Mitsuru Nasu
Dr. James M. Anderson
University of California
Berkeley, CA 94720

Statistical Testing Procedures Applied to Analytical Camera Calibration of Non-Metric Systems

The stability of the analytical calibration of two non-metric cameras was evaluated by employing Hotelling's T^2 test.

Introduction

The objective of this study is to investigate a practical method for making valid statistical significance tests of repeated camera calibrations in order to evaluate system stability. Application of a modified form of Hotelling's T^2 statistic for multivariate comparisons is developed and use of this procedure is illustrated by applying the test to eight analytical calibrations of a Leica 35 mm camera and five analytical calibrations of a Rolleiflex camera equipped with a Planar lens.

Analytical Camera Calibration Procedure

The mathematical model utilized for camera calibration consists of forming a pair of linearized (Taylor series, 2nd and higher order terms neglected) collinearity condition equations, with calibration parameters added, for each ray from exposure station i to object points

Abstract: *Camera calibration parameters for non-metric systems are most easily determined by analytical methods. In non-metric systems, the possibility exists that these parameters may vary from exposure to exposure. Proper evaluation of system stability should include a practical but rigorous statistical test to supplement intuitive analysis of the calibration data. A modification of Hotelling's T^2 test is developed and applied to the results of repeated analytical calibrations for two non-metric cameras. The stability of the two cameras is judged to be fair and the Hotelling's T^2 test is found to be a useful supplement to intuitive evaluation of the calibration data.*

j. Coefficients and discrepancy terms in these equations are functions of known object point coordinates, measured photographic coordinates, and approximations estimated for unknown exposure station and calibration parameters. Using redundant object points, the resulting system of equations is solved by the method of least squares. The normal equations for a calibration using photograph i are

$$\underset{m,m}{\boldsymbol{N}_i} \; \underset{m,1}{\boldsymbol{\delta}_i} = \underset{m,1}{\boldsymbol{C}_i} \tag{1}$$

in which

$$\underset{m,m}{\boldsymbol{N}_i} = \text{normal equation coefficient matrix,}$$

$\underset{m,1}{\boldsymbol{\delta}_i}$ = corrections to m unknown estimated parameters for which the estimates are $\underset{m,1}{\hat{\boldsymbol{X}}^\circ}$, and

$\underset{m,1}{\boldsymbol{C}_i}$ = normal equation constant vector.

Equations (1) are solved for $\boldsymbol{\delta}$ to yield

$$\underset{m,1}{\boldsymbol{\delta}_i} = \underset{m,m}{N_i^{-1}} \underset{m,1}{\boldsymbol{C}_i} \tag{2}$$

and adjusted parameters are

$$\underset{m,1}{\hat{\mathbf{X}}_i} = \underset{m,1}{\hat{\mathbf{X}}_i^\circ} + \underset{m,1}{\boldsymbol{\delta}_i} \tag{3}$$

The solution is iterated using approximations for unknown parameters until convergence occurs. The final adjusted parameters are given by Equation (3) and the estimated covariance matrix of these parameters is

$$\underset{m,m}{\hat{\boldsymbol{M}}_i} = \hat{\boldsymbol{m}}_{io}^2 \underset{m,m}{\boldsymbol{N}_i^{-1}} \tag{4}$$

in which $\hat{\boldsymbol{m}}_{io}^2$ is the variance of unit weight for the adjustment.

Now, assume that $i = 1, 2, \ldots, k$ photographs, taken under varying conditions, are available to permit k determinations of calibration parameters which are included in the vector $\hat{\mathbf{X}}_i$. It is desired to have a statistical test which will allow evalution of possible significant differences in the calibration parameters.

Statistical Tests for Comparison of Calibration Results

Approximating the t-statistic with normal statistics could be used to test the homogeneity of single variables (e.g., f, x_p, y_p, . . . , etc.). However, in analytical camera calibrations high correlations are known to exist among the calibration parameters so that a simultaneous comparison of all parameters is necessary.

One method of simultaneous comparison is by use of the F-statistic in which variances of an individual sample and from pooled data are compared. Unfortunately, tabulated values of the F-statistic are almost asymptotic to its value for an infinite number and 200 degrees of freedom yielding a test with very high power. This factor can lead to indications of significant differences when in a practical sense none should exist. Another disadvantage of this procedure is the difficulty of locating the parameter which causes the hypothesis to be rejected. Hotelling's T^2 statistics are applicable in multivariate comparisons and are discussed next.

Hotelling's T^2 Statistics for Compariing Vectors of Mean Estimates

Suppose $\mathbf{X}$ is the true vector or hypothetical value and $\hat{\mathbf{X}}$ is the estimate of $\mathbf{X}$. Then the quadratic form

$$T^2 = [\hat{\mathbf{X}} - \mathbf{X}]^T \underset{m,m}{\boldsymbol{M}^{-1}} [\hat{\mathbf{X}} - \mathbf{X}] \tag{5}$$

is the equation of an hyperellipsoid centered at $\boldsymbol{X} = \hat{\mathbf{X}}$ in m dimensional parameter space where $\boldsymbol{M}$ is the $m \times m$ covariance matrix of the estimate, the value T^2/m is distributed as $F_{m,n-m}$, and n is the number of samples. Thus, a multivariate confidence region is given by the ellipsoid

$$(\hat{\mathbf{X}} - \mathbf{X})^T \underset{m,m}{\boldsymbol{M}^{-1}} (\hat{\mathbf{X}} - \mathbf{X}) = mF_{m,n-m,\alpha} \tag{6}$$

The probability that this region does not include the true value $\boldsymbol{X}$ is α.

The covariance matrix of the estimated vector is given by $\underset{m,m}{\hat{\boldsymbol{M}}}$ in Equation (4), so that Equation (6) can be re-written as

$$(\hat{\mathbf{X}} - \mathbf{X})^T \frac{\mathbf{N}_i}{\hat{m}_{oi}^2} (\hat{\mathbf{X}} - \mathbf{X}) = mF_{m,n-m,\alpha} \tag{7}$$

which is the fundamental equation when all m components are subjected to the statistical test.

In order to test the hypothesis

$$H_o : X = X_H \tag{8}$$

compute the quantity

$$\frac{T^2}{m} = \frac{1}{m} (\hat{\mathbf{X}} - \mathbf{X}_H)^T \frac{\mathbf{N}_i}{\hat{m}_{oi}^2} (\hat{\mathbf{X}} - \mathbf{X}_H) \tag{9}$$

and test as $F_{m,n-m,\alpha}$. If $\frac{T^2}{m}$ exceeds $F_{m,n-m,\alpha}$, one may reject the hypothesis, H_o, at the α significance level.

Application to Analytical Camera Calibration

In the problem of camera calibration

$$\mathbf{X}_i = [(X,Y,Z)_{oi}, (\omega,\phi,\kappa)_{oi}, (f,x_p,y_p,k_1,k_2,k_3,p_1,p_2)_i]^T$$

and

$$\hat{\mathbf{X}}_i = [(\hat{X},\hat{Y},\hat{Z})_{oi},(\hat{\omega},\hat{\phi},\hat{\kappa})_{oi}, (\hat{f},\hat{x}_p,\hat{y}_p,\hat{k}_1,\hat{k}_2,\hat{k}_3,\hat{p}_1,\hat{p}_2)_i]^T \tag{10}$$

in which $(X,Y,Z,\omega,\phi,\kappa)_{oi}$ are the true values for exposure station parameters, $(f, x_p, y_p, k_1, k_2,k_3,p_1,p_2)_i$ are the true calibration parameters according to the model as developed by Brown[1], and the vector $\hat{\mathbf{X}}_i$ represents estimated values for exposure station and calibration parameters for the i^{th} frame.

The homogeneity of the $\hat{\mathbf{X}}_i$'s obtained from several frames or calibrations with the same camera is to be tested in order to evaluate camera stability. Note that common calibration parameters are assumed for all frames but it is expected that each frame will have different exterior orientation parameters. These exterior orientation parameters are dummy variables which are not to be compared with each other in the statistical testing. Theoretically, errors in camera calibration should not be studied without considering errors in the exterior orientation, due to high correlations which exist among these parameters. Since the T^2 statistic is the most feasible method for this problem of multivariate comparison, a more generalized form designed to handle dummy variables has been developed.

Consider a general hypothesis which says something about the values of the parameters, either all of them or a subset of them. Suppose the linear hypothesis can be written as[2]

$$\underset{b,m}{Q}\ \underset{m,1}{\mathbf{X}} = \underset{b,1}{\mathbf{Z}} \tag{11}$$

The hypothesis is of rank b which we assume to be $b \leqq m$. The null hypothesis is that the true values of $\mathbf{X}$ satisfy these equations. In this study, where $\mathbf{X}$ includes six orientation and eight calibration parameters,

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & & & & & 0 & 0 & 1 & & & & & & 0 \\ \cdot & & & & & \cdot & \cdot & & & & & & & \cdot \\ \cdot & & & & & \cdot & \cdot & & & & & & & \cdot \\ \cdot & & & & & \cdot & \cdot & & & & & & & \cdot \\ \cdot & & & & & \cdot & \cdot & & & & & & & \cdot \\ \cdot & & & & & \cdot & \cdot & & & & & & & \cdot \\ \cdot & & & & & \cdot & \cdot & & & & & & 1 & 0 \\ 0 & & & & & 0 & 0 & & & & & & 0 & 1 \end{bmatrix} \tag{12}$$

Then the following equation can be written

$$\frac{1}{b}(\mathbf{Z} - Q\,\hat{\mathbf{X}})^T (Q^T\hat{M}Q)^{-1} (\mathbf{Z} - Q\,\hat{\mathbf{X}}) \tag{13}$$

TABLE 1. CAMERA AND PHOTOGRAPHIC DATA

Camera	Photo Numbers	Lens Speed Seconds	Aperture	Focus	Nominal f (mm)	Aver. Photo Scale
Leica	1					
	2					
	.					
	.	1/30	f/8	∝	35	1:600
	.					
	8					
Rolleiflex	2-1, 2-2	1/125	f/8	∝		
	2-5, 2-6	1/60	f/11	∝	75	1:250
	2-9, 2-10	1/60	f/16	∝		

TABLE 2. COMPUTATIONAL MODES.

Comp. Mode	No. of Photos per Calibration	Parameters Calibrated & Calibration Program
I	1 Photo	$f,x_p,y_p,k_1,k_2,k_3,p_1,p_2$ TURTLE
II	1 Photo	$f,x_p,y_p k_1,k_2$ TURTLE
III	Sim. Adj. pooled data all frames	$f,x_p,y_p,k_1,k_2,k_3,p_1,p_2$ TURTLE
IV	Sim. Adj. pooled data all frames	f,x_p,y_p,k_1,k_2 TURTLE

TABLE 3. LEICA CAMERA ABSOLUTE CALIBRATION DATA.

Mode	Frame	r_i	m_o (μm)	f (c.f.l.) (mm)	m_f (mm)	x_p (mm)	m_{xp} (mm)	y_p (mm)	m_{yp} (mm)
I	1	205	8.8	35.715	0.054	0.237	0.082	0.028	0.076
$f,x_p,y_p,k_1,k_2,$ k_3,p_1p_2	2	193	8.7	35.732	0.054	0.259	0.084	0.001	0.080
	3	171	10.7	35.733	0.066	0.446	0.105	0.054	0.102
	4	179	9.3	35.655	0.060	0.341	0.093	0.000	0.094
	5	197	8.6	35.647	0.053	0.332	0.080	0.029	0.075
	6	199	8.7	35.695	0.053	0.443	0.082	−0.007	0.078
	7	217	8.8	35.676	0.053	0.410	0.082	0.017	0.078
II	8	187	8.7	35.671	0.054	0.359	0.083	−0.041	0.080
III	*	1591	9.0	35.687	0.020	0.348	0.030	0.007	0.029
Averages				35.690	0.052	0.353	0.080	0.010	0.077
II	1	209	8.9	35.721	0.052	0.404	0.037	0.038	0.027
f,x_p,y_p,k_1,k_2	2	197	8.7	35.729	0.051	0.396	0.038	0.043	0.027
	3	177	10.5	35.719	0.064	0.434	0.049	−0.007	0.033
	4	183	9.3	35.673	0.057	0.378	0.046	0.032	0.031
	5	201	8.6	35.653	0.050	0.407	0.037	0.039	0.025
	6	203	8.7	35.700	0.051	0.441	0.037	−0.011	0.026
	7	199	8.7	35.679	0.051	0.432	0.038	0.012	0.026
II	8	191	8.7	35.677	0.051	0.394	0.039	0.011	0.026
IV	*	1595	9.0	35.692	0.019	0.410	0.014	0.019	0.010
Averages				35.694	0.050	0.411	0.037	0.020	0.026

* Simultaneous 8 Frame Adjustment

which is distributed as $F_{b,n-m}$. The hypothesis is rejected if the computed value of this quantity exceeds the tabulated value of $F_{b,n-m,\alpha}$.

Calcuation of (13) is performed by removing all unnecessary rows and columns of the (1) mean vectors to be tested; (2) hypothetical vectors; and (3) covariance matrix, forming a $b \times 1$ vector of estimates and a $b \times b$ covariance matrix.

Multivariate comparison of any portion of the mean vector can be performed by selecting an appropriate matrix Q. Thus, one could compare all parameters or, at the other extreme,

TABLE 4. ROLLEIFLEX PLANAR ABSOLUTE CAMERA CALIBRATION DATA.

Mode	Frame	r_i = n−μ	m_o (μm)	f (c.f.l.) (mm)	m_f (mm)	x_p (mm)	m_{xp} (mm)	y_p (mm)	m_{yp} (mm)
$f,x_p,y_p,k_1,k_2,k_3,p_1,p_2$ I	2-1	199	12.0	75.281	0.100	−0.790	0.127	0.104	0.097
	2-2	191	13.4	75.446	0.111	−0.744	0.138	0.176	0.109
	2-5	208	12.8	75.385	0.104	−0.686	0.133	−0.008	0.103
	2-6	163	14.1	75.279	0.120	−0.726	0.172	0.001	0.137
	2-9	195	13.3	75.399	0.109	−0.787	0.139	0.035	0.107
I	2-10	193	14.9	75.285	0.116	−0.827	0.154	−0.145	0.119
III	*	1191	13.5	75.347	0.045	−0.758	0.058	0.022	0.046
	†			†75.346	0.101	−0.760	0.132	0.026	0.103
f,x_p,y_p,k_1,k_2 II	2-1	202	12.2	75.248	0.097	−0.471	0.044	0.165	0.067
	2-2	194	13.5	75.397	0.106	−0.531	0.048	0.220	0.076
	2-5	211	12.9	75.364	0.100	−0.522	0.046	0.107	0.068
	2-6	166	14.0	75.269	0.115	−0.596	0.064	0.024	0.078
	2-9	198	13.4	75.369	0.105	−0.479	0.048	0.045	0.072
II	2-10	196	15.0	75.262	0.111	−0.535	0.054	−0.112	0.079
IV	*	1194	13.6	75.319	0.043	−0.520	0.020	0.073	0.030
	†			†75.318	0.097	−0.522	0.046	0.075	0.041

* Simultaneous 6 Frame Adjustment
† Mean values

TABLE 5. LEICA AND ROLLEIFLEX PLANAR: SIGNIFICANCE TESTS STANDARD DEVIATIONS OF UNIT WEIGHT.

	Sample Tested	K	C	$\chi^2_{1-\alpha,k-1}$ 0.95	0.975	Comments
Leica	Mode I	8	14.4	14.1	16.0	No sig. diff. 97.5% conf. int.
	Mode II	8	12.38	14.1	16.0	No sig. diff. 95% conf. int.
	Modes I & II Pooled	16	26.81	26.3	28.8	No sig. diff. 97.5% conf. int.
Rolleiflex Planar	Mode I	6	10.92	11.1	12.8	No sig. diff. 95% conf. int.
	Mode II	6	9.86	11.1	12.8	No sig. diff. 95% conf. int.
	Modes I & II Pooled	12	20.84	21.0	23.3	No sig. diff. 95% conf. int.

K = number of adjustment computations
$C = \Sigma(r_i)\cdot \ln m^2 - \Sigma(r_i \cdot \ln m^2_{oi})$ in which
$m^2 = \dfrac{\Sigma(r_i m^2_{oi})}{\Sigma r_i}$

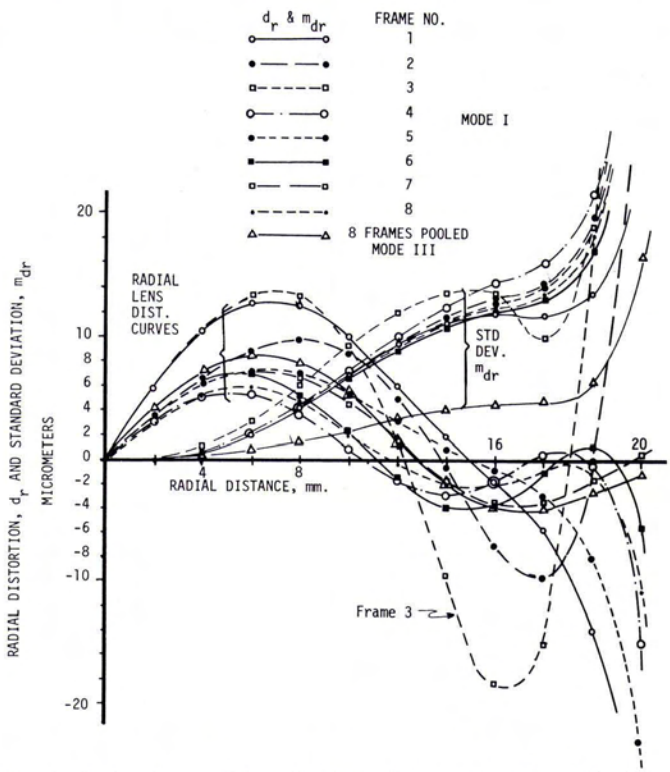

FIG. 1. Leica: Symmetric radial distortion curves and standard deviations of radial distortion.

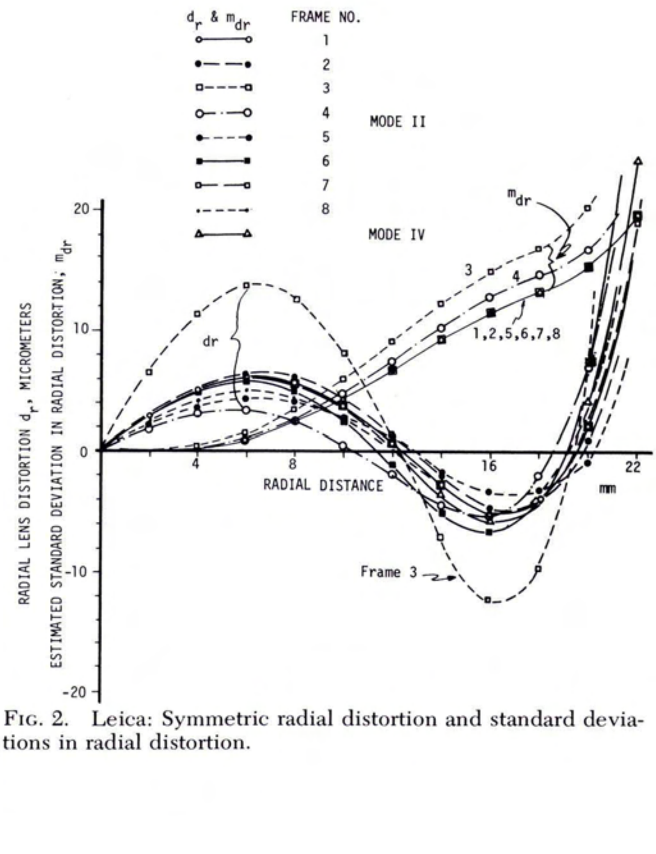

FIG. 2. Leica: Symmetric radial distortion and standard deviations in radial distortion.

make the statistical test of a single parameter which is simply the square of the usual student's t-statistic.

Homogeneity of estimates is tested by "indirect comparison" of group estimates with their hypothetical values. Theoretically, a more correct method would be a "multiple direct comparison" described in detail in References 3 and 4. This direct comparison is very complicated and appears impractical for the present situation. The method of multidimensional indirect comparison can be applied to practical camera calibration problems and is utilized for this study.

Data and Calibration Programs for Application of T^2 Tests

A Leica 35 mm camera (on loan from the U.S. Geological Survey, Menlo Park, California) and a Rolleiflex with Planar lens were calibrated using the Birge Hall Test Field as established by Van Roessel on the University of California, Berkeley campus.[5] The Birge Hall Test Field contains 110 object points of known spatial positions and has relief which constitutes approximately 75% of the object distance.

Data chosen for testing consist of calibrations with eight Leica frames and five Rolleiflex Planar exposures. Table 1 illustrates camera and photographic conditions. Photographic coordinates were measured on the Mann Monocular Comparator at the U.S. Geological Survey in Menlo Park, California for the Leica frames and the PSK stereo-comparator at the Geometronics Division of the California Department of Transportation for the Rolleiflex Planar frames.

A general triangulation-calibration program (Program TURTLE) based on the collinearity condition with added parameters (Brown's distortion model, Equation 10)[1] was employed

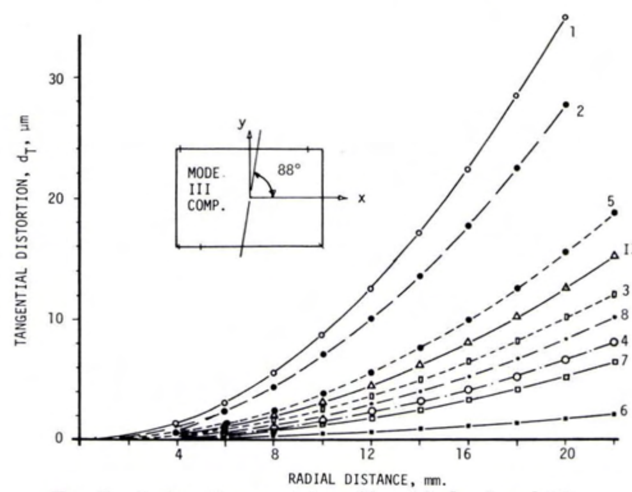

FIG. 3. Leica: Tangential profiles, Modes I and III.

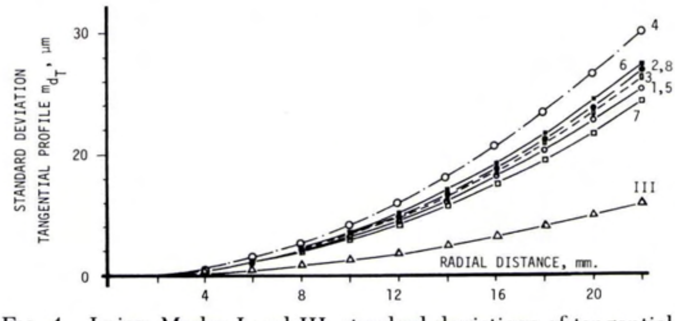

FIG. 4. Leica: Modes I and III, standard deviations of tangential profiles.

for the calibrations. Four computational modes involving two combinations of added parameters plus calibrations with individual and pooled photographs were utilized with Program TURTLE. Table 2 shows these four computational modes.

Statistical Tests for Homogeneity of Data

Estimated unit standard deviations are given for the Leica and Rolleiflex cameras in Tables 3 and 4 respectively. The homogeneity of these estimates was evaluated using Bartlett's test which allows testing all estimates simultaneously.[2] Results of Barlett's test applied to sixteen Leica and twelve Rolleiflex calibrations are summarized in Table 5.

No significant differences are indicated in the unit variances of the calibration adjustments so that the data can be assumed to be homogeneous.

Calibration Results

Calibrated focal lengths (c.f.l.), principal point coordinates (x_p, y_p), and associated estimated standard deviations (m_f, m_{xp}, m_{yp}) are listed in Tables 3 and 4 for the Leica and Rolleiflex cameras.

Symmetric radial lens distortion curves and associated estimated standard deviations are illustrated for Modes I and III, II and IV in Figures 1, 2, 5, and 6, respectively, for the Leica and Rolleiflex Planar calibrations.

Tangential profiles and estimated standard deviations in tangential distortion are shown in Figures 3 and 4 and 7 and 8 for the Leica and Rolleiflex, respectively.

Statistical Comparison Using the T^2 Test

The procedure consists of selecting hypothetical value(s) for the unknown parameter(s) and then testing parameter(s) from the least squares estimates for each frame individually.

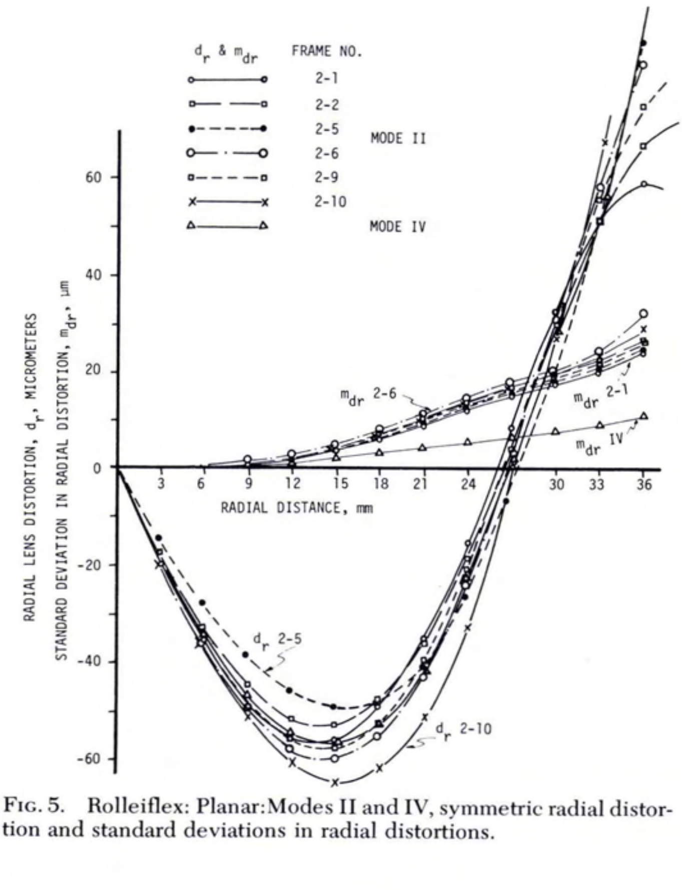

Fig. 5. Rolleiflex: Planar: Modes II and IV, symmetric radial distortion and standard deviations in radial distortions.

The estimates of the multiframe calibrations (Modes III and IV) were chosen as hypothetical values for the unknown parameters in the analysis which follows.

T^2 Analysis of Leica Calibration Results

Tables 6(a) and (b) show the results for T^2 tests of the Leica calibration data computed with Modes I and II. Conclusions drawn from these tests are:

- Mode I, all parameters calibrated.
  —No significant interframe differences noted.
- Mode II, f, x_p, y_p, k_1, k_2 calibrated.
  —Frame Number 3 is rejected.

Reference to Table 3 and Figures 1 and 2 reveals that Frame 3 has:

(a) an estimated standard deviation for the adjustment which is approximately 10% larger than the average (Table 3) (note, however, there was no significant difference indicated by the Bartlett test).
(b) slightly different values for c.f.l. but similar results for x_p and y_p when compared with other frames (Table 3).
(c) a somewhat unique radial lens distortion curve (Figure 1) when all parameters are calibrated.
(d) very unique lens distortion curve for Mode II (Figure 2).
(e) a tangential profile similar in shape to other calibrations (Figure 4).

These valuations of the plotted data plus the statistical test indicate that Frame Number 3 is significantly different in its radial lens distortion curves. Acceptance of the null hypothesis for all frames in Mode I computations is probably caused by interaction among large variances and covariances resulting from highly correlated parameters.

T^2 Analysis of the Rolleiflex Planar Results

Tables 6(a) and (b) also contain results of T^2 tests applied to the Rolleiflex Planar calibrations. Conclusions drawn from these tests are:

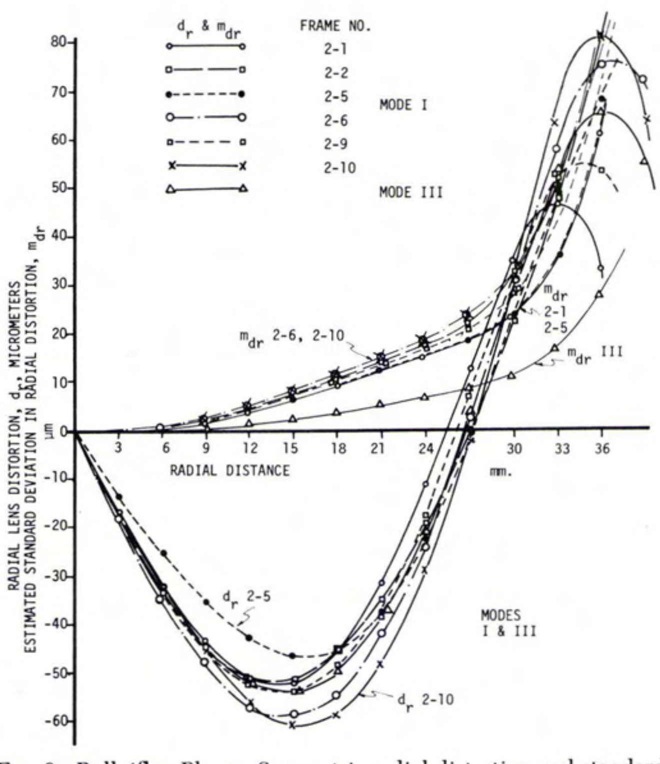

Fig. 6 Rolleiflex Planar: Symmetric radial distortion and standard deviations in the radial distortions.

- Mode I, all parameters calibrated.
  —Frame 2-5 is rejected.
- Mode II, f, x_p, y_p, k_1, k_2 calibrated.
  —Frame 2-5 rejected.

Examination of tabular and graphical representations of the Rolleiflex Planar data in Table 4 and Figures 5, 6, 7, and 8 reveals only slightly different lens distortion characteristics for Frame 2-5. A possible source of difficulty could be y_p (Table 4), which is somewhat irregular, working together with some part of the radial distortion to cause rejections. As a further check T^2 tests were run with only the lens distortion data [Table 6(c) and (d)]. Results from these tests indicate homogeneity of the lens distortions in all cases.

To summarize, the data for the Rolleiflex camera are slightly unhomogeneous due to some randomly occurring systematic perturbation such as film distortion and lack of flatness of film in the focal plane during exposure. Note that frames 2-1, 2-2; 2-5; 2-6; and 2-9, 2-10 have three different sets of photographic conditions (Table 1). However, there is no evidence to indicate significant differences among these groups. Contrast analysis would be necessary for further study of these data.

Conclusions

Intuitive evaluation of calibration data plus statistical Hotelling's T^2 tests shows that the two non-metric cameras possess a fair degree of stability when repeated calibrations are performed using several independent exposures.

One of eight Leica calibrations was rejected on the basis of the T^2 test. However, when all

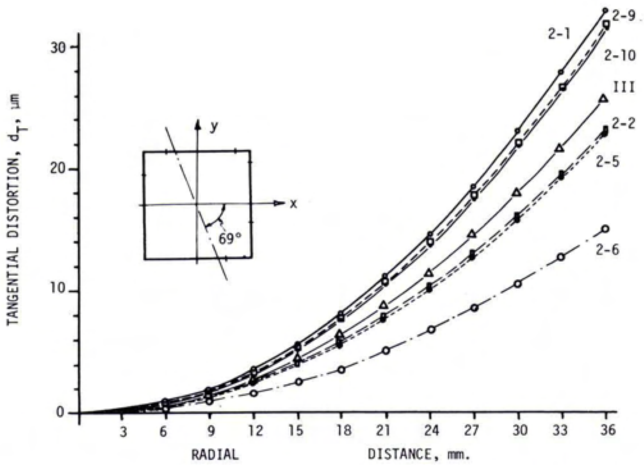

Fig. 7. Rolleiflex Planar: Modes I and III, tangential profile.

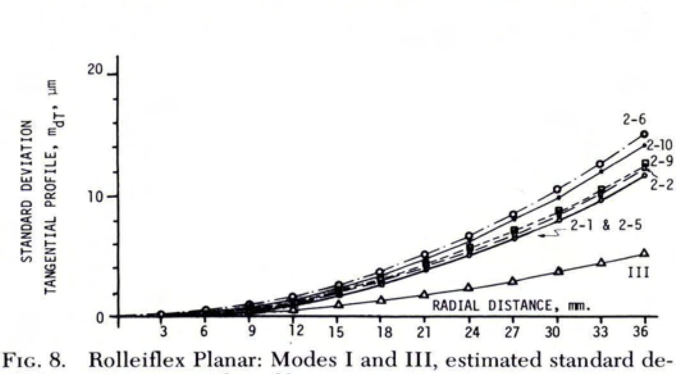

Fig. 8. Rolleiflex Planar: Modes I and III, estimated standard deviations, m_{dt} tangential profiles.

TABLE 6. T^2 TESTS, ROLLEIFLEX AND LEICA.

		(a) Mode I $m = 8, n-m = 200$				(b) Mode II $m = 5, n-m = 200$			
Frame	Camera	$\frac{T^2}{m}$	> or <	$F_{m,n-m}$ $\alpha = 0.01$	H_o: $\hat{X} = X_H$	$\frac{T^2}{m}$	> or <	$F_{m,n-m}$ $\alpha = 0.01$	H_o: $\hat{X} = X_H$
1	Leica	0.63	<	2.51	Accepted	0.36	<	3.02	Accepted
2		1.28	<	2.51	Accepted	0.86	<	3.02	Accepted
3		2.07	<	2.51	Accepted	5.23	>	3.02	Rejected
4		0.23	<	2.51	Accepted	0.31	<	3.02	Accepted
5		0.23	<	2.51	Accepted	0.32	<	3.02	Accepted
6		0.50	<	2.51	Accepted	0.49	<	3.02	Accepted
7		0.12	<	2.51	Accepted	0.10	<	3.02	Accepted
8		0.27	<	2.51	Accepted	0.12	<	3.02	Accepted
2-1	Rolleiflex	1.36	<	2.51	Accepted	1.90	<	3.02	Accepted
2-2		1.39	<	2.51	Accepted	1.42	<	3.02	Accepted
2-5		4.13	>	2.51	Rejected	6.73	>	3.02	Rejected
2-6		0.35	<	2.51	Accepted	0.43	<	3.02	Accepted
2-9		0.95	<	2.51	Accepted	1.54	<	3.02	Accepted
2-10		1.02	<	2.51	Accepted	1.51	<	3.02	Accepted
		Rolleiflex, Lens Distortion Only							
		(c) 7th order polynomial $m = 5, n-m = 200$				(d) 5th order polynomial $m = 2, n-m = 200$			
2-1	Rolleiflex	1.30	<	3.02	Accepted	2.70	<	4.61	Accepted
2-2		0.57	<	3.02	Accepted	0.49	<	4.61	Accepted
2-5		1.69	<	3.02	Accepted	3.52	<	4.61	Accepted
2-6		0.26	<	3.02	Accepted	0.19	<	4.61	Accepted
2-9		0.30	<	3.02	Accepted	0.32	<	4.61	Accepted
2-10		0.87	<	3.02	Accepted	1.98	<	4.61	Accepted

parameters were calibrated (Mode I), fairly large variations occurred among all the radial lens distortion curves (Figure 1). Consequently, acceptance of the null hypothesis for these cases must be regarded with suspicion and the T^2 test applied only to lens distortion data to verify or reject its homogeneity. Distortion curves for Mode II show smaller variations (with exception of Frame 3) so that conclusions drawn from analysis of these curves support the results of the T^2 test. Variations in the distortion curves (Mode I) and the contradictory acceptance of the null hypothesis in this case are most probably due to high correlations which exist among both calibration and exterior orientation parameters plus a poor distribution of object points in the test field.

Slight unhomogeneity of the Rolleiflex data was indicated for the T^2 test of Modes I and II calibrations in spite of good agreement among distortion curves for all frames in each Mode (Figures 5 and 6). Thus, in this case the T^2 test provided a false rejection when considered without supporting evidence. As in the Leica calibrations, highly correlated parameters are the most likely cause of this false rejection. T^2 tests applied only to the lens distortion data resulted in no rejections and verified the conclusions drawn from intuitive analysis of the distortion curves.

Hotelling's T^2 test is a useful supplement to a thorough evaluation of all camera calibration data. It allows simultaneous testing of all or any desired combination of parameters. However, care must be exercised in utilization of results from T^2 tests since high correlations among the parameters (exterior orientation as well as calibration) can lead to apparently false acceptance or rejection of the null hypothesis.

It should be emphasized that this combination of intuitive evaluation of all calibration data plus statistical testing is very important in order to minimize occurrence of mistakes in decision making related to analysis of calibration and other adjustment problems.

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- Combining remote sensing information with other map information
- Appraisal of costs and time required for interpretation, display, and mapping of remote sensing data.

Titles and a brief abstract should be mailed by June 15, 1976, to:

Dr. Robert B. McEwen
U.S. Geological Survey
National Center, #510
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