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The Kern PG-2 as a Monocomparator*

Mean discrepancies between Kern PG-2 and precise monocomparator measurements ranged from 3 to 9 micrometres.

INTRODUCTION

AT THE University of Wisconsin-Madison, a considerable amount of research is being conducted that utilizes a P-30 phototheodolite and other small-format cameras such as 70mm-format Hasselblads. For analytical applications, a digitized DBA Multilaterative Comparator is available for photo coordinate measurements. However, without special apparatus, this comparator is not particularly adaptable to the handling of small format photographs, especially when printed on a film base.

ABSTRACT: *Methods are described for using a digitized Kern PG-2 stereoscopic plotting instrument as a monocomparator. The system has been calibrated specifically for measuring photo coordinates of exposures made by terrestrial cameras and other small non-metric cameras. The accuracy of the procedure is demonstrated through the presentation of sample results wherein PG-2-derived photo coordinates are compared to values obtained for the same points by using precise monocomparators.*

Preliminary investigations indicated that if systematic errors which were known to exist in the measurement system of a digitized Kern PG-2 stereoplotter were properly compensated for, the instrument could be conveniently and satisfactorily used as a monocomparator. This paper describes the methods used to determine the magnitude of systematic errors in the PG-2 measurement system, and discusses procedures employed in correcting for these errors. The successful application of the PG-2 as a monocomparator is then demonstrated by giving results of photo coordinate measurements for several photos, and comparing them with comparator-derived values.

ASSESSMENT OF SYSTEMATIC ERROR

A precise Zeiss Jena grid plate, on loan from the U.S. Department of Transportation, was used to assess the magnitude of systematic errors in the PG-2 measurement system. The x and y coordinates of grid intersections of this plate were accurate to approximately plus or minus one micrometre. The grid plate was placed in each of the PG-2 plate carriers and readings were taken monoscopically on grid points in the model area. Readings were taken on a total of 66 points in a quadrille pattern having a 20mm spacing over an area 100mm by 200mm on each carrier.

In order to determine the systematic errors

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in a reproducible measurement system, the PG-2 was "zeroed" prior to measurement. To do this the dials for omega, phi, and kappa, of the plate carriers were set to zero, $b\phi$ was zeroed, and OMEGA and PHI of the X-Y measurement plane were set to zero. Also, all four principal distance settings, and bx, Z, and the elevation setting of the tracing stand were set to certain values which could be reset each time the instrument was to be used as a comparator.

The grid plate was measured in each of the left and right carriers in order to evaluate the systematic errors of both positions. Each time the plate was measured, three readings were taken on each grid intersection and the mean value was adopted. For consistency, pointings were made always by approaching grid intersections from the lower right side.

A number of independent sets of readings, spaced over several weeks, were taken so that the stability of the system over a time period could be evaluated. The grid plate was oriented as shown in Figure 1, so that its center point (a) was very near the center-cross of the PG-2 plate carrier. It was also set so that the x and y axes of the grid plate were very nearly parallel with the x' and y' axes respectively of the carrier (see Figure 1). Readings of the digitizer are in "counts", and for the adopted instrument settings, each "count" was roughly equivalent to four micrometres on the plate.

The X-Y model space coordinates obtained for the grid intersections were then transformed by scaling, translating, and rotating to place them in a system parallel to the x-y grid plate system. (This procedure was performed without redundancy intentionally to preserve the discrepancies of the measurement system.) A scale factor in millimetres per "count" was established by dividing the grid plate length of line ab (which was 100 millimetres by the same length in "counts" as determined from the Model X-Y coordinates. All X-Y grid coordinates were then converted to the X'-Y' scaled millimetre system on the basis of this factor. Next the scaled coordinates were translated to an X''-Y'' system with origin at point a by subtracting X_a' and Y_a' from all grid X'-Y' coordinates. Finally the X''-Y'' system was rotated through angle α to convert it into the x-y grid plate system. Angle α was calculated as $\tan^{-1}(Y_b''/X_b'')$.

Transformed x-y coordinates obtained as described above were compared to their respective grid coordinates to ascertain the magnitudes and pattern of the discrepancies. Figure 2 illustrates the discrepancies ob-

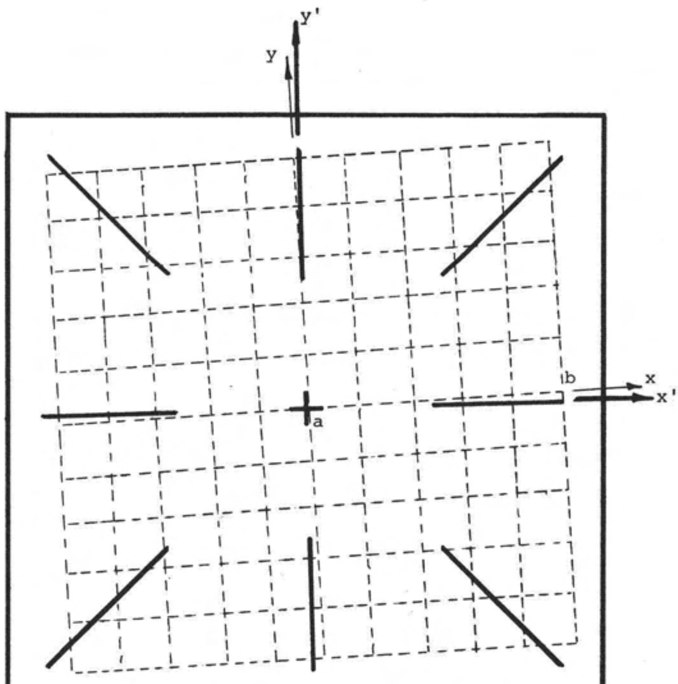

FIG. 1. Orientation of grid plate in PG-2 plate carrier.

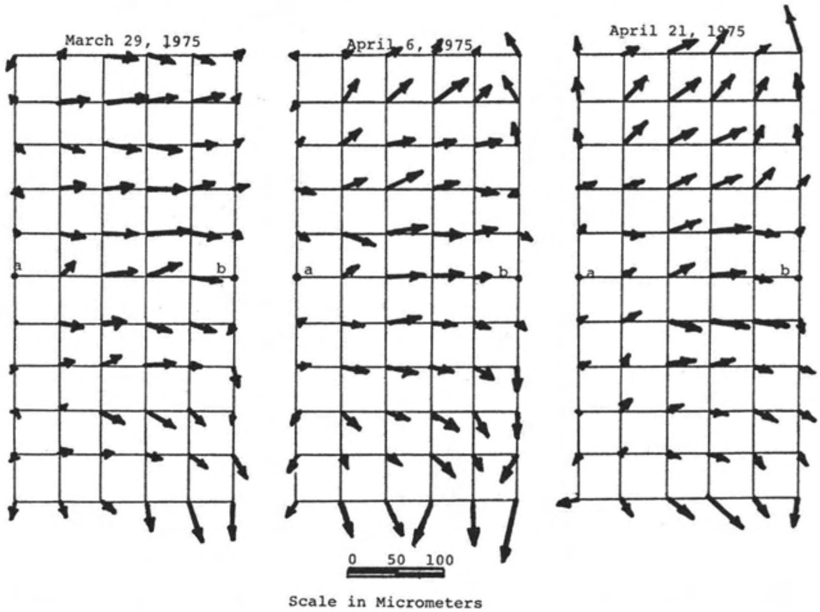

FIG. 2. Discrepancy vectors for left plate carrier of PG-2, three different dates.

tained for the left plate carrier for three independent sets of measurements taken over a period of approximately a month. The systematic nature of the measurement system is immediately apparent. A similar set of figures indicating a slightly different systematic pattern of errors was obtained for the right plate carrier.

Although no attempt has been made in this study to identify the magnitudes of each contributing error source, the combined systematic errors are expected to stem from the following:

- Non-parallel X-Y measurement plane and carrier-plate plane,
- Non-uniform scale in the separation of the X and Y projection systems of the PG-2,
- Non-orthogonalities and curvature of the ways of the X-Y Coordinatograph,
- Non-linearities in the encoding system,
- Eccentricities of the gimbal joints, and
- Non-uniform stresses and inconsistent mechanical restitution in the instrument depending upon X-Y model position.

Error Modelling by Polynomial

The error pattern in the model space appeared to be a non-linear function of both X and Y coordinates. Therefore, a higher order polynomial was chosen to model the errors. Several polynomials were tested and the following one of third-degree consistently produced a good error model and was therefore adopted:

$$x = X + A1 + A2\,(X) + A3\,(X^2) + A4\,(Y) + A5\,(Y^2) + A6\,(X)\,(Y) + A7\,(X^2)\,(Y) + A8\,(X)\,(Y^2) + A9\,(X^3) + A10\,(Y^3).$$

$$y = Y + B1 + B2\,(X) + B3\,(X^2) + B4\,(Y) + B5\,(Y^2) + B6\,(X)\,(Y) + B7\,(X^2)\,(Y) + B8\,(X)\,(Y^2) + B9\,(X^3) + B10\,(Y^3).$$

Where: x and y are refined coordinates,
X and Y are measured coordinates,
$A1$-$A10$ are polynomial coefficients for refinement in the x direction,
$B1$-$B10$ are polynomial coefficients for refinement in the Y direction.

Each measured grid intersection generates one x and one y equation. Ten measurements provides a unique solution, but 66 points were observed, and thus the method of least squares was used to calculate the polynomial coefficients. Refined and measured coordinates for several independent sets of measurements for each carrier were incorporated into this computation. Separate coefficients were determined for each plate carrier. The polynomial coefficients, once determined, were then available for use in converting measured coordinates to refined coordinates for any plates which were measured in the PG-2 by using the same settings and procedures as were used for measuring the grid plate.

For the data used in calculating polynomial coefficients for the left plate carrier, the maximum, RMS, and average discrepancies of grid intersection coordinates were 57, 26, and 21 micrometres, respectively, in *X* coordinates and 82, 28, and 22 micrometres, respectively, in *Y* coordinates. After processing through the correction polynomial these were reduced to maximum, rms, and mean values of 15, 5, and 4 micrometres, respectively, in *X* coordinates and 10, 5, and 4 micrometres, respectively, in *Y* coordinates.

Evaluating The PG-2 as a Moncomparator

To test the accuracy of the PG-2 as a monocomparator, several P-30 phototheodolite (100-by-150-mm format) photos and Hasselblad (70-mm-square format) photos were measured and corrected using the polynomial described above. In each case, photo coordinates measured in the PG-2 system were first reduced from "count" units to a scaled millimetre system by multiplying all coordinates by the previously determined scale factor. These scaled coordinates were then translated to an origin at the center-cross of the plate carrier (whose model coordinates had also been measured), and then they were processed through the correction polynomial. Finally they were transformed into their fiducial systems.

Of the accuracy tests conducted, three representative ones are reported here. The first was a comparison of agreement between independent measurements of the same points taken on different dates. The measurements were for PUG marks on two P-30 phototheodolite plates. Table 1 gives the results of the measurements and illustrates the high degree of consistency obtainable with the PG-2 used as a monocomparator. For plate 003, the maximum, RMS, and mean discrepancies between the two sets of *X* coordinates were 10, 5.8, and 5.1 micrometres, respectively, and the maximum, RMS, and mean discrepancies between the two sets of *Y* coordinates were 15, 7.0, and 5.6 micrometres, respectively. For plate 005 these respective discrepancies were 9, 5.6, and 4.6 micrometres for *X* coordinates and 12, 8.5, and 7.6 micrometres for *Y* coordinates.

The second test is a comparison between the Table I sets of image coordinates for P-30 phototheodolite plates 003 and 005 obtained with the PG-2, and photo coordinates obtained with the Wisconsin Department of Transportation's H. Dell Foster mono comparator. Table 2 lists the results. The maximum, rms, and mean discrepancies, respectively, in micrometres for plate No. 003 were 15, 8.9, and 7.2 in *X* coordinates for Set

Table 1. Comparison of Sets of Photo Coordinates of PUG Marks Independently Observed with the PG-2.

Point	Plate 003						Point	Plate 005					
	X Coordinates (mm)		*Y* Coordinates (mm)		Discrepancies (μm)			*X* Coordinates (mm)		*Y* Coordinates (mm)		Discrepancies (μm)	
No.	Set 1	Set 2	Set 1	Set 2	*X*	*Y*	No.	Set 1	Set 2	Set 1	Set 2	*X*	*Y*
1	59.080	59.074	33.498	33.498	6	0	1	53.496	53.494	-32.126	32.136	2	10
2	33.221	33.218	32.139	32.124	3	15	2	55.155	55.158	0.695	0.705	3	10
3	34.357	34.360	2.100	2.095	3	5	3	55.117	55.113	30.590	30.585	4	5
4	35.114	35.120	-30.273	-30.271	6	2	4	28.266	28.267	29.661	29.649	1	12
5	- 0.125	- 0.131	-30.985	-30.979	6	6	5	27.729	27.734	3.125	3.115	5	10
6	1.081	1.083	0.050	0.045	2	5	6	0.592	0.592	-30.096	-30.104	0	8
7	- 0.477	- 0.471	30.242	30.231	6	9	7	- 0.812	- 0.804	1.272	1.283	8	11
8	-27.367	-27.366	31.803	31.794	1	9	8	- 0.499	- 0.506	25.451	25.451	7	0
9	-26.711	-26.703	- 2.548	- 2.551	8	3	9	-57.071	-57.063	-27.878	-27.876	8	2
10	-29.028	-29.038	-30.033	-30.035	10	2	10	-53.137	-53.128	32.278	32.270	9	8
				Max	10	15					Max	9	12
				Rms	5.8	7.0					Rms	5.6	8.5
				Mean	5.1	5.6					Mean	4.6	7.6

Table 2. Comparison of Table 1 PG-2 Measured Data With Coordinates of Same Images Determined Using H. Dell Foster Monocomparator.

Plate 003							Plate 005						
Point	H. Dell Foster Coordinates		Coordinate Discrepancies From Table 1 Values (Micrometers)				Point	H. Dell Foster Coordinates		Coordinate Discrepancies From Table 1 Values (Micrometers)			
No.	*X* (mm)	*Y* (mm)	*X* Set 1	*X* Set 2	*Y* Set 1	*Y* Set 2	No.	*X* (mm)	*Y* (mm)	*X* Set 1	*X* Set 2	*Y* Set 1	*Y* Set 2
1	59.081	33.506	1	7	8	8	1	53.495	−32.137	1	1	11	1
2	33.227	32.131	6	9	8	7	2	55.155	0.694	0	3	1	11
3	34.366	2.100	9	6	0	5	3	55.111	30.582	6	2	8	3
4	35.129	−30.264	15	9	9	7	4	28.263	29.654	3	4	7	5
5	− 0.118	−30.976	7	13	9	3	5	27.734	3.117	5	0	8	2
6	1.094	0.048	13	11	2	3	6	0.590	−30.112	2	2	16	8
7	− 0.477	30.240	0	6	2	9	7	− 0.807	1.278	5	3	6	5
8	−27.355	31.792	12	11	11	2	8	− 0.508	25.458	9	2	7	7
9	−26.702	− 2.555	9	1	7	4	9	−57.068	−27.888	3	5	10	12
10	−29.028	−30.029	0	10	4	6	10	−53.136	32.271	1	8	7	1
		Max	15	13	11	9			Max	9	8	16	12
		Rms	8.9	8.9	7.0	5.8			Rms	4.4	3.7	8.9	6.7
		Mean	7.2	8.3	6.0	5.4			Mean	3.5	3.0	8.1	5.5

1; 13, 8.9, and 8.3 in *X* coordinates for Set 2; 11, 7.0, and 6.0 in *Y* coordinates for Set 1; and 9, 5.8, and 5.4 in *Y* coordinates for Set 2. For plate No. 005 the maximum, RMS, and mean discrepancies, respectively, in micrometres for *X* coordinates were 9, 4.4, and 3.5 for Set 1; and 8, 3.7, and 3.0 for Set 2. For *Y* coordinates the respective discrepancies were 16, 8.9, and 8.1 for Set 1; and 12, 6.7, and 5.5 for Set 2. The overall averages for the two plates for maximum, RMS, and mean discrepancies, respectively, for *X* and *Y* coordinates in the two sets were 11.6, 6.8, and 5.8 micrometres.

In the third accuracy test, photo coordinates of natural images on a 70mm Hasselblad photo were obtained with the PG-2 and compared to photo coordinates obtained for the same points using the DBA Multilaterative comparator. Table 3 lists the results obtained for two representative photos. The maximum, RMS, and mean discrepancies, respectively, for plate 501 were 16, 8.2, and 6.2 micrometres in *X* coordinates and 20, 9.5, and 8.1 micrometres in *Y* coordinates. For Plate 505 these respective values were 12, 7.4, and 6.2 for *X* coordinates and 17, 9.7, and 9.1 for *Y* coordinates. As would be expected, because the image base was polyester and because natural images are not as discreet as PUG marks, a slight reduction in accuracy is noted in Table 3.

Summary

The data presented in the three tables indicates that a rather high degree of accuracy in photo coordinate measurement is possible when using a mechanicial projection stereoplotter. Mean discrepancies indicated in the tables range between 3.0 and 9.1 mi-

Table 3. Comparison of Photo Coordinates of Natural Images Determined from a DBA Comparator and PG-2.

	Photo 501					
	DBA Coordinates		PG-2 Coordinates			
Point	*X*	*Y*	*X*	*Y*	Δ*X*	Δ*Y*
48	− 3.995	− 4.355	−3.011	−4.356	0.016	0.001
47	3.289	− 4.316	3.295	−4.305	0.006	0.011
46	8.239	− 4.316	8.229	−4.326	0.010	0.010
35	18.044	−12.601	18.043	−12.606	0.001	0.005
1013	−38.333	42.120	−38.323	42.111	0.010	0.009
710	−18.051	19.526	−18.051	19.533	0.000	0.007
58	− 3.942	3.773	−3.940	3.776	0.002	0.003
74	24.359	19.190	24.373	19.170	0.013	0.020
65	17.474	11.472	17.471	11.466	0.003	0.006
83	28.854	26.722	28.853	26.713	0.001	0.009
				Max	16	20
				rms	8.2	9.5
				Mean	6.2	8.1

	Photo 505					
	DBA Coordinates		PG-2 Coordinates			
Point	*X*	*Y*	*X*	*Y*	Δ*X*	Δ*Y*
210	0.948	−17.666	0.940	−17.671	0.008	0.005
39	4.750	−10.106	4.750	−10.105	0.000	0.001
48	12.365	− 2.206	12.358	−2.198	0.007	0.008
47	18.405	− 1.568	18.404	−1.583	0.001	0.015
46	22.665	− 1.166	22.660	−1.177	0.005	0.011
24	38.884	−15.844	38.891	−15.835	0.007	0.009
1013	−14.057	33.736	−14.058	33.719	0.001	0.017
69	6.441	11.442	6.453	11.436	0.012	0.006
58	12.262	5.126	12.251	5.134	0.011	0.008
56	24.516	6.557	24.511	6.548	0.005	0.009
65	30.757	14.916	30.768	14.927	0.011	0.011
				Max	12	17
				rms	7.4	9.7
				Mean	6.2	9.1

crometres, which is suitable for many analytical applications. Of course, it must be conceded that the monocomparator-derived photo coordinates presented in Tables 2 and 3 are themselves subject to some error, but nevertheless they should be accurate enough for the comparisons made herein.

The PG-2, when used as a monocomparator, is accurate, fast, and very convenient, especially for small-format photos which fit within the model area on the plate carrier. Observing time varies depending upon the number of points to be measured, but in general about 10 minutes is required for the average plate having four fiducials and perhaps 10 other points to measure. Output is direct onto punched cards which can then be fed to the computer to give refined coordinates.

For semi-analytic aerotriangulation, the systematic errors which have been shown to exist in the model space will certainly cause distortions in *X*, *Y*, and *Z* coordinates of independently read stereomodels. These three-dimensional systematic errors can be compensated for, however, in a manner similar to that described. Further research on this subject, to evaluate what aerotriangulation accuracy improvements, if any, can be realized from this technique, is now being pursued at the University of Wisconsin-Madison.

The techniques described are applicable to any mechanical projection stereoplotting instrument, and the level of accuracy achieved in this study with the PG-2 should be attainable with other instruments of comparable precision. It should be stressed, however, that the systematic errors discussed in this paper apply only to the PG-2 stereoplotter in the photogrammetric laboratory of the University of Wisconsin-Madison. Unique systematic errors should exist for all mechanical projection stereoplotting systems, and the exact magnitudes and patterns must be determined individually for each.

Acknowledgments

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