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Densification of Urban Geodetic Nets*

Employing aerial photography of the City of Atlanta at a scale of 1:17,500, horizontal coordinates of ground points were determined with a standard error of better than 7 cm.

Abstract: *The development of the bundle adjustment with self-calibration advances photogrammetric technology to the stage where the densification of urban geodetic networks can be accomplished to requistie accuracies from photogrammetric blocks of relatively small scale (1:17,500). This was demonstrated in a recent pilot project conducted in the City of Atlanta. Field checks showed that photogrammetrically determined coordinates of monuments had standard errors of better than 7 cm horizontally. The production of such high accuracies from photographs of such small scale renders the photogrammetric approach to densification overwhelmingly attractive economically in comparison with other methods of comparable accuracy. Although densification constitutes the indispensable first step to the development of the modern cadastre based on description of property by coordinates, progress to date in this regard has been disappointingly slow. We hope that the application of advanced photogrammetric technology will serve to stimulate progress.*

Introduction

The system of geodetic monumentation in most urban areas in the U.S. is widely acknowledged to be inadequate for modern needs. In many communities the problem is not so much one of quality as of quantity, for local first-order networks tied to the national grid may already be in place. By and large, the spacing of stations in such networks ranges from 3 to 5 miles. Only a relatively small percentage of local surveys are tied to the national network, mainly because of the expense entailed in traversing the several miles typically necessary for a proper connection closing on two different points. As a result, most urban areas are beset with a multiplicity of local survey datums perpetuated by diverse interests ranging from various utilities and public works departments all the way down to private surveyors. The consequences of this Balkanization of urban surveying networks have been costly, to say the least. Delays, disputes, misunderstandings, inadvertent destruction or misuse of property, infringements, interruptions of projects, litigation, inequitable taxation, inefficiencies, and undue hardships are only some of the negative consequences. Perhaps the only positive consequences consist of the short term convenience and economic advantage accruing to special public and private interests from narrow localization of surveying operations. Yet, just such considerations have traditionally predominated, and it has been estimated that cur-

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rently only about one urban survey in four is tied to the national network.

To bring order from the present chaos in a manner acceptable to all interests, two major measures must be undertaken. The first consists of implementing in urban areas an appropriate level of densification of the handful of locally available first-order monuments tied to the national grid—a nominal spacing of one new monument every one-half mile would be sufficient to assure large-scale voluntary use of a densified net. The second consists of enactment of appropriate legislation requiring that all surveys of legal standing be tied directly to the densified network. As a practical matter, the second measure cannot be taken until the first has been implemented—otherwise, unacceptable economic burdens would ensue.

A few far-sighted communities already have taken action along the above lines. In Monroe County, New York, for example, a program has been underway for several years leading to an exemplary system of monumentation. When completed, permanent monuments will have been installed at minimum intervals of nominally one-half mile along all roads under the county's jurisdiction—a total of well over 3,000 monuments in an area of some 670 square miles. Yet, this project which began in 1968 and is still underway by means of conventional traversing will ultimately consume a decade and will end up costing over 1.2 million dollars. It has thus become clear that unless a far more timely and economical means for densification is forthcoming, it is unlikely that many other communities will embark on similar undertakings. In recent years two candidates of great promise in this regard have emerged, namely, photogrammetric triangulation and inertial surveying. This present paper is concerned primarily with the former, though brief consideration will be accorded the latter at the conclusion of the paper.

Why Densify?

Before technical aspects of photogrammetric densification are discussed, it is appropriate that further consideration be given to reasons for densification. Perhaps the most compelling reason is that densification is the indispensable first step to the creation of a modern computerized land data system. This fundamental fact was brought out in paper after paper published in the *Proceedings of the North American Conference on Modernization of Land Data Systems: A Multi-Purpose Approach.* This conference was held in Washington, D.C., on April 14-17, 1975, under the joint sponsorship of the following participating agencies:

> American Bar Association
> American Congress on Surveying and Mapping
> Canadian Institute of Surveying
> American Land Title Association
> American Society of Photogrammetry
> International Association of Assessing Officers
> National Association of County Recorders and Clerks
> National Research Council (Canada)
> Department of Energy, Mines and Resources (Canada)
> Service du Cadastre, Ministere des Terres et Forets
> Comision de Estudios del Territorio Nacional
> Oficina de Catastro, Gobierno del Edo de Mexico
> Department of Commerce (U.S.)
> Department of Interior (U.S.)
> Department of Agriculture (U.S.)
> Department of Housing and Urban Development (U.S.)

The conference, referred to as MOLDS (Modernization of Land Data Systems), was convened for reasons set forth in the introduction made by the Chairman of the Executive Committee, Capt. John D. Phillips of the NOAA/Federal Geodetic Control Commission. In the section of his paper entitled "*Why the Conference*", Capt. Phillips states:

> Over the years there has been a great deal of discussion and concern with the existing situation in the field of land use and inventory systems and real property and conveyance records in the North American countries. The importance of general land records was recognized by many countries a long time ago. The Economic Council of the United Nations and the Pan American Institute of Geography and History on several occasions has called on national authorities to implement modern systems to satisfy the multiple requirements of a modern society. The North American countries have been lax in this particular area. This lack is causing serious damage, delays and economic losses—especially when integrated programs are conceived to combat environmental social and land use problems and deficiencies. This Conference, the first under the (North American Institute for Modernization of Land Data Systems) MOLDS umbrella is to draw attention to the need, define parameters, review progress and lay the foundation countries to establish compatible national land data systems.

Of the many significant papers presented at this prestigious conference, one of the most important is the Preliminary Report of the Ad Hoc Committee of representatives of the

American Bar Association and the American Congress of Surveying and Mapping. The committee report was presented jointly by William A. Chatterton and John McLaughlin and is entitled: *"Towards the Development of Modern Cadastral Standards."* Of particular cogency with regard to densification are the proposed Cadastral Survey Standards put forth under Section 5.3 and 5.4 which state:

> 5.3 The committee has concluded that "all descriptions of land parcels, easements, and other interests in land required for conveyancing, taxation, and any other purposes should be ultimately tied to the state/ provincial plane coordinate system." Only in this way can the consumer be afforded acceptable protection. As a means of promoting the use of the coordinate system, the committee further agreed that: "upon the verification of the state plane coordinates system within a given region, areas should be proclaimed as designated survey areas within which:
>
> a) All property surveys carried out for agencies having the power of eminent domain would be required to be tied to the coordinate system;
> b) all property surveys carried out to create subdivisions of greater than 20 lots would be required to be tied to the coordinate system;
> c) all property surveys carried out within 800 meters of a coordinate monument would be required to be tied to the coordinate system;
> d) and any other property survey, at the owner's discretion, could be tied to the coordinate system."
>
> 5.4 Minimum standards of survey practice within the designated survey areas, including accuracy of measurement and the provision of evidencial documentation, would be established and regulated by an appropriate state/ provincial agency. Accuracy specifications for property surveys should be developed in terms of error ellipses for monumentation and tolerance lanes for boundaries. At this time, however, this committee acknowledges prevalence of relative precision criteria and have agreed that all property surveys within the designated survey areas should satisfy the minimum relative precision criteria of 1 (1cm + 0.0002 * Distance in meters).

As a practical matter, these key recommendations of the committee can be implemented only if a uniform system of monumentation of sufficient density has been established in a given area. In order for recommendation (c) of 5.3 to have any substance it is clear that 'coordinate monuments' must be provided at minimum spacings of about 800 m or nominally one-half mile (this interpretation assumes that, in general, the distance from a point to a monument is reckoned along a grid of streets rather than along a mathematical straight line). Unless densification is established to such a level, there is no economically practical possibility of carrying out the pivotal recommendation of the proposed standards, namely, that "all descriptions . . . should ultimately be tied to the state/provincial plane coordinate system." Hence the truth of the earlier statement that densification is the indispensable first step to the creation of a modern land data system. Auxiliary benefits include

> (a) substantial savings in costs of traversing to tie special surveys into the national geodetic system;
>
> (b) immediate and adequate availability of geodetic control sufficiently accurate for virtually all future mapping projects anywhere within the area;
>
> (c) the practical and economical realization of the direct tie-in of all future surveys of the facilities of public and private utilities (electric, telephone, water, gas, sewer, public transportation, public works agencies, etc.) to a common unified datum (although the desirability of this is universally recognized, it is presently prohibitively expensive precisely because of lack of densification);
>
> (d) the economical and timely reconciliation of discrepancies or conflicts arising from different existing surveys and/or maps involved on projects involving a diversity of agencies or interests;
>
> (e) the practicability of establishing appropriate parameters for the transformation to a unified system of numerous 'floating datums' established by various agencies and groups over the years; and
>
> (f) easing of the ultimate transition to a computerized land data system by early phase-in of legal requirements for specification of property descriptions by coordinates (e.g., by requiring that all conveyance of property to new owners during, say, a five-year transitional period be supported by a survey tied to the state place coordinate system and paid for by the new owner).

With such compelling reasons for densification, it is both surprising and dismaying that so little has yet been accomplished in this regard throughout the country. Perhaps continuing pressures from such groups as MOLDS coupled with technical advances (such as those to be reported here) will increasingly stimulate recalcitrant communities into undertaking basic programs of densification.

Benefit/Cost Analysis for Densification

The general desirability of densification is not a matter of contention among informed parties. However, it would be helpful to all concerned if some quantitative measure of its value could be established. For proposed public projects this is normally accomplished through an analysis of benefit-to-cost ratios associated with the project. The ratio is defined by the standard formula

$$\frac{B}{C} = \frac{\text{Benefit}}{\text{Cost}} = \frac{\sum_{t=1}^{T} \frac{B_a}{(1+i)^t}}{K + \left[\sum_{t=1}^{T} \frac{0}{(1+i)^t} \right]}$$

in which

B_α = value of annual benefits resulting from the project,

T = the useful life of the project (or time horizon),

i = the cost of money (interest rate),

O = annual operating or maintenance costs, and

K = the initial capital investment cost.

Naturally, the greater the benefit-to-cost ratio, the more economically desirable the project. Although ratios as small as 1.2:1 have been successfully used to justify multimillion dollar projects, a ratio of 1.4:1 is a more acceptable threshold and a ratio of greater than 2:1 is considered to be most compelling.

As applied to densification, the main difficulty in a benefit/cost analysis is in quantifying the annual benefits B_α: the other parameters T, i, O, and K can be estimated or postulated with reasonable confidence. Fortunately, a detailed study has already been performed on one well-defined benefit, namely, the miles of traverse saved by virtue of a specified level of densification. This study was prepared by Phillip C. Johnson of the National Geodetic Survey and is entitled "*A Measure of the Economic Impact of Urban Horizontal Geodetic Control Surveys*," (published by the U.S. Department of Commerce, August 1972).

In his study, Johnson first developed a mathematical model for the estimation of the miles of traverse saved by increasing the number of approximately evenly spaced control monuments in a specified area from an initial number N to an increased number N+N′. It is enlightening to recount the main points of Johnson's analysis as applied to a specific example, namely, Monroe County, New York (area 673 square miles; county seat, Rochester). As mentioned earlier, this county has an active and extensive program of geodetic densification underway by means of ground survey. Monroe County was selected by Johnson for analysis because accurate cost data were available on the first phase of the project in which the initial average spacing of monuments of 5.8 miles (some 20 monuments) was decreased to an average spacing of 1.1 miles (some 532 new monuments). Eventually, well over 3000 monuments are to be installed of nominal intervals of one-half mile. From analysis of questionnaires returned by the surveying profession, Johnson was able to estimate that of the total of some 3140 traverses performed in Monroe County in 1970 some 973 (or 31 percent) were tied into the national geodetic system. By using his model, Johnson was then able to compute that, with the spacing of monuments decreased from 5.8 miles to 1.1 miles, an annual savings of 2570 miles of traverse could be realized. This translated into an annual cost savings (B_α) of \$630,781 (at 1970 prices). As for costs, the total capital investment (K) for the project of densification was an actual \$555,000, and the annual operating and maintenance costs (O) were estimated to be \$30,912. With the time horizon (T) taken conservatively at 15 years and the cost of money (i) taken as 10 percent, the benefit/cost ratio becomes

$$\frac{B}{C} = \frac{\sum_{t=1}^{15} \frac{\$631{,}000}{(1+.10)^t}}{\$555{,}000 + \left[\sum_{t=1}^{15} \frac{\$\ 31{,}000}{(1+.10)^t} \right]} = \frac{6.07}{1.00}$$

This means that by the fifteenth year of the project's economic life the community would have realized a return of \$6.07 for every \$1.00 invested. Indeed, the breakeven point of the project would be reached before the end of the first year. The economic impact of the project is therefore overwhelmingly favorable. Cost/benefit ratios for other time horizons and interest rates are summarized by Johnson in Table 1.

One point concerning the foregoing analysis should be particularly emphasized: one and only one benefit of densification was considered, namely, savings in traversing for the 31 percent of surveys historically tied to the national network. In actuality, numerous other benefits from densification are to be expected, as discussed earlier. How much, for example, might be saved by the likelihood that with the higher density of monuments a far greater percentage of surveys (than 31 percent) would be tied to the national network? Or what would the savings be from the speed-up of initiation (and

TABLE 1. SENSITIVITY OF BENEFIT-COST RATIOS TO CHANGES IN INTEREST RATE AND TIME HORIZON (FROM JOHNSON (1972)).

Interest Rate	Time Horizon					
	5	10	15	20	30	50
	Benefit Cost Ratios					
5.00	3.96	6.13	7.47	8.35	9.40	10.28
7.50	3.75	5.64	6.72	7.38	8.09	8.55
10.00	3.56	5.20	6.07	6.56	7.02	7.26
12.50	3.37	4.81	5.50	5.86	6.16	6.27

hence completion) of projects dependent on surveying; or savings to aerial mapping projects which would not require special surveys to bring in ground control; or savings likely to be realized from the long run decrease in litigation ultimately to be expected? The quantification of these and other benefits were, of course, beyond the stated scope of Johnson's investigation and, we trust, will be addressed in future studies. However, from a cursory consideration of additional benefits it would not seem to be out of line to expect that their inclusion would lead to at least a doubling of the numerator of the *B/C* ratio. If so, a benefit/cost ratio of perhaps 12: 1 could be justified.

Another avenue exists for further improvement of the benefit/cost ratio, namely, though the reduction of the denominator of the ratio. In Johnson's analysis, costs were based on the assumption that densification was to be accomplished by means of conventional ground traversing. However, as pointed out in Brown (1973), the emergence of analytical photogrammetry now provides an acceptable alternative that, in projects of sufficient scope, can reduce overall costs to one third of those to be expected from conventional traversing. Conservatively, then, the prospect exists for reducing the total denominator of the *B/C* ratio by a factor of two. Accordingly, a strong case can be made that the benefits of densification are likely to exceed costs by a factor of better than 20 to 1 when the photogrammetric approach is employed. By normal standards such a ratio is astronomical. Nationwide, it means that tens of millions of dollars are, in effect, being lost annually because of the lack of sufficiently densified geodetic networks.

ACCURACIES NECESSARY FOR DENSIFICATION

Before any project of densification can be undertaken, an answer must be provided to the fundamental question of what accuracies are to be required. Unfortunately, no definitive standards have been adopted in this country that serve to clarify this matter. However, such standards are in effect in Western Europe and, in particular, in West Germany where land values are especially high. This topic is addressed by Wahl (1975) in another paper presented at the 1975 MOLDS conference referred to earlier. A pertinent extract from this paper is—

> Until now such evasive terms like "acceptable" or "admissible" errors, discrepancies, or differences between the results of two different surveys or the calculated and measured values, were used without giving any numerical values. The numerical values for such "admissible errors", called "tolerances", are established by almost all surveying instructions for the various measured elements, like angles, distances and calculated areas. The logical supposition is that the tolerances were established accordingly to the postulated requirements of accuracy. In reality, the Cadastre in his development always accepted the already existing National Network of Control Points as its basis, principally due to the economy and time-saving of such arrangement. Consequently, the conventional cadastral tolerances were in concordance with the accuracy of the National Network of Control Points. In newly formed cadastral systems, very often with independent networks of control points, the required accuracy and the optimal surveying method with its corresponding tolerances can be chosen. We will limit ourselves to the discussion of tolerances of distances and areas, the most important for the various users of the cadastral information. As an example, the conventional German Cadastre presents the tolerance for the difference of the results of two surveys of a distance between two boundary points, or for the difference between a measured and the calculated by coordinates distance, by means of the following formulas:
>
> $$D_I = \pm (5 + 0.03\, s + 0.8\sqrt{s})\ \text{cm}$$
>
> $$D_{II} = \pm (5 + 0.05\, s + 1.2\sqrt{s})\ \text{cm}$$
>
> where
>
> s = *the difference in* m,
> D_I = *the tolerance for normal conditions of the terrain,*
> D_{II} = *the tolerance for difficult conditions of the survey.*
>
> The numerical values for different distances are presented in the following table :
>
> | s in m | 20 | 50 | 100 |
> |---|---|---|---|
> | D_I in cm | 9 | 12 | 16 |
> | D_{II} in cm | 11 | 16 | 22 |
>
> It has to be explained that these values present the maximum admissible discrepancy between two determinations of distance be-

tween two boundary points for two different surveys, or one survey and the distance calculated by coordinates. This maximum admissible discrepancy between two determinations, or "limit of error", (in German Grenzfehler), is accepted as the triple value of the mean square error of the surveying method, and its probability of occurrence is very limited, about 1 in 370 surveys. Also the significance of the three terms in the formulas has to be understood. The first term, ± 5 cm, is the limit error of the identification of the centre of the boundary monuments. The 2nd term represents the influence of not eliminated systematic errors of the survey, and the 3rd term the influence of the random errors. Also these two factors could be diminished by the use of more exact instruments and methods. As a result, more rigorous tolerances could be established.

As an example of such more rigorous tolerances, the newer tolerances for the survey-lines of the Cadastre of Hamburg, used before the introduction of the Computational Cadastre, are presented. Here already in the determination of the points of the Control Network, an accuracy of an average mean square error of the coordinates of $\mp$ 7 cm was achieved.

As explained in Wahl's paper, the specified tolerances represent 3 sigma values for the differences between two repetitions of the survey between a pair of points. It follows that if σ denotes the standard deviation of an individual survey, the tolerance is equal to $3\sqrt{2}\sigma$. Thus, with spacing between control monuments of s = 800 m (or one-half mile), one obtains for the conventional tolerance for normal conditions of terrain

$$\begin{aligned} 3\sqrt{2}\,\sigma &= D_I \\ &= \pm[5 + 0.03\,(800) + 0.8\,\sqrt{800}]\ \text{cm} \\ &= \pm[5 + 24.0 + 22.6]\ \text{cm} = 51.6\ \text{cm}. \end{aligned}$$

From this the admissible standard deviation σ of the individual survey of the distance between two monuments with separation of 800 m becomes

$$\sigma = 12.2\ \text{cm or } 0.40\ \text{ft}$$

or taken one step further, the standard deviation σ_o of the horizontal coordinates (X or Y) of the individual monuments must be

$$\sigma_0 = \sigma/\sqrt{2} = 8.6\ \text{cm} = 0.28\ \text{ft}.$$

Interestingly, this figure corresponds almost precisely to one half the value to be expected from the application of the formula proposed by Chatterton and McLaughlin (1975), namely, ± (1 cm ± 0.0002 × Distance in meters), when applied to a distance of 800 m. Accordingly, if a value of 8.6 cm were adopted in a program of densification as the acceptable standard deviation for the coordinates of the typical monument, it would mean that the accuracy of the monuments would be twice as great as the accuracy demanded of the most distant point ultimately to be referred to the monument in subsequent surveys. This seems to be an altogether reasonable requirement, although some might prefer a tighter specification. However, it seems doubtful whether a specification much tighter than ±5 cm could be justified on economic and practical grounds.

Urban Densification by Conventional Traversing

In order to place densification by photogrammetric means into proper perspective it is appropriate that consideration first be given to certain aspects of densification by means of conventional traversing. To this end, the specific and admittedly idealized situation pictured in Figure 1a is analyzed. Here, the four corner points A,B,C, and D are considered to consist of established, first-order monuments of the national network and, for present purposes, are considered spaced at 8 km intervals. The intervening points pictured as small dots represent locations of new monuments to be established by densification and are assumed to be evenly distributed at 800 m intervals. The lines connecting the dots define the assumed course of the traverse in accordance with one straightforward design. Here, traverses are first made between A and B and C and D respectively. With the intermediate points on these two vertical segments thus established and considered to be known, the remaining points are established as indicated by means of horizontal traverses joining opposing points on segments AB and CD. It is further assumed that the traverse is performed in accordance with first order standards as defined in Table 2. Thus, each 800 meter segment is measured to an "average accuracy" (which will be assumed to represent one standard deviation) of 1 part in 50,000. The matters of primary interest in this exercise concerns the accuracies to be expected for the coordinates of the newly established monuments and the cost and time likely to be required for a typical project.

A rough estimation of the accuracy to be expected for, say, the mid-point M of the densified net may be arrived at by use of a rule-of-thumb for an open traverse, according to which

$$\sigma = n\,\sigma_0 + n^{1/2}\,s\,\sigma_p$$

wherein

σ = standard deviation of coordinate of terminus of traverse,

n = number of intermediate segments,
σ_o = estimate of bias of typical segment,
s = length of typical segment (same units as σ_o), and
σ_p = proportional accuracy of random error in typical segment.

Thus, if M were reached via L in an open traverse starting, say, from A, n would be equal to 10. A reasonable value for σ_o is 1 cm or 0.01 m. Then, since s is equal to 800 m and σ_p for a first order traverse is 1:50,000, it follows that σ turns out to be equal to

$$\sigma = 10\,(0.01\text{ m}) + \frac{10^{1/2} \times 800\text{ m}}{50{,}000} = 0.15\text{ m} = 15\text{ cm}.$$

However, in the postulated net, M is not reached by a simple traverse from A, but is also reached via links to B, C, and D. Hence, the accuracy of M following an appropriate adjustment will be improved over that of the simple open traverse by a factor approaching 2. Accordingly, a value of roughly 8 cm would be a reasonable expectation for the standard deviation of the coordinates of the midpoint of the net. Now, this is in line with the value of 8.3 cm arrived at earlier as a reasonable specification for densification to the 800 meter level. One might thus conclude that a traverse executed according to the design just described would be perfectly satisfactory. However further probing will disclose that the survey has a certain weakness. If the rule-of-thumb for traversing accuracy were applied to points Q (reached from A via P) and R respectively, one would obtain σ_Q = 9.9 cm and σ_R = 9.6 cm. These would be improved by a factor of roughly $\sqrt{2}/2$ by virtue of the closing of the traverse on D (closings on B and C are too remote to be of much value), and hence they become σ_Q = 7.0 and σ_R = 6.8 cm, respectively. Since Q and R are reached by independent routes, the errors in their coordinates are essentially mutually independent. The expected standard deviation of the distance between points Q and R is, therefore, given by $[(7.0)^2 + (6.8)^2]^{1/2}$ = 9.8 cm, a value acceptable, yet unsettling, for one finds that the proportional accuracy of the distance between Q and R

TABLE 2. STANDARDS OF SURVEYING ACCURACY.*

Order of Accuracy	Minimum Accuracy	Average Accuracy
A. Triangulation and Trilateration		
	1:Part in	1:Part in
First-order class I	100,000	150,000
First-order class II	50,000	100,000
First-order class III	25,000	75,000
Second-order class I	20,000	50,000
Second-order class II	10,000	20,000
Third order	5,000	10,000
B. Traverse		
First order	25,000	50,000
Second order	10,000	20,000
Third order	5,000	10,000

* Taken from "Standards and Specifications for supplemental Horizontal Control Surveys", a paper presented by J. F. Dracup to the 1970 Annual Meeting of ACSM.

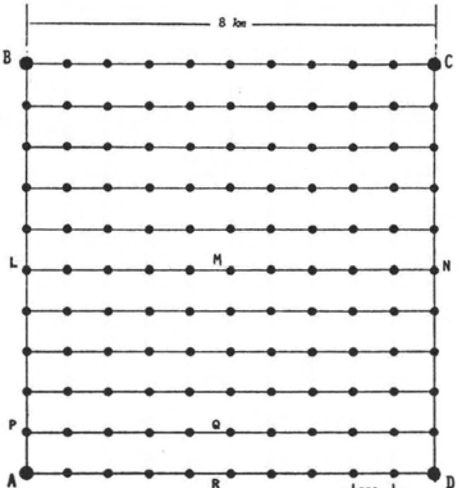

FIG. 1a. One possible, minimally redundant scheme for densification by traversing.

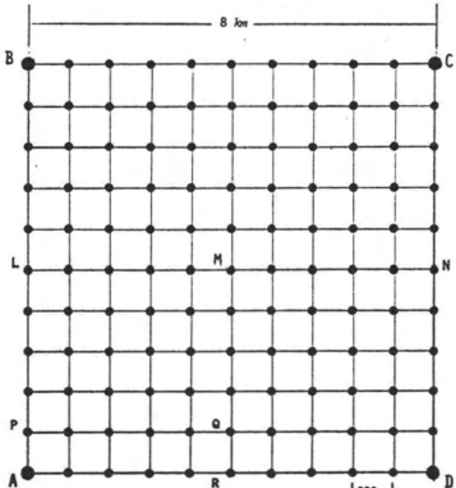

FIG. 1b. An alternate, moderately redundant scheme leading to realization of homogeneous accuracies.

turns out to be only 1 part in 8200 which is equivalent to what is to be expected from a third-order traverse between Q and R. This demonstrates that even a superficially logical first-order traverse can actually yield the equivalent of third-order results for contiguous stations that are not directly connected with each other.

What the foregoing signifies for urban densification is clear. To produce homogeneous accuracies, a traverse should ideally follow a highly redundant scheme akin to that (Figure 1b) wherein each interior station is reached by a pair of independent traverses. Moreover, to be comparable in rigor to photogrammetric method presently to be described, the entire net should be adjusted as a whole, rather than being subject to simple, piece-wise adjustment. This would not be a particularly formidable demand for the 121 station net of Figure 1b. However, such a net would ordinarily constitute only a single 'cell' of the overall net required for the coverage of a typical urbanized county. If an overall area of, say, 40 by 40 km (roughly 25 by 25 miles) were considered to constitute a typical project, densification to the 800 m level would generate a total of 51 by 51 = 2601 stations and would entail some 4000 km of traversing in accordance with the redundant scheme of Figure 1b. The rigorous least squares adjustment of the net would lead to a system of normal equations on the order of 5204 by 5204. With proper ordering of unknowns, the coefficient matrix of the normal equations could be made to assume a banded form which would render the solution of the system feasible, though nonetheless still formidable.

At a rate of \$250 per kilometer the total cost of the postulated 4000 km traverse would approach \$1,000,000. In addition, at \$75 each, the cost of fabrication and installation of monuments would total about \$195,000. Analysis, data reduction, and reporting would require at least another \$150,000 bringing the total project cost to the neighborhood of \$1,345,000. With average progress of 2.4 km per party day over reasonable terrain the first-order traverse would consume some 1600 productive party days. With due allowance for weather and other delaying factors, this could be expected to be pushed towards 2000 party days. This corresponds to an eight-year project for one party or to a two-year project for four parties. From six months to a year should be added for the accomplishment of the final overall adjustment. Accordingly, with four field parties an optimistic estimate for the total duration of the project would be 30 calendar months.

Although various assumptions in the above breakdown can be challenged, it is believed that the net estimated totals are fairly consistent with expectations of current practice. As such, they provide a reasonable basis for subsequent comparisons with densification by the photogrammetric method.

Densification by Photogrammetric Triangulation

Technical aspects of the application of photogrammetric triangulation to densification have been thoroughly covered in previous papers of the author (Brown, 1971, 1973, 1974, 1976). Therefore, only a few of the major considerations will be touched on here. Of paramount importance is the consideration that accuracies of the highest order are to be obtained only by means of the bundle adjustment with self-calibration. Here, no compromises are made in the name of expediency. The adjustment involves the simultaneous, least-squares triangulation of all bundles of rays from all exposure stations to all measured ground points in a process which also (a) recovers the elements of orientation of all participating exposures, (b) adjusts the control survey (in accordance with its postulated accuracy), and (c) estimates coefficients of error models that describe the residual systematic errors affecting the plate coordinates. The general system of normal equations generated by this process is of order $6m + 3n + q$ where m, n, and q denote respectively, the number of photos, the number of measured ground points, and the number of error coefficients. The $3n$ unknowns corresponding to the ground points can be eliminated by the algorithm originated in Brown (1958) to produce a reduced system of normal equations of order $6m + q$. Through proper ordering of the photos constituting a typical block this reduced system, can, in turn, be made to assume a banded-bordered form with the q unknowns of the error model being confined to the border. As pointed out in Brown (1968), such systems can be solved efficiently by means of an algorithm termed recursive partitioning. What this would entail for a photogrammetric block covering the 40 by 40 km area considered in the preceding section merits special attention. First, by virtue of analytical error propagation based on realistic assumptions it can be deduced that an appropriate photogrammetric block for the project is one consisting of 26 strips of 26 photos (a) with forward and lateral overlap of

60 percent (b) with a horizontal/vertical control point located near the nadir of every fifth photo on the perimeter (a total of 18 points, in all, spaced at 8 km intervals around the perimeter), and (c) with a photo scale of nominally 1:17,500. The layout of such a block is pictured in Figure 2. If the given control were regarded as flawless and film measuring accuracies ranging quadratically from 3 μm at the center of the frame of 6 μm at the corners were achieved, the desired accuracy of 8.3 cm could be expected to be modestly bettered with excellent uniformity throughout the block, *provided* the adjustment were accomplished by means of the bundle adjustment with self-calibration. If a total of 18 error coefficients were exercised for self-calibration, the reduced banded-bordered system of normal equations would be of order $6 \times 676 + 18 = 4074$ with bandwidth of 424 elements and borderwidth of 18 elements. Solution of this system by recursive partitioning would be about $[4074/(324 + 18)]^2 = 142$ times faster than a conventional reduction. The problem could thus be solved by recursive partitioning on a medium-scale computer in a matter of a few hours, whereas a prohibitive three to four hundred hours would otherwise be required. It is this consideration that makes the bundle adjustment with self-calibration computationally feasible.

It should be pointed out that the improvement to be expected from the incorporation of self-calibration into the bundle adjustment is very considerable, ranging in general from a factor of 2 to 3. As compared with widely employed methods of analytical triangulation based on piecewise polynomial adjustment of strips (e.g., the well-known Schut method), the bundle adjustment with self-calibration can be expected to produce results of at least five times greater accuracy.

It has been demonstrated both theoretically and practically that the bundle adjustment of a block with reasonably tight perimeter control can be expected to yield accuracies that are essentially uniform through the interior of the block. The implications of this are at first difficult to accept by persons accustomed to thinking in terms of proportional accuracies customarily used to characterized ground surveying. In the hypothetical 40 by 40 km block just considered the expected standard deviation of X and Y would be close to 7.5 cm for virtually all points. Converted to proportional ac-

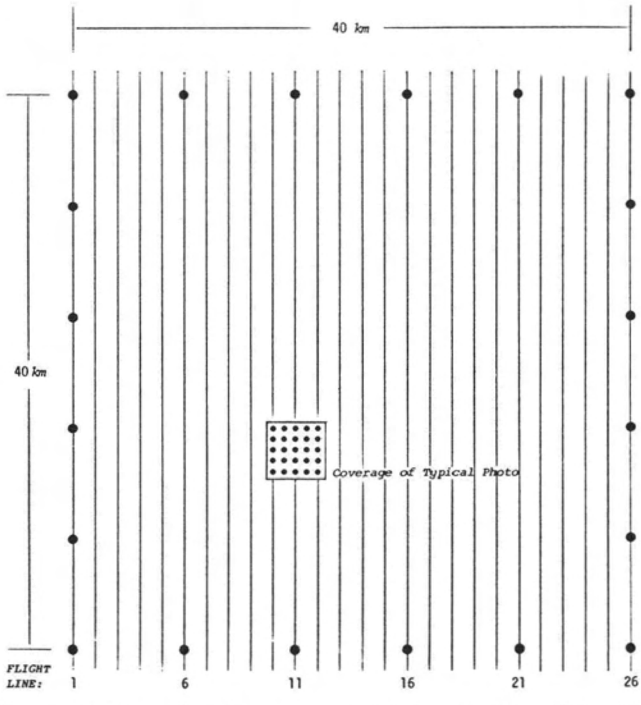

FIG. 2. Layout of photogrammetric block covering a 40 by 40 km area used as the standard in analyses of comparative costs of densification by different methods.

curacies for various distances, this corresponds to the values given in Table 3. Thus, in conventional terms, accuracies of photogrammetric triangulation range from those to be expected from third-order traverses (for relative positions of adjacent points) to accuracies far beyond normal expectations from a first-order traverse for relative positions points separated by 20 km or more. It is the homogeneity of the absolute accuracies of coordinates that makes the photogrammetric method so potentially attractive from the theoretical point of view in applications to geodetic densification. The fact that *proportional* accuracies range from adequate to spectacular is of secondary concern.

It is to be emphasized that the strict validity of the results given in Table 3 depends on the assumption that the postulated peripheral control is totally free of error. In actuality this is not the case, and the photogrammetric adjustment will tend to distribute errors in the control framework to points throughout the block in a smoothly flowing apportionment. However, by virtue of its theoretical potential for producing uniform accuracies, the photogrammetric method is inherently capable of extracting full benefits from control surveys of the highest possible accuracy. Accordingly, photogrammetric densification should ideally be supported by the very best control surveys afforded by the state-of-the-art.

In a later section an estimate will be made of the time and cost required for the photogrammetric densification of the hypothetical 40 by 40 km area which was analyzed in the preceding section in connection with densification by traversing. It will be useful beforehand, however, to review the design and outcome of an actual project of photogrammetric densification.

Actual Demonstration of Photogrammetric Densification

When it comes to claims made by photogrammetrists, all surveyors are from Missouri—they have to be shown. This skepticism is well and good, for our industry is still dominated by obsolete technology insofar as photogrammetric triangulation is concerned. As a result, negative experiences continue to accumulate and, to outsiders, all of photogrammetry seems to be tarred with the same brush. Yet the difference between run-of-the-mill photogrammetric triangulation and the most advanced photogrammetric triangulation (the bundle method with self-calibration) is comparable to the difference between ground surveying by third-order and first-order methods. After years of frustration in attempting to promote urban densification by photogrammetry, the author welcomed in late 1974 the opportunity and challenge for a demonstration afforded by the Georgia Coordinate Mapping Committee. Arrangements were made under the auspices of this group for the cooperative undertaking of a small pilot project in the city of Atlanta. The major active participants in this project were DBA Systems Inc., Airborne Data Inc., the City of Atlanta, and the Georgia Department of Transportation (DOT). The major goals of the project were to

(a) determine the feasibility of urban densification of the existing geodetic net to an rms accuracy of 0.25 ft (7.6 cm) by means of photogrammetric triangulation of 1:17,500 scale photos made by DBA's super wide angle camera with a proprietary platen reseau (for correction of film deformation) and operated from a gyroscopically stabilized camera mount ASTACS proprietary to Airborne Data (ASTACS is short for Automatically Stabilized Airborne Camera System);

Table 3. Proportional Equivalents of Uniform Accuracies of Coordinates Generated from Adjustment of Hypothetical 676 Block of 1 : 17,500 Scale Photos Covering a 40 × 40 km Area.

Distance Between Points	Standard Deviation of Distance*	Proportional Accuracy
(1) 800 m (adjacent points)	0.106	1 : 7,547
(2) 1600 m (every other point)	0.106	1 : 15,094
(3) 3200 m (every fourth point)	0.106	1 : 30,189
(4) 20,000 m (center of block to edge)	0.106	1 : 188,679
(5) 28,284 m (from center of block to corner)	0.106	1 : 266,830
(6) 40,000 m (from one side of block to opposite side)	0.106	1 : 377,358
(7) 56,569 m (across diagonal of block)	0.106	1 : 533,670

* This figure is essentially the same for any new monument with respect to any other new monument and is equal to the standard deviation of the coordinates (i.e., 0.075 m) multiplied by $\sqrt{2}$.

(b) determine in conjunction with (a) the feasibility of a previously untried method of offset targeting in place of direct targeting of monuments; and
(c) determine the feasibility of generating digitized planimetric maps to a scale of 1″ = 100 ft along with topographic maps having 2 foot contour intervals from the same (1:17,500 scale) super wide angle photographs to be used for geodetic densification (thereby demonstrating possible dual utility of the photographs).

The major responsibilities of the participants were

(a) targeting and monumentation—DOT and City of Atlanta;
(b) special surveying for extra control and field checks for verification of accuracies—DOT;
(c) providing super wide angle camera with proprietary reseau platen—DBA Systems;
(d) photographic flights employing ASTACS stabilized camera platform—Airborne Data;
(e) performance of analytical aerotriangulation using bundle adjustment with self-calibration—DBA Systems;
(f) generation of digital tapes for computerized plotting of mapping data—Airborne Data;
(g) production of mapping products from digital tapes—City of Atlanta;
(h) general coordination of project—Airborne Data.

In fulfilling each phase of the project each participant bore his own expenses. In this paper emphasis will be placed mainly on those aspects of the pilot project concerned with geodetic densification by photogrammetric triangulation.

The pilot project encompassed the 3 by 4 mile area shown in Figure 3. At nominal half-mile intervals within this area potential locations for permanent monuments were selected—a total of 63 locations in all. Actual locations of monuments were established by field reconnaissance to be as close as possible to the ideal locations laid out on a map. As can be seen from Figure 2 which depicts the actual locations of the monuments, it proved possible to adhere rather closely to the planned layout despite the wooded, hilly, and built-up character of the landscape.

Photogrammetric projects in urban areas requiring the installation of a large number of movable panels to serve as photo-identifiable targets can be expected to be beset with all sorts of difficulties that are well known to the industry. It was thus felt that an improved approach to targeting was needed in order to enhance the practicability of photogrammetric densification. In an attempt to satisfy this need, the method of *dual offset targeting* was formulated. As applied to the Atlanta project it was implemented as follows: Near each of the selected locations of the monuments two circular targets of 32-inch diameter were painted in fluorescent orange along the center of the adjacent roadway. The separation of the targets was preset to be precisely 30 feet and the center of each target was marked by a surveyor's nail. The monument itself, whether actual or potential (some were put in after the photography), was located off the roadway about equidistant from the two painted targets as shown in Figure 4. In those cases where monuments were actually implanted, the offset distances of the monument from the centers of the two painted targets were measured along with the differences in height between the monument and the centers of the targets. From these easily made offset measurements and the coordinates (ultimately to be established photogrammetrically) of the neighboring targets it becomes a simple matter to compute the precise coordinates of the monument itself. This means that the permanent monument need not actually be installed prior to the photography. It can be installed at any desired later time as long as the centers of the associated offset targets remain recoverable. This constititues a significant advantage of the offset target method, for the installation of monuments is a far more time-consuming and tedious task than the painting of dual targets in the roadway. Moreover, because of the designed proximity of each eventual monument to corresponding pairs of painted monuments, the necessary offset measurements can be made quickly, easily, and inexpensively by relatively unskilled personnel. Three other advantages of this approach are noteworthy: (a) vandalism of movable targets placed over monuments is a major problem in urban areas, whereas targets painted in the roadway are virtually immune to disruption; (b) such targets by virtue of their location along the center of the road are less likely than the monument itself to be obscured in the photography by nearby trees or structures; and (c) painted targets are extremely durable and are photographically useful for many months after initial installation. Still another advantage of offset targeting will be brought out later.

In addition to the 63 pairs of offset targets just discussed, a total of 32 painted, single targets was established close to the boundary of the block to serve as *wing pass points* in order to help strengthen the photogram-

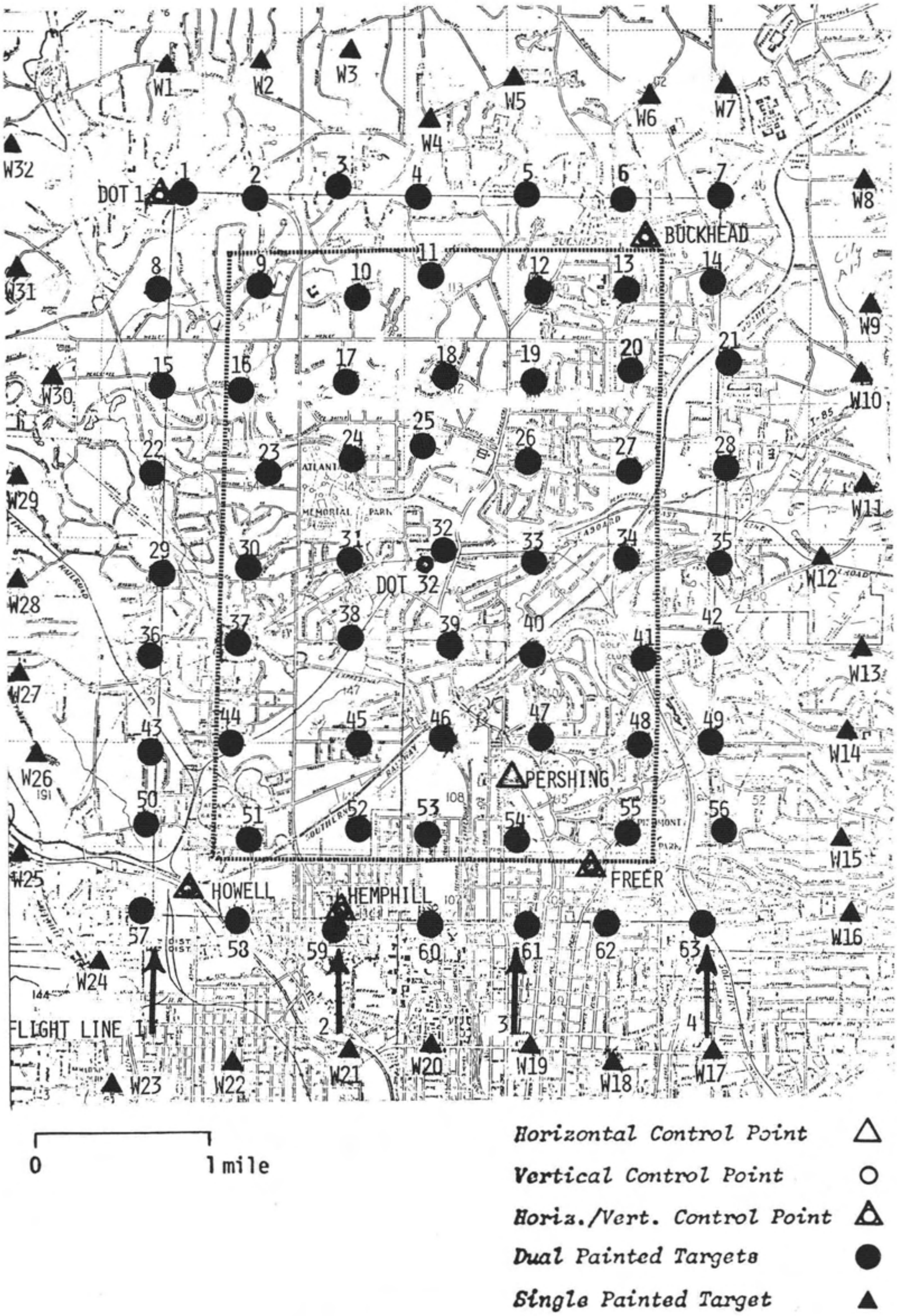

FIG. 3. Layout of Atlanta project.

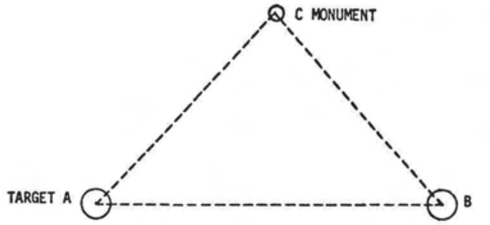

FIG. 4. Illustrating process of location of monument by means of measurements from nearby offset targets. Distance *AB* between centers of offset targets is pre-established to a convenient fixed value (30 ft in the Atlanta Project). Monument *C* is located approximately equidistant from *A* and *B* and subtends an angle close to the optimum value of 90°. After the monument has been so installed, distances *BC* and *AC* are taped and height differences betwen *A* and *C* and *B* and *C* are measured. From these quantities together with the photogrammetrically determined coordinates of *A* and *B* the coordinates of the monument can be computed.

metric adjustment. Horizontal control was provided by six paneled, first-order points indicated in Figure 2. With the exception of the station Pershing, vertical control coincided with horizontal control. In addition to this vertical control, a single vertical point (DOT 32) was epecially established near the center of the block by the Georgia DOT.

Photographic coverage of the test area was taken at a scale of 1:17,500 (altitude $\approx$ 4900 ft) with DBA's Zeiss RMKA 8.5/23 camera using the DBA 61-point reseau platen. With both forward and side overlap of 60 percent, each photo contained a rather uniform pattern of 25 points (or pairs of points). The photographs proved to be of excellent quality, and none of the painted targets was obscured. This demonstrated that near optimal distribution of targets is altogether practical in an urban area even when photography is performed with a super-wide-angle camera and thus answered one of the questions raised before the test.

At this point it is appropriate to explain the reason for the choice of a super-wide-angle mapping camera (f = 85mm) rather than the more conventional wide angle mapping camera (f = 150mm). When both cameras are flown at the same scale, they yield essentially equivalent accuracies in planimetry. However, because of its much more favorable base/height ratio, the super-wide-angle mapping camera can in theory be expected to produce heights that are nearly twice as accurate as those produced by a wide-angle mapping camera. This means that once the accuracy for height has been specified, a wide-angle mapping camera must be flown at twice the scale as a super-wide-angle mapping camera in order to produce equivalent accuracies in height; this in turn means that the photographic block generated by the former will contain four time as many photos as the block generated by the latter. Hence, the super-wide-angle camera hypothetically enjoys an enormous potential economic advantage over the normal wide-angle camera. In view of these considerations and because it was generally agreed that heights of an accuracy comparable to those to be expected for horizontal positions would be a most useful byproduct of the process of densification, the decision was made to employ a super-wide-angle camera in the Atlanta project. It remained to be seen, however, whether the theoretical superiority of the super-wide-angle camera with regard to accuracy of heights could indeed be realized in practice in such an exacting project.

As mentioned earlier, a *reseau* platen was employed in conjunction with the camera. This platen was developed by DBA Systems expressly for applications of photogrammetric triangulation demanding accuracies of the highest possible order. As the photograph is exposed, the *reseau* platen projects an array of 61 evenly distributed points through the backside of the film. These points, or *reseau* images, consist of small, well-defined dots of about 80μm diameter at the center of 'locating' circle of 500μm diameter. The precise locations of the *reseau* images are known by virtue of prior calibrations derived from exposures made on glass plates. The displacements of the *reseau* images on a given frame of film from their calibrated positions thus provide direct measures of film deformation and permit appropriate corrections to be made for the removal of this major source of systematic error. To the extent that film deformation is constant for all frames in a block, the bundle adjustment with self-calibration can actually account for the major part of the error. However, as Carman and Martin (1968) have shown, unless several anticipatory frames are sacrificed immediately prior to the onset of the photography of a given strip, the first several frames of the strip can display a degree of film deformation substantially different from the norm for the block. Thus, the *reseau* platen serves to eliminate errors introduced by any maverick frames. In the application to photogrammetric densification it also serves several other purposes that will be considered later.

As mentioned earlier, ground control for the 28 photo block was limited to the 6 horizontal and 6 vertical points indicated in Figure 2. The photogrammetric reduction was performed by DBA using the proprietary program for bundle adjustment with self-calibration, COMBAT II. Typically, a set of 125 points was measured on each frame. These consisted of 25 pairs of offset targets, 61 *reseau* images, and 4 fiducial marks. The coordinates of the monuments were computed from the triangulated coordinates of the offset targets by means of offset measurements supplied to DBA by the Public Works Department of the City of Atlanta.

After completion of the photogrammetric reduction DBA sent to the Georgia DOT the coordinates of all of the offset targets along with the computed coordinates of all installed monuments. By prior agreement it was then up to the DOT to perform a field check of the results. Except for the stipulation that only first-order methods were to be used, no restrictions were placed on DOT concerning the number of points to be checked or their dispostion. After examining the results submitted by DBA, DOT made a selection of the points to be checked and conducted the necessary surveys. A total of 18 horizontal checks were made by means of a first-order traverse. These corresponded to six triplets, each consisting of two painted targets and the associated monument. In addition, a total of nine vertical checks (i.e., three triplets) were made by leveling. The discrepancies between the photogrammetric and ground surveys of the check points were computed by the DOT and are summarized in Table 4. Separate breakdowns are given for targets and monuments, for the latter were not triangulated directly but, as mentioned above, were computed from the coordinates of the targets and the offset measurements. Except for being converted from feet to centimeters, the entries in Table 4 are exactly as provided by the DOT.

It is seen from Table 4 that root-mean-square (RMS) errors for the offset targets are 9.4, 6.9, and 4.2 cm in X, Y, and Z, respectively (these correspond to 5.4, 3.9, and 2.4 μm at photo scale). For the monuments the corresponding figures are 7.0, 4.1, and 2.8 cm, respectively (or 4.0, 2.4, and 1.6 μm at photo scale). The relatively higher accuracies obtained for the monuments are consistent with expectations, because errors in the coordinates of the offset targets are essentially averaged in the subsequent reduction leading to the coordinates of the monument from its offset measurements (errors in the offset measurements themselves are too small to be of any significance). With optimal geometry wherein offset targets are equi-distant from the monument and subtend a 90° angle, the theoretical improvement in accuracies for the horizontal coordinates of the monument over those of the offset targets is about 25 percent; for the vertical coordinate it is about 30 percent. This improvement in accuracies thus constitutes still another advantage of the offset target method.

Because of the smallness of the sample of check points (especially for vertical coordinates) one should not draw overly strong conclusions from these results. In particular, the extraordinarily good results for vertical accuracies must partially be regarded as fortuitous because they exceed theoretical expectations by almost a factor of two. The analytical error propagation performed in conjunction with the photogrammetric triangulation and carried through the computation of the positions of the monuments yielded the following results: offset targets, $\sigma_x,\sigma_y \approx 5.8$ to 7.3 cm (3.3 to 4.2 μm at photo scale), $\sigma_z \approx 6.4$ to 7.9 cm (3.7 to 4.5 μm); monuments, $\sigma_x,\sigma_y \approx 4.3$ to 5.8 cm (2.4 to 3.3 μm), $\sigma_z \approx 4.0$ to 5.5 cm (2.3 to 3.1 μm). Except for the overly optimistic outcome for vertical coordinates, results from the check points are seen to be fairly consistent with theoretical expectations. One rather firm conclusion to be drawn from the Atlanta Project is that the potential superiority of the super-wide-angle camera with regard to accuracies of vertical coordinates can indeed be realized through the use of the bundle adjustment with self-calibration.

In view of these results, the Atlanta project must be considered an outstanding and unqualified success. Photogrammetric accuracies from 1:17,500 scale photos of 7.0 cm (RMS) or better were demonstrated. The super-wide-angle camera was shown to be surprisingly well suited to the task of densification in urban areas. Accordingly, heights of useful accuracy can henceforth be deemed an attractive dividend of photogrammetric densification. The combination of the *reseau* platen and the bundle adjustment with self-calibration produced accuracies far in excess of those to be expected from other photogrammetric approaches. Finally, the utility, economy, attractiveness, and effectiveness of the method of offset targeting were solidly established. From a detailed analysis of all aspects of the project, a number of measures that could lead to greater efficiencies and still higher accuracies have been uncovered. It is therefore be-

TABLE 4. DISCREPANCIES AT CHECKPOINTS OF ATLANTA PROJECT.

				Mean Discrepancies (cm)		
		Point		ΔX (East)	ΔY (North)	ΔZ (Up)
H	O	10A		−16.0	0.7	–
O	F	10B		−17.4	3.0	–
R	F	11A		15.4	3.3	–
I	S	11B		1.1	− 7.3	–
Z	E	12A		4.7	4.6	–
O	T	12B		6.2	0.3	–
N		18A		3.4	9.7	–
T	T	18B		11.8	10.4	–
A	A	39A		− 3.4	0.2	–
L	R	39B		0.5	−15.7	–
	G	41A		2.1	− 1.0	–
C	E	41B		− 7.0	− 6.0	–
H	T		*RMS:*	9.4 cm	6.9 cm	–
E	S			(5.4 μm)*	(3.9 μm)*	
C						
K						
P	M	10		−14.0	4.3	–
O	O	11		5.5	− 2.3	–
I	N	12		3.9	1.1	–
N	U	18		6.9	5.1	–
T	M	39		− 2.2	− 7.3	–
S	E	41		− 1.7	0.3	–
	N		*RMS:*	7.0 cm	4.1 cm	
	T			(4.0 μm)*	(2.4 μm)*	
	S					
V	O	34A		–	–	0.9
E	F	34B		–	–	4.3
R	F	39A		–	–	−9.1
T		39B		–	–	1.4
I	T	47A		–	–	0.5
C	A	47B		–	–	1.8
A	R		*RMS:*	–	–	4.2 cm
L						(2.4 μm)*
C	M	34				2.5
K	O	39		–	–	−3.9
P	N	47		–	–	1.5
T			*RMS:*	–	–	2.8 cm
S						(1.6 μm)*

* At plate scale.

lieved that coordinates having standard deviations on the order of 5 cm are a reasonable expectation from future projects of a similar nature.

POSSIBLE BYPRODUCTS OF PROJECTS OF PHOTOGRAMMETRIC DENSIFICATION

The photographs used for densification have potential utility for other purposes. Two will be considered briefly here. The one concerns planimetric and topographic mapping and the other concerns the establishment of a computerized photogrammetric data base.

It is clear that the photos used for densification also can be used for mapping. What is perhaps not so clear is that such photos have a special potential for extraordinary accuracies in this application, particularly when the possibilities of their use in conjunction with an analytical plotter are examined. This stems from two considerations. First, the corrections for film deformation (from the reseau) and for residual systematic error (from error coefficients resulting from the bundle adjustment with self-calibration) can be directly exploited in the analytical plotter, thereby leading to direct improvement of accuracies. But beyond this is the fact that well-defined and ideally distributed points generated by 15 pairs of offset targets will appear in the typical model. Because the X,Y,Z coordinates of these points are known to extremely high accuracies by virtue of their prior use in densification, they can be exploited to generate analytical corrections to remove most of the residual deformations of the stereoscopic model. This can be accomplished in the following manner. If X,Y,Z denote the coordinates of a point as established in the process of densification and X',Y',Z' denote the corresponding coordinates of the points in the model of the analytical plotter, the systematic discrepancies between the two sets of points can be approximated by empirical polynomial expressions of the form

$$\begin{aligned}\Delta X &= X - X' \\ &= a_0 + a_1X + a_2X^2 + a_3XY + a_4Y^2 \\ &\qquad + \text{possible higher order terms} \\ \Delta Y &= Y - Y' \\ &= b_0 + b_1X + b_2X^2 + b_3XY + b_4Y^2 + \ldots \\ \Delta Z &= Z - Z' \\ &= c_0 + c_1X + c_2X^2 + c_3XY + c_4Y^2 + \ldots .\end{aligned}$$

As truncated, each of these functions involves a total of six unknown coefficients. It follows that from a minimum of six suitably distributed control points, a solution can be made for the error coefficients. However, since an average of some 15 pairs of control points would generally be available in each model, the coefficients describing the deformation of the model can be established with great accuracy from a strongly overdetermined, least-squares solution.

Once appropriate error coefficients for a given model have been generated as just described, they can be applied in the analytical plotter to effect the continuous correction of all points in the model. Because the influence of systematic error is thereby largely removed, the effective C-factor of the plotter is appreciably improved over its normal rating. When the foregoing process is used in conjunction with an advanced analytical plotter operating on photographs of outstanding quality and resolution, it may not be too far fetched to expect the realization of C-factors approaching 4800 for super-wide-angle photographs. This would mean that contours as fine as 1 foot could safely be plotted from photographs having a scale of only 1:17,500. In this connection it should be pointed out that as part of the Atlanta project a map with 2-foot contours was compiled by Airborne Data on a Kern PG-2 from a model developed from a pair of the 1:17,500 scale photos used for densification. The accuracy of the map was fully authenticated by means of field checks performed by the City of Atlanta. In this application the C-factor of the PG-2 (a second-order analog instrument) was boosted to 2400 from its normal rating of 1700 (for super wide angle photographs). Hence, the prospect for attainment of previously unheard of accuracies from the compilation by analytical plotters of models generated from photographs used for geodetic densification is promising indeed. The economic implications of an improvement in accuracies by a factor of two would be enormous, for it would mean that a mapping project could be performed from one fourth as many photos as would otherwise be employed. In many projects of densification where conjoint mapping with 1- to 2-foot contours is required, the savings to be expected from the implementation of the foregoing measures could be sufficient to pay for the densification itself.

The second potentially important use for the photographs generated for densification, namely, the establishment of an urban photogrammetric data base, would exploit the wealth of information captured in the photographs themselves. The concept of the photogrammetric data base is well advanced in military applications (McIntosh, 1974) and could equally well be exploited in civilian applications. This is rendered feasible by virtue of the fact that precise values for the elements of orientation of all photos and all necessary supplemental calibration data are available from the output of the photogrammetric adjustment performed for densification. This, in turn, means that by capturing such information on a storage medium accessible to a computer, one can at any later date determine precise coordinates of any photo-identifiable object located on a pair of overlapping frames. This entails the following steps:

(1) the object (or objects) to be located are first identified on two or more photos of the data base;
(2) with the aid of a relatively simple and inexpensive monocomparator, the photo coordinates of the object are measured along with those of the four surrounding reseau images (this is done on at least two different photos);
(3) the measured data from step (2) along with the identification of the photos and reseau images that were used are then entered into a computer (which could well be an inexpensive minicomputer) which then accesses the appropriate calibration data and performs a least-squares triangulation and associated error propagation; and

(4) the computer reads out the triangulated coordinates of the object along with estimates of accuracy.

The photogrammetric data base could, in principle, be employed directly by any urban agency having access to a copy of the data base and access to a suitable comparator for performing the measurements. The system of marks imprinted by the *reseau* platen on each photo greatly simplifies matters in two ways. First, it permits the use of a relatively compact and inexpensive comparator having a measuring range limited to the span of the *reseau* marks (typically 35 by 35 mm). Second, it permits an unlimited number of copies of the photographic data base to be reproduced by virtue of the fact that deformations introduced by reproduction can be automatically compensated for from the reduction of measurements of *reseau* images.

Fully and properly utilized, a computerized photogrammetric data base along the lines just described could well become indispensable to many urban agencies, particularly for preliminary engineering studies. Accurate coordinates could be obtained quickly and inexpensively for such photo-identifiable objects as utility poles, corners of buildings, manhole covers, radio and TV towers, water hydrants, street and sidewalk intersections, chimneys, steeples, water towers, intersections of fence lines, oil storage tanks, railroad facilities, and so on. From these, distances, bearings, areas, heights, clearances, and other useful measures can be developed.

Both the potential mapping and the data base applications of photographs employed for densification demand photographic resolution of the highest order. Here again the *reseau* platen can be useful, for it admits without accompanying metric degradation the possible use of thin-based (2.5 mil), high-resolution, reconnaissance type film such as Kodak Panatomic X. This film has been used in a *reseau* camera to very good effect in recent experiments in Australia reported by G. Roberts (private communication, 1976).

Practical Utilization of Photogrammetric Densificaton

The ultimate products of photogrammetric densification consist of X,Y,Z coordinates of monuments. In addition, coordinates can be generated for prominent, well-defined landmarks (towers, steeples, etc.). When a given monument is intervisible with another monument or with an appropriate triangulated landmark, it is possible to compute an azimuth reference for the station. However, it is not normally to be expected that this will be possible at each photogrammetrically established station. Indeed, a sizeable proportion of such stations will not have an azimuth reference. This is sometimes raised as an objection to photogrammetric densification, for azimuth references are a standard product of conventional surveying. However, it is so commonplace for such references to be destroyed over the course of years that a number of strategems or "tricks of the trade" have been developed to circumvent the loss of azimuth reference. Several of these are given in an NGS publicaiton authored by Dracup (1976). But beyond this the fact is that azimuth references are by no means necessary for the practical utilization of highly densified networks, nor are they even especially desirable. An appreciation for this statement merely requires a rethinking of the ultimate purpose of a highly densified net: it is simply to permit the timely and economical location of points within a local and relatively small cell (e.g., 800 by 800m). The traditional solution to this problem is accomplished by means of the method illustrated in Figure 5a. Here, it is assumed that the location of the point P is to be obtained by means of a traverse starting from Monument A and closing on Monument B. Because P is not necessarily directly intervisible with A or B, a few intermediate stations must be established, as shown. The measurements consist of the starting and closing azimuths at A and B, the distances between adjacent stations, and the angles subtended at each station by contiguous stations. In general, it can be seen that if a total of n intermediate stations are involved in the traverse, this process will generate a system of $2n + 3$ observational equations in $2n$ unknowns (an X and Y for each of the n stations). Thus there are three redundant equations and these provide a desirable internal check. But what if the two reference azimuths were not available? There would still be one redundant equation, and the problem could still be solved (actually, two solutions would then exist, one the mirror image of the other, and in practice there would be no difficulty in making the correct selection). It follows that the pair of reference azimuths are not strictly necessary and serve mainly to provide a useful internal check. But this can be accomplished in other ways, as is illustrated in Figure 5b. Here, the reference azimuths at A and B are dropped and a tie-in to a third monument C is made. Because the coordi-

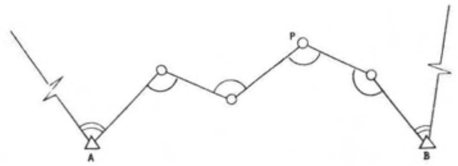

FIG. 5a. Schematic of conventional traverse with azimuth references at termini.

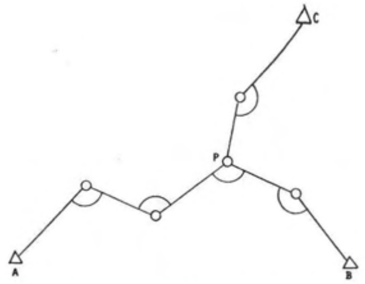

FIG. 5b. Schematic of alternative traverse without azimuth references but tied to three control points.

nates of C are known, this serves to restore the redundancy to the level of Figure 5a. Moreover, it can be argued that a check made by tying into three points is stronger and more effective than one based on azimuth closures. In addition, chances are that at least one of the three monuments A,B,C will have an azimuth reference, thereby further increasing the level of redundancy. As for the extra work involved in a three point tie, it is to be remembered that the distances involved in the problem at hand are small, typically only a few hundred meters. Then, too, most projects can be expected to require the location of several points, not just one. Under these circumstances, the extra work is not particularly significant.

We hope that the foregoing considerations will lay to rest the "azimuth objection" to photogrammetric densification once and for all. In the urban application it is a spurious objection that cannot survive scrutiny.

Cost and Time Estimates for Photogrammetric Densification

From guidelines drawn from the Atlanta Project it becomes possible to arrive at a reasonable estimate for the time and cost required for photogrammetric densification of the hypothetical 40 by 40 km area considered earlier. As with estimates based on traversing, the assumption will be made that monuments are to be installed at 800 m intervals, thereby generating a total (rounded) of 2600 monuments. The breakdown of major cost elements for the hypothetical project shown in Table 5. It will be noted that Items 2 and 3 together amount to \$247,000 or just over half the total cost of the project. Both of these items are of such a nature that they could be performed by the public works department of the community under the guidance of the photogrammetric contractor. Moreover, only the installation of the offset targets need be performed prior to the photographic flights. Installation of monuments could be performed, if desired, over a period of a few years following the photographic flights (for this purpose it is necessary only that the centers of the offset targets be recoverable). Such considerations could be helpful in the budgeting process, for it would mean (in the project under consideration) that only \$241,000 of the \$488,000 would need to consist of "outside funds" to be committed to the photogrammetric contractor. Moreover, of the remaining \$247,000 all but \$52,000 could be deferred to later fiscal years. Finally, a project of this nature would entail a natural and productive opportunity for support by public service employment, should such be sought.

TABLE 5. BREAKDOWN OF COSTS OF HYPOTHETICAL PROJECT OF PHOTOGRAMMETRIC DENSIFICATION.

Item	Estimated Cost
1. Planning, Reconnaissance and Coordination	30,000
2. Installation of Offset Targets: 2600 pairs @ \$20 each	52,000[1]
3. Installation of Monuments: 2600 @ \$75 each	195,000[2]
4. Photographic Flights: 676 photos @ \$60 each	41,000
5. Photogrammetric Data Reduction: 676 photos @ \$200 each	135,000
6. Project Management, Reporting and Miscellaneous	35,000[3]
TOTAL	\$488,000

[1] Includes cost of materials and cost of making offset measurements.
[2] Includes cost of materials.
[3] Includes cost of photographic prints and diapositives.

If the installation of the monuments is not considered, the time required for the hypothetical project would break down roughly as (a) planning and reconnaissance, 60 days; (b) installation of offset targets, 60 days (3 crews); (c) photographic flights, 10 days; (d) data reduction 120 days; and (e) reporting, 60 days. This totals to 310 working days or about 15 calendar months.

As compared with projections for densification by first-order traversing, the photogrammetric approach would cost \$488,000 versus \$1,345,000 and could be completed in 15 calendar months as opposed to 30 calendar months (for traversing by four field parties). After final adjustment, both methods could be expected to produce coordinates having relatively uniform standard deviations in the vicinity of 5 cm. However, relative accuracies of contiguous stations would be appreciably better in the case of densification by traversing (about 2 cm vs 7 cm). Offsetting this is the fact that heights accurate to 5 cm would be produced by the photogrammetric survey, whereas no heights of worthwhile accuracy would be produced by traversing. Moreover, as discussed earlier, mapping products and a photogrammetric data base could be developed from the very photographs employed for densification. When all factors are thus duly considered, the conclusion is overwhelming and inescapable that the task of densification can best be accomplished by photogrammetric means.

Inertial Surveying

As mentioned at the outset, another emerging technology that may well prove suitable in applications to urban densification is inertial surveying. The topic of inertial surveying is addressed in papers by Ball and Ives (1975), Huddle and Mancini (1975), and Gregerson (1975). The heart of this new method of surveying is an inertial navigational system originally developed for aircraft navigation. At present, units produced by Litton Corp. under the trademark *Autosurveyor* are commercially available for leasing from Span International Inc. of Phoenix, Arizona. Current leasing rates are understood to be in the vicinity of \$500 per hour. While this is expensive, indeed, the productivity of the system is potentially enormous. Major applications of the Autosurveyor to date have been in Alaska and Canada in the execution of long traverses, a task at which the unit excels. In operation, the Autosurveyor is simply initialized at a known station, stopping, as well, at fairly regular intervals of about four minutes for what is termed a *zero velocity update*. The fact that the unit is stationary during a zero velocity update provides information that is processed by the onboard computer in a real-time reduction that serves to estimate and partially compensate for the drifts of the inertial unit. Further compensation for drifts is made through apportionment of closing errors on the end point. In applications to singly run, long traverses the Autosurveyor has demonstrated accuracies of 1:20,000 to 1:30,000 after apportionment of closing error. However, by performing the traverse in both direct and reverse courses in an uninterrupted run and performing what amounts to a self-calibration of the unit from discrepancies arising from the two courses, Gregerson has been able to boost accuracies to the 1:50,000 to 1:100,000 range. In other words, it has been demonstrated that the Autosurveyor can, through proper procedure and data reduction, perform long traverses to first-order standards. In the short traverses of interest to urban densification (e.g., 800 m between stations) the basic level of the random error characteristic of the readout of Autosurveyor amounts to some 6 to 8 cm and is thus a significant handicap. Nonetheless, Autosurveyor (or its eventual successors) clearly has great potential for urban densification and should be accorded due consideration.

As a preliminary step in this direction, a cursory cost/time analysis has been undertaken using the same 40 by 40 km area considered in earlier exercises. The following working assumptions for inertial densification were adopted:

(1) first-order control points are available at 8 km intervals throughout the 40 by 40 km area (this divides the area into 25, 8 by 8 km units);
(2) along each of the four sides of each 8 by 8 km unit a direct and reverse traverse is performed (this involves a total of 960 km of traversing);
(3) from the reduction of (2), preliminary positions are established for monuments at 800 m intervals along the sides of the 8 by 8 km units;
(4) within each 8 by 8 km unit direct and reverse runs are made as indicated in Figure 1b of segments joining corresponding monuments on opposing sides (this divides each unit into 800 by 800 m cells and entails a total of 288 km of traversing per unit, or a grand total of 25 by 288 = 7200 km of traversing); and
(5) the preliminary locations from (3) and (4) are refined in a rigorous least-squares adjustment solving simultaneously for the x,y,z coordinates of all $51 \times 51 = 2601$ stations (a

total of $3 \times 2601 = 7803$ unknowns) plus appropriately revised sets of error coefficients for each of the 510 segments traversed (if 4 coefficients are each exercised for X,Y, and Z, respectively, this would generate $3 \times 4 \times 510 = 6120$ additional unknowns and the total system of normal equations would be of order 13,923).

It will be noted that the final simultaneous adjustment stipulated in step (5) follows the same exacting standards of rigor as were adopted in the corresponding conventional and photogrammetric analyses. It is believed that only in this way would the Autosurveyor stand a chance of approaching its two competitors in overall accuracy. Thus, the data reduction for the Autosurveyor is fully as extensive and as formidable as for the other two methods and accordingly can be expected to cost in the neighborhood of \$150,000. The costs associated with planning, reconnaissance, and monumentation can be assumed to be essentially equivalent to those of the photogrammetric method method and thus total \$30,000 + \$195,000 = \$225,000. Program management, reporting, and miscellaneous expenses are also likely to correspond to those of the photogrammetric method and thus amount to about \$35,000. All that remains, then, is to estimate the productivity of the Auto-surveyor. Here, a reasonable estimate might well be ten stops per hour, each stop being at a monument and also serving as a zero velocity update. On this basis, a double run along an 8 km segment (a total of 21 stops at monuments) could be accomplished in a 2-hour period, and the total time required for the direct and reverse traverse of all 510 segments in the project would amount to 1020 hours. With a modest allowance of 5 percent for reruns this would be increased to 1071 hours. At the current lease rate of approximately \$500 per hour, the cost of the Autosurveyor would thus amount to \$536,000. When this is added to the other figures, one arrives at a grand total of \$946,000, or approximately twice as much as the \$488,000 estimated for photogrammetric densification but less than the \$1,345,000 estimated for conventional densification.

Of course, the figure just arrived at is largely dependent on a specifically assumed level of productivity of the Autosurveyor and on its current lease rate. At the assumed productivity of ten stations per hour, it can be calculated that a lease rate of about \$70 per hour would be required for inertial densification to compete with photogrammetric densification. On the other hand, if assumed productivity were increased to 15 stations per hour, a lease rate of about \$105 per hour would be competitive. While both of these figures are far below current rates, it must be appreciated that high technology is typically expensive in its early stages but is also subject to rapid cost reduction with acceptance in the marketplace and development of competitive pressures. Accordingly, a fivefold reduction in hourly rates over, say, a five-year period may not be too much to expect. Until such a reduction is achieved, it seems unlikely that inertial surveying will be competitive with photogrammetric surveying in large-scale projects of urban densification. Even then, photogrammetry may hold a decisive edge in projects where weight is given to mapping byproducts or to establishment of a photogrammetric data base.

Conclusions

At present, photogrammetric technology at its most advanced level can achieve accuracies in urban densification approaching a standard deviation of 5 cm in all three coordinates from reduction of super wide angle photographs of 1:17,500 scale. Benefit-to-cost ratios of such densification probably exceed 20 to 1.

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