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Scanning Electron Micrography and Photogrammetry

The collinearity condition of photogrammetry and several distortion functions are employed to model the distortions in a scanning electron microscope system.

INTRODUCTION

THE BASIC GEOMETRIC PROPERTY of photographs is the "central (perspective) projection", i.e., the points of an object are imaged with straight rays, all of which pass through the "perspective center" (See Figure 1). Fundamental to this is the assumed condition that an object point, the perspective center and the corresponding image point, all three, lie in a straight line. This is known as the collinearity condition. This assumed condition, however, does not always hold true. A variety of disturbances may cause the actual rays to deviate from the collinearity condition. Many of these disturbances are found to be a systematic character, i.e., they repeat regularly and can be determined in advance by the calibration procedure, and can be subsequently corrected for. There are other, irregular, types of disturbances which are random in character. They limit the quality of the measurement procedure. However, they also can be estimated statistically and determined as accurately as possible through calibration.

ABSTRACT: *With the growing need for serious quantitative investigations with scanning electron microscope (SEM) micrographs, more attention is being drawn to statistically sound metric calibration of the SEM system and to a general understanding of the inherent distortions in the SEM micrographs. By using the basic concept of collinearity condition with respect to perspective and parallel projections and an advanced photogrammetric self-calibration technique, one obtains the patterns of scale, radial, tangential, and spiral distortions. These are discussed and the corresponding mathematical models are presented. Results of recent research at The Ohio State University are presented.*

The parameters that define the actual relative relationship between the object points, their photographic image plane, and the perspective center are called the elements of interior orientation (I.O.). These normally include: (1) the camera constant (also called the focal length or principal distance); (2) locations of fiducial marks on the image (focal) plane to define the principal point (foot point of the plate perpendicular from perspective center) and references for distortions on the photograph; and (3) all distortion parameters (in the interior of the photographic system).

If sufficient information is known about the I.O. elements along with the information about film shrinkage and other parameters external to the imaging system affecting the trajectory of rays, it is possible to reconstruct the bundle of rays in reverse order by measuring the coordinates of image points on the photograph.

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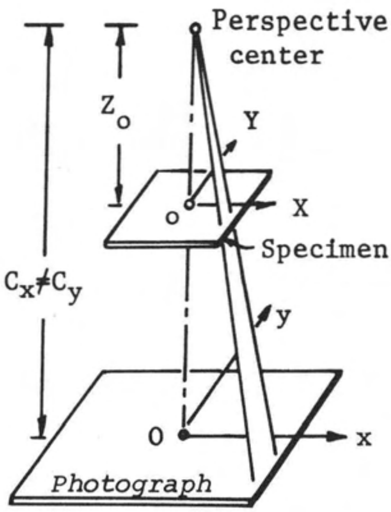

FIG. 1. Perspective projection.

If such information is available from two or more photographs taken of the same object with different camera orientations, it is possible to reconstruct two or more such rays such that they intersect at the corresponding "model" point, indicating its location in three-dimensional space. When one knows the elements of exterior orientation (E.O.), viz., (1) the locations of the perspective center at the various instants of exposure (with respect to the object) as well as (2) the angular orientations of the photo plane (or the camera axis), it is possible to obtain all quantitative information of the photographed object by way of using the refined photo-coordinates.

An accurate method of determining E.O. parameters is to use stereo (or multiple) imagery of an area which includes sufficient number of control points (with known coordinates). Working with I.O. parameters and computed in reverse order by a procedure known as space resection, one obtains the E.O. parameters. Afterwards, the E.O. parameters are coupled with the I.O. parameters to determine, through space intersection, the object space coordinates (3-D) of other points previously unknown. This general photogrammetric procedure is now well developed and documented thoroughly and is fundamental to the theory developed in this paper.

A knowledge of the *distortions* in order to refine the photo-coordinates' data is essential for such applications.

BACKGROUND

Klemperer and Barnett[1] observed that magnetic electron lenses show isotropic and anisotropic errors giving distortions of various types:

(a) Barrel and pin-cushion shaped distortions,
(b) Hammock shaped distortions, and
(c) Spiral distortions.

Existence of other types of distortions and their mathematical models in forms that are computer compatible were not adequately known until the works of Maune[2] and Nagaraja[3] were completed. Howell and Boyde[4] observed that the SEM comprises a perspective central projection, which approaches a parallel projection at higher magnifications. At this stage, no distortions were considered. Later on, Boyde *et al.*[5] admitted the existence of various distortions (Also see Hilliard[6]) but ignored them in practice.

By using a very rigorous and statistically controlled self-calibration and adjustment procedure, Maune[2] observed that

(1) The SEM system is better represented by a mathematical model for an effective

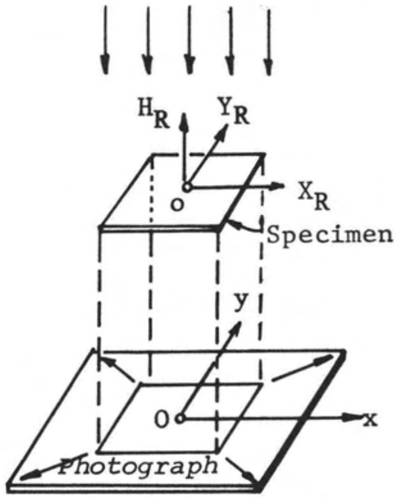

FIG. 2. Parallel projection.

central, perspective projection rather than for a parallel projection (See Figure 2), although not statistically significant when the magnifications are higher than 2000×.

(2) Strict Collinearity Condition is disturbed by four types of systemaic distortions viz., scale (differential), spiral, tangential, and radial. The last one (radial) may be considered as containing both the barrel and pin-cushion types.

In the group effort at The Ohio State University, Nagaraja[3] observed, beyond what was reported by Maune[2], the following:

(1) The difference between the perspective projection and the parallel projection can be mathematically modelled.
(2) The scale distortions can be contained directly in the mathematical model for the projection (parallel or perspective).
(3) The tangential distortion can be effectively contained in the mathematical model of the spiral distortion for most applications.
(3) The effects of radial and spiral distortions are best corrected by use of polynomials derived from theoretical considerations.

This is an attempt to consolidate the above and submit the developed acceptable mathematical models relevant to the use of quantitative observations made on SEM micrographs.

A thorough understanding of any working system can be made through calibration. This depends on the instrument and its complexities. Further, the measurements for calibration must be referable to a standard. The standard used in this case is a replica grid (carbon replica, mounted on a 200-mesh copper grid, made from a master diffraction grating with 2160 lines per mm in both x and y directions) and, being two dimensional, the procedure of self-calibration was used. In this procedure the calibration parameters are recovered analytically, through rigorous computations, without the necessity for absolute 3-D control in the object space. The geometric configuration obtained from four exposures made of the same grid (object) with one tilt and four rotations (approximately 90° apart) is shown in Figure 3. Without absolute knowledge of the object space and perspective center coordinates, there is a well-known projective compensation which strongly correlates the image and object distances. Similar correlations exist, e.g., between the principal point and the perspective center coordinates, between the element ω(X-tilt) and the y-coordinates, etc. Many of them are uncoupled through the use of such highly convergent photography. This being the case, the self-calibration technique is ideally suited for any SEM system where the specimen can be tilted and rotated in such a way as to provide any desired orientation.

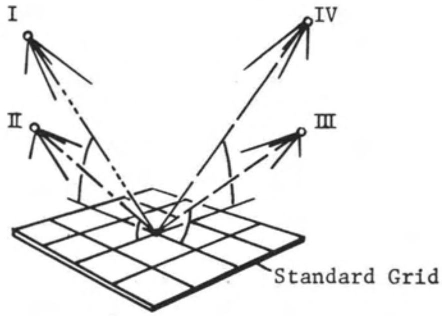

FIG. 3. Self-calibration set-up.

Mathematical Models

Before considering the mathematical models, let us consider what we require of such models. We would like to have equations that will describe the effect of the various types of distortions as exactly as possible. In addition, we want the equations to be simple enough that we can obtain closed solutions (in order to refine the coordinates observation data) without undue difficulty. These two requirements are not usually compatible. One is, therefore, forced to compromise with models for useful (may not be exact) results. Furthermore, instead of applying only one very complicated equation for all distortions it is convenient to apply several equations sequentially. This approach is simple, easily understood, and applicable by the user who may even never see the equipment or understand its intricacies. On the other hand, such sequential approach implies primary, secondary, tertiary, etc. effects or importance. Also, simple expressions (like fractions) or polynomials are easy to work with and it is desirable that the mathematical models fall into such categories.

The above considerations were behind the development of the mathematical models for the SEM distortions. The basic mathematical considerations are that the coordinates and rotations in the system be right-handed; that the sequence of rotations be φ primary, ω secondary, and $\varkappa$ tertiary, the selected origin of the system be in the specimen (o in Figures 1 and 2).

PARALLEL PROJECTION

The parallel projection (See Figure 2) system is expressed by:

$$\begin{bmatrix} x \\ y \\ 0 \end{bmatrix} = \begin{bmatrix} K_1 & 0 & 0 \\ 0 & K_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_R \\ Y_R \\ H_R \end{bmatrix} \tag{1}$$

where x and y are the photo coordinates; K_1 and K_2 are the scale factors along the X_R and Y_R directions, respectively; and X_R, Y_R, and H_R are coordinates of the point in the fixed coordinate system. Furthermore,

$$\begin{bmatrix} X_R \\ Y_R \\ H_R \end{bmatrix} = \begin{bmatrix} c\varkappa & s\varkappa & 0 \\ -s\varkappa & c\varkappa & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\omega & s\omega \\ 0 & -s\omega & c\omega \end{bmatrix} \begin{bmatrix} c\varphi & 0 & -s\varphi \\ 0 & 1 & 0 \\ s\varphi & 0 & c\varphi \end{bmatrix} \begin{bmatrix} X-X_o \\ Y-Y_o \\ H-H_o \end{bmatrix} \tag{2}$$

where c and s suffixes mean cosine and sine, respectively, of the rotation angles $\varkappa$ around H, ω around X, and φ around Y; and X_o, Y_o, and H_o are the coordinates of the selected origin, o, in the object based system.

If we define the datum for measuring heights by setting $H_o=0$ and after appropriate substitutions and simplifications, the general photo projective equations are—

$$
\begin{aligned}
x &= K_1 (X - X_o)(\mathrm{c}\varphi\ \mathrm{c}\varkappa + \mathrm{s}\omega\ \mathrm{s}\varphi\ \mathrm{s}\varkappa) + K_1 (Y - Y_o)(\mathrm{c}\omega\ \mathrm{s}\varkappa) \\
&\quad + K_1 H (-\mathrm{s}\varphi\ \mathrm{c}\varkappa + \mathrm{s}\omega\ \mathrm{c}\varphi\ \mathrm{s}\varkappa) \\
y &= K_2 (X - X_o)(-\mathrm{c}\varphi\ \mathrm{s}\varkappa + \mathrm{s}\omega\ \mathrm{s}\varphi\ \mathrm{c}\varkappa) + K_2 (Y - Y_o)(\mathrm{c}\omega\ \mathrm{c}\varkappa) \\
&\quad + K_2 H (\mathrm{s}\varphi\ \mathrm{s}\varkappa + \mathrm{s}\omega\ \mathrm{c}\varphi\ \mathrm{c}\varkappa)
\end{aligned}
\tag{3}
$$

PERSPECTIVE PROJECTION

In addition to the basic assumptions made in the parallel projection, we may add that for the perspective projection (See Figure 1)

(a) There exists a perspective center;

(b) Different camera constants C_x and C_y (conceptually the same as different focal length values in x and y directions, as in line focus lenses) can be considered, in keeping with the current practices; and

(c) Logical substitution are K_1 for C_x/Z_o and K_2 for C_y/Z_o.

The above will give (see Nagaraja[3] for the details of derivation) from Equations 1 and 2

$$
\begin{aligned}
x &= K_1 X_R \left[1 + (1/Z_o) \left\{(X - X_o)(\mathrm{c}\omega\ \mathrm{s}\varphi) + (Y - Y_o)(-\mathrm{s}\omega)\right.\right. \\
&\quad \left.\left. + (H - H_o)(\mathrm{c}\omega\ \mathrm{c}\varphi)\right\} + \dots\right] \\
y &= K_2 Y_R \left[1 + (1/Z_o \left\{(X - X_o)(\mathrm{c}\omega\ \mathrm{s}\varphi) + (Y - Y_o)(-\mathrm{s}\omega)\right.\right. \\
&\quad \left.\left. + (H - H_o)(\mathrm{c}\omega\ \mathrm{c}\varphi)\right\} + \dots\right]
\end{aligned}
\tag{4}
$$

Note here that the first terms of Equation 4 constitute Equation 1. These can be written in the more compact form

$$
\begin{aligned}
x &= K_1 X_R \left[1 + \Delta + \Delta^2 + \dots\right] \\
y &= K_2 Y_R \left[1 + \Delta + \Delta^2 + \dots\right]
\end{aligned}
\tag{5}
$$

The validity inherent in the above approximations has been satisfactorily studied by Nagaraja[3].

Distortion Corrections

PERSPECTIVE DISTORTION

Departures from the parallel projection condition are termed systematic distortion errors. By considering Equations 1 and 4, in view of Equation 5, one can easily write

$$
\begin{aligned}
x_{\text{parallel}} &= x_{\text{pers}} - x_{\text{pers}} \left[\Delta + \Delta^2 + \dots\right] \\
y_{\text{parallel}} &= y_{\text{pers}} - y_{\text{pers}} \left[\Delta + \Delta^2 + \dots\right]
\end{aligned}
\tag{6}
$$

Representative values of Δ and Δ^2 can be easily obtained through self-calibration procedures as have been presented by Nagaraja[3].

RADIAL DISTORTION

Considering a polynomial approach to radial distortion, it can be expressed by (as in conventional photogrammetry)

$$\Delta r = k_o r + k_1 r^3 + k_2 r^5 + \dots$$

where r is the radial image distance. The first term is equivalent to a magnification (scale) change, which has been considered earlier. Now, neglecting terms of 5 and higher order in r,

$$\Delta r = k_1 r^3.$$

For the components of this distortion,

$$\Delta x = \Delta r\ (x/r) = k_1\ r^3\ (x/r) = k_1\ (x^3 + xy^2)$$
$$\Delta y = \Delta r\ (y/r) = k_1\ r^3\ (y/r) = k_1\ (x^2y + y^3) \tag{7}$$

Now, in keeping with Klemperer and Barnett[1], and in view of the two different scale factors, along X and Y, the following can be considered as very logical for use in practice:

$$\Delta x = D_1x^3 + D_2xy^2$$
$$\Delta y = D_3y^3 + D_4x^2y \tag{8}$$

SPIRAL DISTORTION

Spiral or rotational distortion results in tangential displacement of the image. The term "tangential" of the aerial camera case is different from this. Klemperer and Barnett[1] give an expression which agrees with the calibration results obtained in this research, i.e., $\Delta d = C\ (y'_o/y)\ r^3$ where Δd is the lateral displacement, C is the spiral distortion coefficient, y'_o/y is the magnification, and r is the distance of the object point from the principal electron axis (analogous to radial distance). This is simplified by considering a new spiral distortion coefficient, $S = C\ (y'_o/y)$, giving

$$\Delta d = S \cdot r^3. \tag{9}$$

Its components are (See Figure 4)—

$$\Delta x = \Delta d \cdot \sin\theta = S\ (y/r)\ r^3 = S\ (x^2y + y^3)$$
$$\Delta y = \Delta d \cdot \cos\theta = S\ (x/r)\ r^3 = S\ (x^3 + xy^2) \tag{10}$$

Considering that the scales along x and y are different, these can be modified to write

$$\Delta x = S_1\ x^2y + S_2y^3$$
$$\Delta y = S_3x^3 + S_4\ xy^2 \tag{11}$$

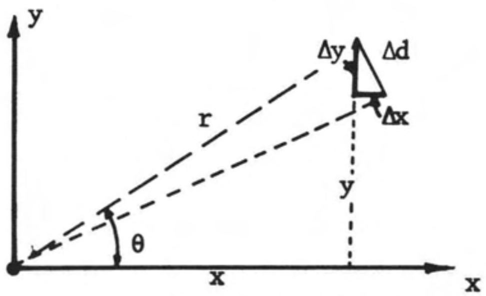

FIG. 4. Components of spiral distortion.

Simplified, they are—

$$\Delta x = S_x\ (x^2y + y^3) \tag{12}$$

$$\Delta y = S_y\ (x^3 + xy^2)$$

The basic equations as given above may be written in a number of ways. For example, K_2 (scale factor along Y_R) can also be represented as K_1R_1, where R_1 would be the ratio of the magnification in the direction of the Y-axis to that of the X-axis. Some of the important considerations in choosing the independent parameters are the physical reality, the computational advantages, and the obtainable accuracy of the final computed 3-D coordinates (vital for quantitative work).

Twelve different equations with different sets of parameters were tried and numerically (statistically) analyzed. This led to the selection of the particular set of equations which seem to be the best for use in handling SEM micrographs.

These involve the following parameters:

K_1; K_2; φ; D_1; D_2; S_x; D_3; D_4; S_y; X_{o1}; Y_{o1}; ω_1; $\varkappa_1$; $K_1 + \Delta K_1$; $K_2 + \Delta K_2$; X_{o2}; Y_{o2}; H_{o2}; ω_2 and $\varkappa_2$.

Details of this numerical analysis have been presented by Nagaraja[3]. Graphical representation of the distortion patterns are given in Figure 5.

Results

One must not ignore the fact that the quality of the initial output of the particular SEM, which are the raw data, would greatly influence the final results. In the present case, the studies were made with raw data obtained from a Materials Analysis Co. Model 700 (MAC 700) SEM with respect to magnification 5000× (this being the optimum scale known to be used generally by the particular research group at OSU).

The results indicate that the standard deviation of a photo-coordinate observation is about

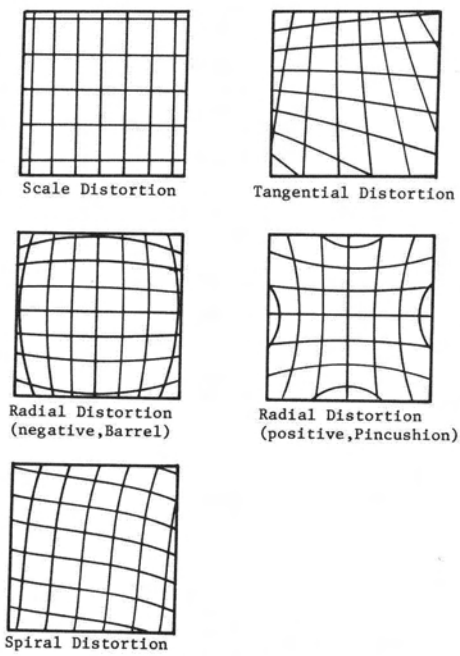

Fig. 5. Distortion patterns.

TABLE 1. STANDARD DEVIATIONS (σ), VALUES IN NM

	10° convergence			45° convergence		
	σ_X	σ_Y	σ_H	σ_X	σ_Y	σ_H
Maximum	42	46	2086	19	25	340
Minimum	21	27	351	12	14	237

115μm. This is about the limit of resolution on the film for the particular SEM, at 5000x magnification, corresponding to about 23 nm on the specimen. This is the practical limiting factor for precision measurements with micrographs from this particular SEM.

The studies are based on grid intersection points selected at random, based on uniformly distributed random numbers generated on a Hewlett Packard desk calculator. Coordinate observations were made on a Wild A7 autograph instrument used as a comparator having a least count of photo-coordinates of 3 μm (standard deviation of $\pm$ 7 μm on the micrograph).

Scale distortion (contained in the perspective distortion) is by far the most significant one. The differential scale difference has been in some cases [8] as much as 10 per cent, between K_1 and K_2. A satisfactory correction for scale distortion alone reduced the standard deviation of the photo-coordinates from nearly 1500 μm to about 550 μm. After accounting for other distortions, this standard deviation reduced to around 100 μm, which seems to be the limit in this case, in view of the standard deviation of around 115μm for a photo-coordinate observation (reported above).

The user need not attempt to separate the effects of the various components of the SEM. He also may not have any interest in knowing the contribution of each separate type of distortion. He has, however, rather serious interest in the data refined for the various inherent distortions. In general, the accuracy of 3-D coordinates obtainable from a given photogrammetric procedure depends on: Stability of the system, observational error, geometry of intersection (primarily, angle of convergence), image quality (including resolution), shape of the object (including surface features), and the instrument used for photogrammetric observation.

In this light, the final analyses of stereo-pairs of two different convergence angles (10° and 45°) gave the results presented in Table 1.

Furthermore, accuracy of the heights at the center (Y_i = 0) of the stereo model is always better than at other locations. The accuracy of the heights are better when there is no scale change (i.e., $\Delta K_2 = 0$). Larger magnifications (i.e., K_1, and K_2 being comparatively large) give better accuracy in heights.

The degree of stability of the SEM system is very important but can be easily determined in evaluating the performance. Maune[2] and Ghosh[7] have presented ideas on this. Image quality and other factors are beyond the scope of this paper and require future research.

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