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Inadequacy of the Scattering Coefficient

The definition of the beam volume scattering coefficient is deficient because it does not take into account such parameters as beam size.

It is accepted practice to employ the beam volume scattering coefficient, s, in describing the exponential distance attenuation of a collimated monochromatic beam of light in an isotropic, optically homogeneous, negligibly absorbing, transmitting medium[1-5], that is,

$$N = N_o\, e^{-sx} \tag{1}$$

where N_o is the initial flux or radiance in the collimated beam and N is the flux or radiance in the beam at distance x.

Abstract: *The beam volume scattering coefficient, defined conventionally without regard to beam size, cross-section geometry or initial flux, is examined critically. Simple stochastic consideration of the scattering of two collimated, square cross-section light beams in an isotropic, optically homogeneous medium indicates that the scattering coefficient is not defined specifically enough to characterize the scattering or light attenuation property of a transmitting medium.*

Since Equation 1 is usually introduced with the promulgated definition of scattering coefficient, the equation can serve as a point of departure for determining the validity of the coefficient. There are investigators who either make measurements in special ways that tend to make Equation 1 appear valid or limit the application of the scattering coefficient to particular types or ranges of scattering. The unique efforts of these investigators is not under consideration. Brought into question here is the formal definition of scattering coefficient found in standard references[1-5], implying, without any qualification, that this coefficient characterizes the inherent scattering property of a light transmitting medium.

The beam volume scattering coefficient definition follows from Equation 1.

$$s = -\frac{dN}{N dx} \tag{2}$$

The initial collimated beam cross-section magnitude, geometry, and flux distribution N_o are not specified or restricted in any way in the references cited or in other standard references defining the light scattering coefficient. Nor are either single or multiple path, or narrow or wide angle scattering events, or any combination of them specified or sometimes even mentioned in any of the definitions. It is thus predicated in these typical standard references that the initial collimated beam cross-section may be of arbitrary size, geometry, and flux distribution and scattering may be single or multiple path, or

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narrow or wide angle. The availability of these many options can lead to certain incorrect conclusions that apparently have not been appreciated.

To avoid any misunderstanding, I present S.Q. Duntley's clear definition of scattering[6]: "Scattering refers to any random process by which the direction of individual photons is changed without any other alteration."

Reverting to Equations 1 and 2, the exponential attenuation law states that in an isotropic, optically homogeneous transmitting medium, the fraction (or per cent) of light energy lost per unit length by scattering from an originally collimated monochromatic beam is constant, beam cross-section size, geometry, and flux distribution being unprescribed.

Applying the definition for scattering to an isotropic, optically homogeneous medium, we have the stochastic fact that the scattering angle, change in path direction of a photon when scattered, is a random variable with an RMS value (standard deviation since the mean is 0).

The import of the definition of the scattering coefficient given by Equation 2 is immediately apparent. The beam volume scattering coefficient s is the probability per unit beam length, dN/Ndx, that a photon will be scattered out of the beam. Only scattering-out-of-the-beam events are counted. If the direction of a photon's path is changed but it remains in the beam, no contribution is made to the beam volume scattering coefficient as defined in the references cited.

Given a fixed RMS value of photon scattering angle (a consequence of isotropy and optical homogeneity) and the same initial flux, N_o, in two collimated monochromatic geometrically similar beams of different cross-section magnitude, the probability per unit length, $-dN/Ndx$ ($=s$), of a photon scattering-out-of-the-beam event occurring is greater in the narrower beam than in the broader one. This fact is readily established.

Figure 1(a) shows a volume, configured as a rectangular parallelepiped, of an optically homogeneous, light scattering medium. The volume has unit length, say one metre. A square beam of light in the form of axially collimated photons enters the square face *A* and can leave the parallelepiped, in part or entirely, through the rear face *B* or any of the four sides or any combination of them. The square cross-section collimated entering beam will be considered unscattered if all the photons entering front face *A* pass through rear face *B*. Photons leaving the parallelepiped through any of the four remaining sides will be considered scattered out of the beam in one metre. To facilitate presentation, scattering will be assumed to be narrow angle. This assumption is in accord with the scattering coefficient definition[1-5].

The processes discussed normally involve millions or billions of photons per second. In the interest of simplification, small numbers of photons will be employed to develop the thesis. The number of photons per second may, of course, always be multiplied by a constant; that is, 4 photons may become 4×10^6 photons, 10 photons then become $10 \times$

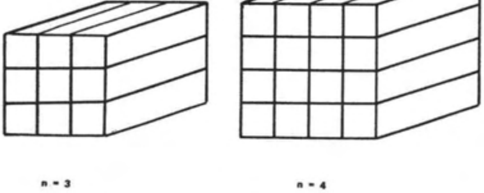

FIG. 1. Various beam sizes for n = 1, 2, 3, 4.

10^6 photons, etc., if these changes make the magnitudes seem more realistic. The simplification causes no loss of generality.

Assume a square cross-section beam of 10 axially collimated monochromatic photons per second impinging on face *A* with 6 photons per second passing through rear face *B*. Then 4 photons per second leave through the four sides of one-metre length. Since the transmission medium is assumed isotropic and optically homogeneous, we may say that of the 4 photons per second per metre narrow angle scattered out of the beam, one photon per second on the average passes, for reasons of symmetry, through each of the four one metre long rectangular sides.

From Equation 2, we have

$$s = \frac{\text{probability of a scattering-out-of-the-beam event occurring}}{\text{unit beam distance}}$$

Alternatively, we may write

$$s = \frac{\text{number of photons scattered out of the beam per second per metre}}{\text{total number of photons entering front face per second}}$$

Substituting,

$$s_1 = \frac{4 \text{ photons sec}^{-1} \text{ m}^{-1}}{10 \text{ photons sec}^{-1}}$$

$$s_1 = 0.40 \text{ m}^{-1}$$

Now consider Figure 1(b) where 40 axially collimated photons per second form a larger square cross-section beam in the same transmission medium. Only the beam cross-section size is changed. The incoming photons are uniformly distributed and enter the volume of interest through front face *ABCD*. For uniformity of initial beam flux, let 10 photons per second enter the front faces of each of the four cells. Because of narrow angle scattering and cell adjacency, the beam has a much lower probability of losing photons by scattering through the common cell sides. This probability can be made zero, on the average, by selecting the beam cell dimensions as permitted by the definition of the scattering coefficient. The beam loses one photon per second on the average through each non-contiguous cell side as before. Then 8 photons per second are scattered out of the beam on the average through the 8 exposed sides of the four cells. It then follows that

$$s_2 = \frac{8 \text{ photons sec}^{-1} \text{ m}^{-1}}{40 \text{ photons sec}^{-1}}$$

$$s_2 = 0.20 \text{ m}^{-1}$$

It is apparent that $s_1 \neq s_2$, neither beam volume scattering coefficient value being able to uniquely characterize the scattering property of the transmitting medium. It is easy to show that, in general, the conventionally defined scattering coefficient depends on beam size.

Referring to Figure 1, let

n = number of non-contiguous cell areas on one side of the original beam,
n^2 = number of cells in the initial beam cross-section,
10 = number of axially collimated photons entering each cell of the beam front face,
$10n^2$ = number of axially collimated photons entering the beam,
$4n$ = total number of non-contiguous cell sides in the beam, and
$s(n)$ = scattering coefficient as a function of the number of cells on one beam side.

From the definition of the volume scattering coefficient,

$$s(n) = \frac{4n}{10n^2} \tag{3}$$

$$s(n) = \frac{0.40}{n} \tag{4}$$

Since,

$$s(o) = \lim_{n \to o} \frac{0.40}{n} = \infty$$

and

$$s(\infty) = \lim_{n \to \infty} \frac{0.4}{n} = o$$

it is obvious that the scattering coefficient, s, can be made to have any desired value, i.e., $o < s < \infty$, by selecting the beam size or n, even though beam cross-section geometry (square) and original collimated beam flux distribution (N_o) are kept constant. The latter conditions are, of course, not required by the scattering coefficient definition. Figure 2 is a plot of Equation 4.

It is clear that the beam volume scattering coefficient, s, as defined in standard references cannot properly characterize the scattering property of an electromagnetic radiation scattering medium. As a caveat, it should be mentioned that using the volume scattering coefficient, in combination with the volume absorption coefficient or alone, to calculate light attenuation, may lead to gross errors or very approximate and unrepeatable results, often attributed to poor measurement technique or inconstant measuring conditions.

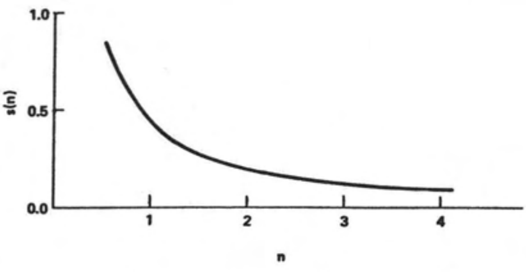

FIG. 2. The scattering coefficient as a function of beam size.

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