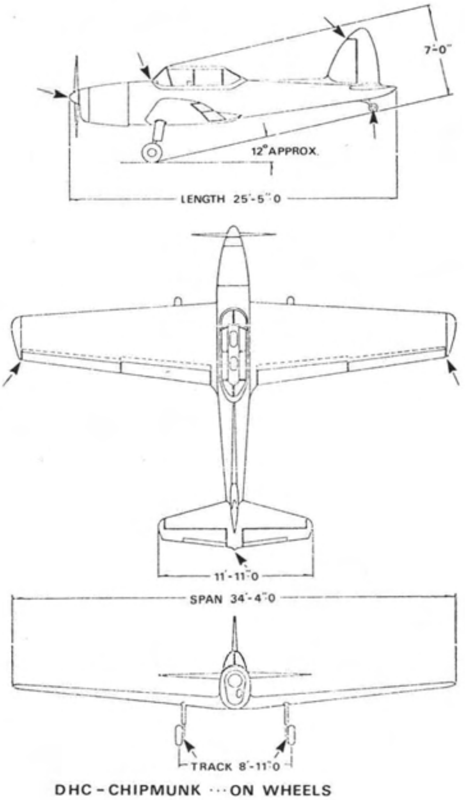

FIG. 5. The coordinates of all points indicated by arrows were scaled off these drawings of the aircraft.

$$\begin{bmatrix} \overline{X} \\ \overline{Z} \end{bmatrix} = \overline{S} \cdot \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{31} & d_{32} & d_{33} \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Use of the scale factor $\overline{S}$ and the matrix D now permitted the determination of the correct location and orientation of the airplane for each single frame.

The results obtained from each seventh frame of the last 154 frames prior to impact, and the aircraft orientation and position at impact of the filmed sequence, are given in Figure 6 which shows in addition to a three-dimensional presentation, three two-dimensional projections. The results were forwarded to the agency investigating this fatal accident and served as primary evidence in a damage suit brought by the late pilot's widow.

The general aircraft position was determined with $m_x = m_z \approx \pm 1.0$ m and $m_y \approx \pm 8.5$ m. The distances y which were derived by means of the scale factor $\overline{S}$ were smoothed out prior to producing Figure 5 while all other values were used as derived from the computations.

Determination of three-dimensional information using a beam splitter attachment. A system which can be used to obtain three-dimensional information from a single photographic exposure is the beam splitter attachment, shown in Figure 7.

It consists of two surface-coated plane mirrors and a surface-coated 90° prism. When mounted in front of the camera lens, two separate bundles of rays are formed so that a photographed object is recorded in two different locations on the photograph. This photograph contains, therefore, the same information as offered by two photographs taken by simultaneously operated, perfectly synchronized cameras, which is of particular interest in photogrammetric studies of high speed processes. The base length of the stereo photographs,

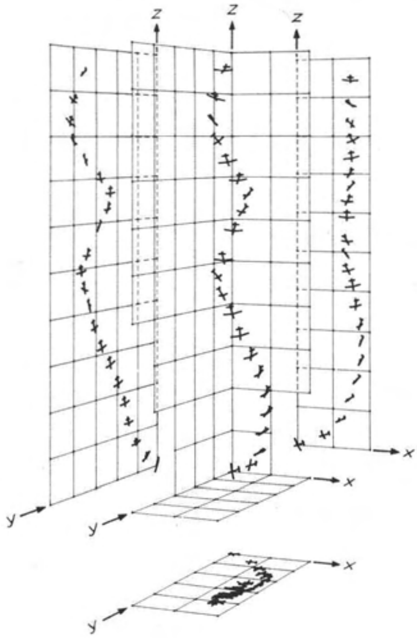

FIG. 6. Three-dimensional presentation of the aircraft movement as derived from the 16 mm movie. The coordinate axes are centered at the aircraft for the instant of impact and each seventh frame going backwards on the film is shown. The grid spacing is 20 m in all three coordinate directions.

obtained by this beam splitter attachment, is approximately the distance between the two outside mirrors. The use of the system is consequently limited to close-range applications involving reasonably short base lengths.

The system was used in the photogrammetric determination of particle velocity vectors in a vertical two-phase solid-gas flow[4]. The determination of the particle velocity was based on double flash, consecutive photographic exposures at a time interval of 10^{-4} sec, using a flash duration of 0.5×10^{-6} sec. The particle flow was photographed in a vertical test column through a plane-parallel glass window. A grid of horizontal and vertical lines at known spacing

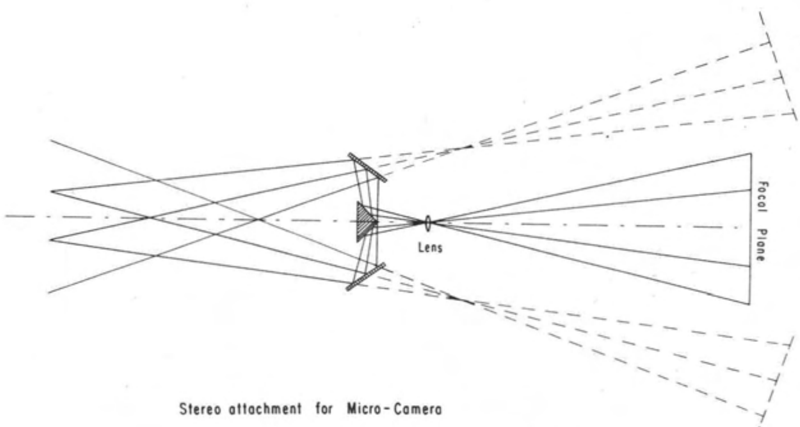

FIG. 7. Beam splitter attachment.

was inserted in the wall of the test section, opposite the window. This grid provided the information needed to correct systematic errors introduced by the observation window and by the beam splitter attachment. It also offered the necessary control for the analytical relative and absolute orientation procedure[5,6]. The effect of the convergence of the stereo images, resulting from the orientation of the outer mirrors of the beam splitter attachment, was corrected during the relative orientation. It was therefore not necessary to calibrate the orientation of the mirrors prior to the photography.

The camera used in this application had been built at the NRC laboratories for close-range application. Its principal distance is 340 mm and it is permanently focussed at the same distance, which results in a photo scale of 1:1. Although this camera can be considered to be a metric camera, the particular arrangement of this test, including the use of the reference grid, would have permitted the use of a non-metric camera.

Based on the grid measurements, the standard error of the photogrammetrically determined coordinates was estimated to be 0.06 mm. Based on this result, the photogrammetric method was judged to be sufficiently precise to measure the three components of the particle velocity in axial, lateral, and transversal directions.

Example for Double Camera Solution

Two Graflex press cameras were used for photogrammetric determination of three-dimensional flow patterns in water tunnel experiments at NRC's National Aeronautical Establishment. In order to provide the necessary means for self-calibration of the cameras and for correcting distortions caused by the observation window and by the water, two regularly spaced grids were used throughout the experiments (Figure 8).

One of these grids was located on the back wall of the tank and the other on the inside of the observation window at the water-glass interface at a distance of 25 cm from each other. Since both grids were in contact with the body of water in the tunnel, it was possible to determine, in each individual exposure, the equivalent principal distance of the cameras for the water medium and to reconstruct the bundle of projecting light rays within the tunnel.

The flow in the tunnel was made visible in the photographs by illuminating tracer particles with a stroboscopic light source. A frequency of 120 cycles/sec was found to give a satisfactory spacing of the particle images for the speed of flow in the tunnel which, during the experiment, was 25 cm/sec. The time base of the stroboscope served to calculate particle velocities from the photogrammetrically determined particle coordinates. The cameras were oriented with their optical axes normal to the observation window. The distance between each camera and this window was 2.1 m, resulting in a photo scale of 1:12. The distance between the two cameras was 50 cm.

The photographs were measured on a Zeiss Jena Stereocomparator 1818; and standard programs for relative and absolute orientation, developed for aerial photogrammetry, were used for calculating the coordinates of the particle images and the grid points[5,6]. Standard errors of the photogrammetrically determined grid points were $m_{x,y} = 0.2$ mm (parallel to the image plane) and $m_z = 1.3$ mm (normal to the image plane). Standard errors of the photogrammetrically determined particle coordinates were calculated from consecutive particle images, assuming that the particle velocity is constant during the short time interval of $0.1 - 0.2$ sec

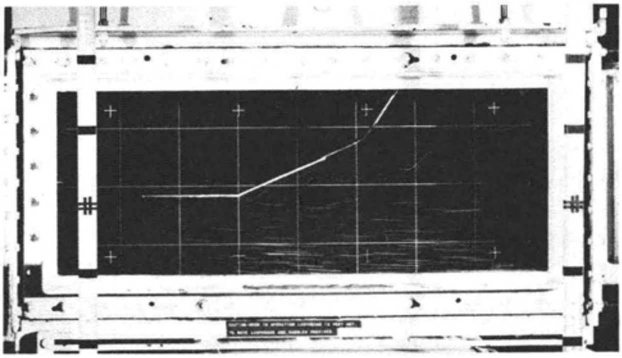

FIG. 8. Watertunnel experiments.

that these consecutive particle images were recorded. A standard error of 0.5 mm was determined for the spatial distance between the particle images. This error is smaller than expected from the standard errors of the grid points. This can be explained by the fact that the smaller distances between consecutive particle images are less influenced by systematic errors (lens distortion, improper film flattening, etc.) than the grid points, which cover a much larger area.

When considering the scale of the photographs (1:12) and the relatively short camera base (base/height ratio = 1:5), the results obtained for the distances between the particle images and the derived velocities correspond to a standard error in x-parallax of 0.008 mm, which can be considered to be satisfactory. No significant improvement could be expected in determining the small distances from photographs taken with metric cameras.

Conclusion

The applications described in this paper were based on photographs taken with non-metric cameras because no suitable photogrammetric camera was available at the time of the experiments for the short object distances and the rapid sequence of exposures required for some of the projects. In addition, some of the work was based on photographs that had been taken for a different purpose before it was decided to use them in a photogrammetric evaluation.

Use of photogrammetric cameras would have resulted in a reduction of the preparatory operations such as the establishment of reference grids, etc. It has, however, been demonstrated that satisfactory results can be obtained from photographs taken with non-metric cameras if the necessary precautions are taken. This is particularly true for photogrammetric determination of short distances, such as the deformation and the velocity vectors, described in this paper; or when photo-coordinates measurements can be referred to a well-defined grid in the object space.

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