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Independent Model Triangulation—an Improved Method

An iterative approach gave better accuracy than noniterative methods.

INTRODUCTION

INDEPENDENT MODEL TRIANGULATION, a semi-analytical method of aerial triangulation, is now being extensively used in many organizations since, unlike analogue aerial triangulation, it does not need a universal instrument with base-in and base-out capability and the models can be observed on Wild A-8, Kern PG-2, and similar instruments. It is often preferred to the fully analytical methods because it involves fewer computations, but it is generally less accurate.

ASTRACT: *The author has tested a proposed method for connection of models to form strips in the semi-analytical method of aerial triangulation known as independent model triangulation. This modified method gives better accuracy as compared to the accuracy achieved by other appraoches tested.*

The main concern in most of the independent model triangulation approaches is the accurate connection of models to form strips, and the accuracy of these methods is to a great extent dependent on the accuracy of this strip formation.

The author has developed an improved method of strip formation from independent models which results in better accuracy as compared to other tested methods. The method derived by the author is an iterative one.

MATHEMATICAL FORMULATIONS

One method investigated uses the simple transformation given as:

$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} = \begin{bmatrix} \lambda & -K & \Phi \\ K & \lambda & -\Omega \\ -\Phi & \Omega & \lambda \end{bmatrix} \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} + \begin{bmatrix} Sx \\ Sy \\ Sz \end{bmatrix} \qquad (1)$$

where X_1, Y_1, Z_1 are the coordinates of model (I); X_2, Y_2, Z_2 are the coordinates of model (II); Sx, Sy, Sz are the three displacements; λ is the scale factor; and K, Φ, Ω are the three rotations.

Instead of the above simple transformation the author has used the equations given below in which the rotation matrix $\boldsymbol{R}$ is orthogonal.

$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} = \lambda \left[\boldsymbol{R}\right] \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} + \begin{bmatrix} Sx \\ Sy \\ Sz \end{bmatrix} \qquad (2)$$

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$$\text{where } [\boldsymbol{R}] = \begin{bmatrix} \cos\Phi\cos K & -\cos\Phi\sin K & \sin\Phi \\ \sin\Omega\sin\Phi\cos K+\cos\Omega\sin K & -\sin\Omega\sin\Phi\sin K+\cos\Omega\cos K & -\sin\Omega\cos\Phi \\ -\cos\Omega\sin\Phi\cos K+\sin\Omega\sin K & \cos\Omega\sin\Phi\sin K+\sin\Phi\cos K & \cos\Omega\cos\Phi \end{bmatrix}$$

The models are connected with the help of conjugate projection centers and at least three points common to the two models. The parameters are solved in a specified number of iterations; three iterations are found to be sufficient.

Procedure

(i) Since rotations K, Φ, Ω are small, the elements of the orthogonal rotation matrix $[\boldsymbol{R}]$ can be replaced by their approximate values and the transformation Equation 2 is reduced to Equation 1

$$\begin{aligned} X_1 &= Sx + \lambda X_2 - KY_2 + \Phi Z_2 \\ Y_1 &= Sy + \lambda Y_2 + KX_2 - \Omega Z_2 \\ Z_1 &= Sz + \lambda Z_2 - \Phi X_2 + \Omega Y_2 \end{aligned}$$

$$\text{or} \quad \underset{(3\times1)}{\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}} = \underset{(3\times7)}{\begin{bmatrix} 1 & 0 & 0 & X_2 & -Y_2 & Z_2 & 0 \\ 0 & 1 & 0 & Y_2 & X_2 & 0 & -Z_2 \\ 0 & 0 & 1 & Z_2 & 0 & -X_2 & Y_2 \end{bmatrix}} \underset{(7\times1)}{\begin{bmatrix} Sx \\ Sy \\ Sz \\ \lambda \\ \kappa \\ \Phi \\ \Omega \end{bmatrix}} \qquad (3)$$

This equation is first used to determine the seven parameters of transformation.

(ii) Next the corrdinates X_2, Y_2, Z_2 of model (II) are transformed in terms of model (I) by using the rigorous Equation 2.

The transformed coordinates AX_2, AY_2, AZ_2 are given below:

$$AX_2 = \mathrm{Sy} + \lambda\,[\cos K\cos\Phi\ \mathrm{X}_2 - \cos\Phi\sin K\ Y_2 + \sin\Phi\ \mathrm{Z}_2]$$

$$AY_2 = \mathrm{Sx} + \lambda\left[(\sin\Omega\sin\Phi\cos K + \cos\Omega\sin K)\,\mathrm{X}_2 + (\cos\Omega\cos K - \sin\Omega\sin\Phi\sin K)\,\mathrm{Y}_2 - \sin\Omega\cos\Phi\ \mathrm{Z}_2\right]$$

$$AZ_2 = \mathrm{Sz} + \lambda\left[(-\cos\Omega\sin\Phi\cos K + \sin\Omega\sin K)\,\mathrm{X}_2 + (\cos\Omega\sin\Phi\sin K + \sin\Omega\cos K)\,\mathrm{Y}_2 + \cos\Omega\cos\Phi\ \mathrm{Z}_2\right]$$

(iii) These transformed coordinates AX_2, AY_2, AZ_2 replace the old coordinates X_2, Y_2, Z_2 and the transformation on parameters are again solved by using the approximate Equation 3.

(iv) The process is repeated till the corrections to parameters become insignificant. Three iterations have been found sufficient.

(v) Along with the solution of parameters, all the remaining points of model (II) which are not common with model (I) are also transformed in terms of model (I) with the equations given in step (ii) above.

(vi) Next, the coordinates of model (III) are transformed in terms of the transformed coordinates of model (II) and so on till all the models are transformed in terms of model (I) and a continuous strip is formed.

Test Study

An experiment was conducted by the author to test the suitability of this method, and also to compare the accuracies obtained from an aerial triangulation using the author's method with the results of a noniterative method.

For this purpose, models formed analytically from error-free simulated photograph coordinates were used. The results of strip formation by the two methods in the form of residuals in X, Y, Z coordinates obtained during linkage of models are given in Table 1.

The strips were then adjusted in terms of known ground coordinates with the help of second degree (X, Y, Z in common) polynomials. The results of strip adjustment are given in Table 2 in the form of root mean square errors and maximum values of residuals of X, Y, Z coordinates in μm on negative scale.

TABLE 1. RESULTS OF STRIP FORMATION FROM BOTH THE AUTHOR'S AND A NONITERATIVE METHOD. Residuals in coordinates.

Models Connected	Points	A Noniterative Method			Author's Method		
		V_x mm	V_y mm	V_z mm	V_x mm	V_y mm	V_z mm
	7	−0.01	0.00	0.01	0.00	0.00	0.00
(1–2) to (2–3)	8	0.00	0.00	−0.01	0.00	0.00	−0.01
	9	−0.02	0.00	0.01	0.00	0.00	0.00
	10	0.03	−0.01	0.01	0.00	0.00	0.00
(2–3) to (3–4)	11	−0.03	0.00	0.02	0.00	0.00	0.01
	12	−0.09	−0.02	−0.03	0.00	−0.01	−0.02
	13	0.00	0.00	0.00	0.00	0.00	0.00
(3–4) to (4–5)	14	0.00	0.00	0.00	0.00	0.00	0.00
	15	0.01	0.00	0.01	0.00	0.00	0.00
	16	−0.13	0.01	−0.01	0.00	0.00	0.00
(4–5) to (5–6)	17	0.02	0.00	0.01	0.00	0.00	0.01
	18	0.16	0.00	−0.01	0.00	0.00	0.00
	19	0.03	0.01	−0.02	0.00	0.00	0.00
(5–6) to (6–7)	20	0.01	0.00	0.00	0.00	0.00	0.00
	21	0.00	0.01	0.02	0.00	0.00	0.01
	22	0.10	0.00	−0.01	0.00	0.00	0.00
(6–7) to (7–8)	23	−0.02	0.00	−0.01	0.00	0.00	0.00
	24	−0.13	0.01	0.01	0.00	0.00	0.00
	25	−0.03	0.00	0.01	0.00	0.00	0.00
(7–8) to (8–9)	26	0.06	0.00	−0.01	0.00	0.00	0.00
	27	0.18	0.00	0.00	0.00	0.00	0.00
	28	0.02	−0.01	0.01	0.00	0.00	0.00
(8–9) to (9–10)	29	0.01	0.00	−0.01	0.00	0.00	0.00
	30	0.01	−0.01	−0.01	0.00	0.00	0.00

TABLE 2. RESULTS OF STRIP ADJUSTMENT OF STRIPS FORMED BY BOTH A NONITERATIVE METHOD AND THE AUTHOR'S METHOD. ADJUSTMENT CARRIED OUT BY SECOND DEGREE POLYNOMIALS (X, Y, Z IN COMMON).

Root-mean-square errors and maximum values of residuals of X, Y, Z coordinates in μm on negative scale.

Strip	No. of control points	$\overline{)X}$ μm	$\overline{)Y}$ μm	$\overline{)Z}$ μm	V_x (max) μm	V_y (max) μm	V_z (max) μm
Formed by noniterative method	9 control points plus 21 check points	35.8	50.2	30.0	100	120	60
Formed by author's method	9 control points plus 24 check points	9.4	11.2	11.2	20	20	20

$$\overline{)X} = \sqrt{\frac{\Sigma V_x^2}{n-1}}\ ;\ \overline{)Y} = \sqrt{\frac{\Sigma V_y^2}{n-1}}\ ;\ \overline{)Z} = \sqrt{\frac{\Sigma V_z^2}{n-1}}$$

Conclusions

(i) Table 1 shows that the method suggested by the author leaves less residuals at common points after model connections and hence the strip formed by this method is less deformed.

(ii) Table 2 shows that the root mean square errors in X, Y, Z coordinates after adjustment of the strip formed by the author's method are 9.4 μm, 11.2 μm and 11.2 μm respectively, whereas for the strip formed by the noniterative method the corresponding mean square errors are 35.8 μm, 50.2 μm and 30 μm, respectively.

Thus, the author's method gives much better accuracy which compares well with the accuracies achieved by simultaneous block adjustment methods. The use of the method is therefore recommended.

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