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A Cost Model for Remote Inspection of Ground Sites*

A model for the cost/effectiveness of remote sensing for inspection of ground sites, given that a good site may be classified as bad or a bad site as good, is presented.

BACKGROUND AND MOTIVATION

THE MODEL DEVELOPED here is an outgrowth of studies conducted at Mathtech on the technology and economics of surface mining in the Commonwealth of Kentucky. Surface mining has been the subject of heated debate between environmentalists and pro mining interests in the United States due to past abuses and the prospect of continued environmental impacts. The result of this controversy has been the enactment of numerous laws designed to curb these excesses while permitting "responsible" mining activities. Critical to the success of this legislation, however, is the capability to monitor and control compliance.

At present these laws have been enforced by inspectors periodically visit each mine. Advances in remote sensing technology now facilitate aerial (satellite or high altitude imagery) inspection for detection of the scale of mining operations, landslides, revegetation failures, unauthorized mining operations, and other prohibited activities.[1,2,3,4,5]

ABSTRACT: *A simple model is developed to assess the cost/effectiveness of remote sensing for inspection of ground activities with application to the inspection of surface mining operations. The model considers both single and multiple tier systems (those in which additional inspection is conducted if a higher order tier indicates a violation), possible trade-offs between α and β errors, the worth of technological improvement, and certain fixed cost aspects of manned inspection. An interesting conclusion of this analysis is that (with current cost factors) satellite and aircraft inspection systems can be cost/effective even with relatively high (0.15 - 0.25) α and β errors.*

This paper presents a framework for determination of the cost/effectiveness of this technology for such applications. The framework is illustrated with several numerical examples. Though imputs to these examples are plausible and correct to within an order of magnitude, they have not been verified experimentally. Hence the emphasis of this paper is on illustrating methodology. With accurate data the model should be useful for policy analysis.

MODEL DEVELOPMENT

Let us assume that in an area to be inspected there are N sites at which surface mining is taking place. Prohibited activities are occurring at N_1 of these sites while, at the others, the prescribed regulations are being met. The exact number and locations are, of course, unknown at each inspection period. The inspectors are responsible for ensuring

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that proper mining practices are maintained and, therefore, need to know the least cost way of performing the investigating activity.

It is assumed for illustrative purposes that manual inspection always results in a correct determination of whether or not prohibited activities are occurring. The cost of manned inspection is relatively high. The cost of inspection using via remote sensing is used, either by spacecraft or aircraft, the possibility of misclassification arises. In this case there exist four possible outcomes:

(1) a site at which a violation exists is so identified (a "bad" site is correctly identified);
(2) a site which is being mined according to proper standards is so identified (a "good" site is correctly identified);
(3) a "bad" site is classified as "good," that is, a problem is undetected; or
(4) a "good" site is classified as "bad," implying unnecessary follow-up inspection.

The last two possibilities are known as the beta, β, and alpha, α, errors, respectively, and are indigenous to any decision procedure where there is less than complete information.

Figure 1 depicts the structures of inspection systems having one-tier and a two-tier structure. The present ground inspection system has a one-tier structure. In this case all sites, whether problem areas or not, are examined by inspectors and, consequently, are all correctly classified. Both the spacecraft/ground and aircraft/ground inspection systems have a two-tier structure. In these cases, however, there is a probability, β, that a decision rule depending on an aerial inspection will judge a problem area as a no-problem area, and a probability, α, that the rule will judge a no-problem area as a problem area. The subscripts, s and a are used to denote parameters associated with satellite and aircraft systems, respectively. Thus, for example, β_s denotes the β error associated with the satellite inspection.

Under the two-tier concept inspectors are dispatched only to areas judged to be suspect by aerial inspection. It is assumed that manned inspection of problem sites is necessary not only to initiate sanctions but also to initiate damage limitation measures. A consequence of the alpha-error is that the expected number of unnecessary manned inspections is $(N - N_1)\ \alpha$. The impact of the beta error is that the expected number of undetected problem areas is $N_1\beta$ (see Figure 1). Both kinds of site misclassifications introduce associated cost penalties. The actual values of the error probabilities depend upon the technical characteristics of the remote sensing system and, consequently, the technology that is available. In the limit we might theoretically design and implement a remote sensing system that, like the ground inspection system, is error free. Of course, the decision to implement such a system would depend on the costs, both non-recurring and recurring, that would have to be paid for such a system.

The structure of a three-tier inspection system is shown in Figure 2. The satellite/aircraft/ground inspection system has this structure. A decision rule provides that aircraft will be called in only after the spacecraft has classified an area as a problem area though other schemes are possible; see the appendix for details. The expected number of problem areas judged as non-problem areas is

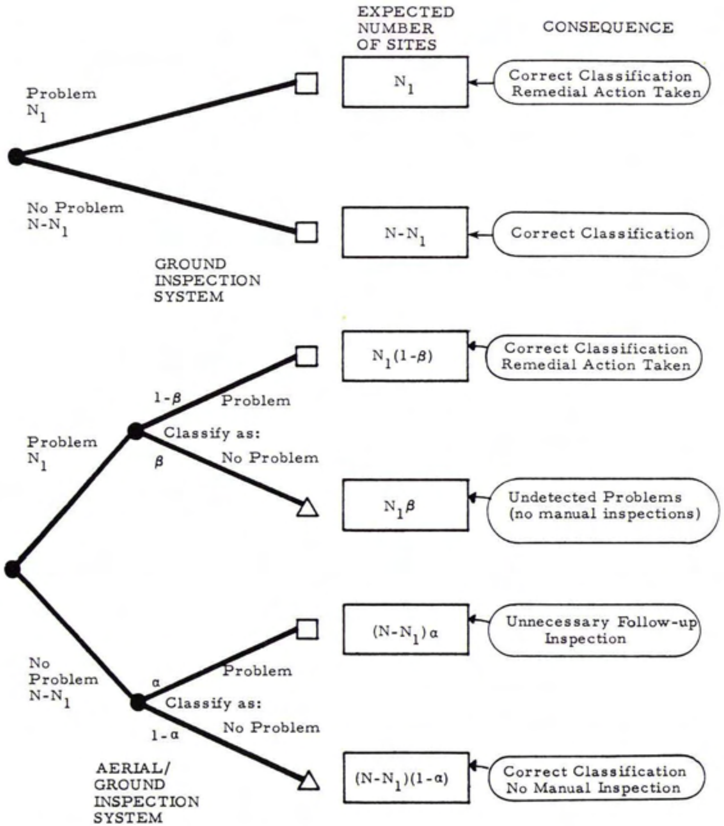
$$N_1\beta_s + N_1(1-\beta_s)\beta_a \tag{1}$$

and the expected number of non-problem areas misclassified as problem areas is

$$(N - N_1)\alpha_s\alpha_a\ . \tag{2}$$

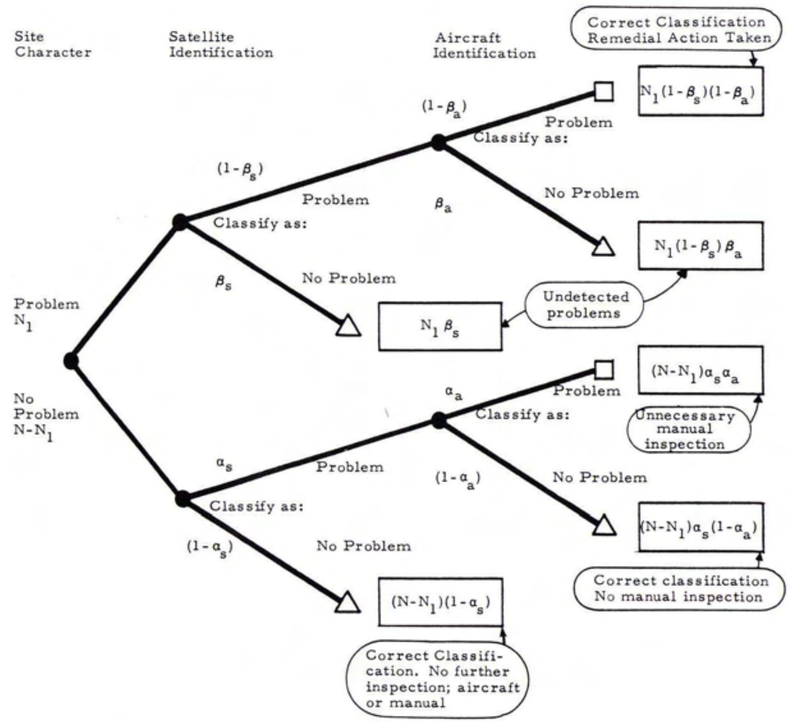
Equations 1 and 2 imply that errors associated with satellite and aerial inspection are statistically independent, i.e., the probability that a follow-up aerial inspection misclassifies a site is not a function of the satellite identification. Should this not be the case, it is necessary to introduce conditional error probabilities, e.g., let α_a be the probability of aircraft false positive given satellite false positive, etc. With this interpretation the expectations shown in Figure 2 remain correct, though appropriate numerical inputs may vary. Analogous to the two-tier systems, the attending misclassification penalty costs are unnecessary aircraft and manned inspection costs and the cost of undetected problem areas.

Other two- and three-tier inspection policies can also be envisioned. Examples include the possibility of multiple follow-up aerial inspections with identical or distinct remote sensing devices whenever an apparent problem is detected. The approach to



NOTE: Nodes represented by triangles indicate that no further inspection is conducted. Nodes represented by squares indicate manual inspection is conducted. The expected number of sites in each category is enclosed in each box.

FIG. 1. Structures for model development, the one- and two-tier inspection system.



NOTE: Nodes represented by triangles indicate that no further inspection is conducted. Nodes represented by squares indicate manual inspection is conducted. The expected number of sites in each category is enclosed in each box.

FIG. 2. Structure of a three-tier inspection system.

analysis of these cases is similar to that presented here for the simple case and will not be explored further.

From the decision models shown in Figures 1 and 2, the cost functions presented in Table 1 may be derived. It was assumed that satellite inspection represents a fixed cost if used; the incremental costs are regarded as zero. For certain kinds of satellite photo interpretation and processing (e.g., preparation of "ink squirt" mosaics, or cluster analysis classification) the variable or per site cost may be appreciable, necessitating a change in term 1 of Table 1 from $C_s X_s$ to $(C_s + C_s' N) X_s$, where C_s' is the variable cost per site and C_s, as before, is the fixed cost component. The aircraft inspection costs, as shown in Table 1, may be derived directly from the structures in Figures 1 and 2, and, as shown, depend on the decision model (i.e., whether or not aircraft inspection follows a determination by satellite inspection that there is a problem area). The number of "tiers," or combinations of inspection schemes, are provided for in the cost model by the binary variables X_s and X_a, their values depending on whether or not spacecraft and aircraft systems are being used, respectively. If spacecraft are used, for example, then X_s would be one. If spacecraft are not used, then X_s would be zero.

The third cost factor, "false negatives," indicates the social cost of a beta-type error. It includes the social and economic cost of nondetection and, by implication the noncorrection of a problem area. Some of the cost of nondetection results from the probability of physical damage, the value of which can be estimated. Other costs, however, are for non-market goods and activities. The values of these goods are difficult to determine and could theoretically range from zero to infinity, depending upon the imputation of the social costs incurred due to misclassification. In this context C_p can be looked upon as a policy variable. High values assigned to C_p will shift inspection policies to those with fewer false negatives.

Some Illustrative Examples

In Table 2, several sets of assumed values are given to the parameters discussed so far, and the alternative inspection policies are compared depending upon the values of the parameters. Policy 1 (P_1) assumes a man-only investigation and, therefore, with an assumed price of \$50 per site, and a thousand sites, there is an invariant cost of \$50,000 to investigate all of the locations. Policy 2 (P_2) assumes that ground investigation occurs only after it is determined by satellite that a site is a problem area. Policy 3 (P_3) assumes that ground inspection occurs only after it is determined by aircraft that a site is a problem area. Policy 4 (P_4) assumes that men are called in to investigate only after it is determined both by satellite and aircraft that a site is a problem area.

In Table 2, the costs of implementing the four inspection plans are computed under two sets of assumptions of α and β errors for aerial inspection. Holding all other parameters constant, it is seen that the costs, and consequently the choices, of the alter-

Table 1. Composite Cost Function for Inspection Policies.

Cost Factor	Value
(1) Satellite Inspection	$C_s X_s$
(2) Aircraft Inspection	$C_a \left[X_a(1-X_s)(N) + X_s X_a\left((N-N_1)\alpha_s + N_1(1-\beta_s)\right)\right]$
(3) False Negatives	$C_p\left[X_s(N_1\beta_s) + (1-X_s)(X_a)(N_1\beta_a) + X_s X_a\left(N_1(1-\beta_s)\beta_a\right)\right]$
(4) Manual Inspection	$C_m\left[(1-X_s)(1-X_a)(N) + X_s(1-X_a)\left((N-N_1)\alpha_s + N_1(1-\beta_s)\right) + X_a(1-X_s)\left((N-N_1)\alpha_a + N_1(1-\beta_a)\right) + X_s X_a\left((N-N_1)\alpha_a\alpha_s + N_1(1-\beta_a)(1-\beta_s)\right)\right]$

where

C_m = cost/site inspected manually.

C_s = cost of satellite inspection.

C_a = cost/site inspected by aircraft.

C_p = cost problem area *not* detected

α = probability "good" area is misclassified as problem area.

β = probability problem area is misclassified as good.

X_s, X_a = integer variables to denote whether satellite or aircraft inspection is used

$$X = \begin{cases} 1 & \text{system used} \\ 0 & \text{system not used} \end{cases}$$

native inspection policies are very sensitive to the alpha and beta risks associated with aircraft and spacecraft. When the alpha risk is relatively high (0.2 as compared with 0.1), then an increased cost would be incurred for re-inspecting sites which are, in fact, not problem areas. Also, there is a high likelihood of incurring the social cost of not detecting problem sites when the beta risk is relatively high. The asterisks in Figure 4 identify the optimal policies in each case. It is seen that even if the alpha and beta risks are relatively high, the three-tier and two-tier inspection systems are economically preferred over manual inspection only. Such computations demonstrate that imperfect systems can be cost effective provided the operating costs and penalty costs are sufficiently small. Whether or not this is true, in fact, will depend upon the particular application of the model and the inputs appropriate to the application. In general, if the number of problem areas, the social cost of misclassification C_p, or the α and β errors are high, the optimal policy is ground inspection only. This results from the expectation of incurring substantial social costs for undetected problem sites. When the alpha and beta risks are relatively low and equal for aircraft and spacecraft systems, the policy P_2, a satellite/ground system, is preferred. This results from the fact that the satellite system costs are less than the aircraft system costs.

Figure 3 maps other information about the systems onto a graph in which the horizontal axis represents the parameter N_1, the number of defective areas, and the vertical axis represents the total cost of the alternative inspection programs. The values of the parameters other than N_1 are given in the top half of Table 2 in runs 1 through 3. The efficiency frontier that has been drawn indicates the lowest cost strategy as a function of the number of defective areas in the actual population. Any policy other than the one indicated for a given value of N_1 is inefficient from an economic standpoint. At values of N_1 less than 15, the three-tier plan, P_4, is the most cost /effective approach. Above that, up to about 39 defective areas, the man/spacecraft approach is the most cost/effective; from 40 to approximately 95, the aircraft/man plan is preferred; and above 95, a man-only plan is the cost/effective approach. The shape of the efficiency frontier depends upon the value of the parameters. At the limiting case of C_p equal to infinity where no beta risks are tolerated, either a man-only system or an enhanced remote sensing system will be chosen, assuming that the technology is available to reduce β_a or β_s to zero. The choice would depend upon the relative costs of these systems.

We have seen that a simple model can be used to assess the economic impact of an

TABLE 2. ILLUSTRATIVE RESULTS WITH SIMPLE SURVEY MODEL.

DIRECT COMMON COST FACTORS

Ground (men) Cost per site $C_m = 50$	Satellite Cost $C_s = 200$	Aircraft Cost per site $C_a = 15$	Cost of Misclassification per site by β-type error $C_p = 2000$

Run No.	Input Factors						Cost of Survey ($1,000)			
	N	N_1	α_s	β_s	α_a	β_a	P_1	P_2	P_3	P_4
1	1000	10	0.20	0.25	0.05	0.15	50	15.5	20.9	11.3*
2	1000	50	0.20	0.25	0.05	0.15	50	36.6	34.5*	41.9
3	1000	100	0.20	0.25	0.05	0.15	50*	62.9	51.4	80.1
4	1000	10	0.10	0.10	0.05	0.05	50	7.6	18.9	5.4*
5	1000	50	0.10	0.10	0.05	0.05	50	17.2*	24.7	19.1
6	1000	100	0.10	0.10	0.05	0.05	50	29.2*	31.9	36.4

Costs of Optimal Choices are Denoted with an Asterisk(*)

N = Total Number of Sites
N_1 = Number of Defective Sites
α = Rate of Occurrence of α-Type Errors
β = Rate of Occurrence of β-Type Errors
$(\ )_a$ = ( ) for aircraft
$(\ )_s$ = ( ) for satellite

P_1 = Ground (men) only
P_2 = Satellite + Ground
P_3 = Aircraft + Ground
P_4 = Satellite + Aircraft + Ground

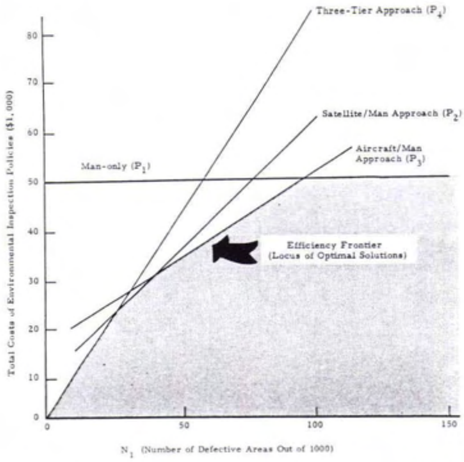


FIG. 3. Costs of alternative policies as a function of the number of defective areas (run 1, 2, 3).

important technical characteristic of remote sensing systems, the system accuracy on the selection of a cost/effective system. Another technical aspect of the remote sensing system which influences the choice of the most cost/effective inspection mode is that of system availability (the fraction of the inspections which can be completed). This system characteristic is influenced by many factors, some of which are related to the system design, but most of which are exogenous to the system, chiefly weather conditions. The potential impact of system availability on the choice of the economically optimum inspection mode is an important issue which is explored in the next section.

THE EFFECT OF SYSTEM AVAILABILITY

Denote by f the fraction of sites which can be examined by remote sensing. On a period to period basis the quantity f is a random variable with values ranging from zero (system failure, complete cloud cover, etc.) to one (system functioning, weather clear). How does this affect the selection of policy choices identified earlier? The answer depends upon what assumptions can be made *vis a vis* the behavior of manual inspection costs with workload. In the previous section a constant cost per site inspected, C_m, was assumed. This assumption would be correct if inspectors had other duties (e.g., consulting, permit application review, research projects, etc.) which could be performed when inspections were not required. In this event the appropriate modification to the cost expression is (here illustrated for the satellite-man system)

$$C_s + \bar{f} N_1 \beta_s C_p + \left[\bar{f}\left((N-N_1)\,\alpha_s + N_1\,(1-\beta_s)\right) + (1-\bar{f})N\right] C_m. \qquad (3)$$

The first term is satellite fixed cost as before. The second term is the expected satellite false negative cost. If $\bar{f}$ is the expected fraction of sites classifiable by satellite, then $\bar{f}N_1$ is the expected number of problems areas among those classifiable, etc. The third term includes the expected number of inspections required from satellite identification, $\bar{f}((N - N_1)\ \alpha_s + N_s\ (1 - \beta_s))$, plus the expected number of sites not observed by satellite which must be normally inspected, $(1 - \bar{f})\ N$. For given values of α_s, β_s. N, N_1, C_s, C_p, and C_m, Equation 3 can be used to calculate a "breakeven" value of f above which the satellite is a cost/effective component of the inspection system.

If, however, inspectors do not have other duties which can absorb the work load fluctuations resulting from f being a random variable, then certain fixed cost aspects of manual inspection must be reflected in the analysis. This section presents a model relevant to this situation. First, some notation is introduced: Let M be the number of inspectors to be employed and r be the number of inspections per period per man in a "regular time" workday. Then an inspection work force of M inspectors can perform rM inspections per period at a total fixed cost of $C_m rM$ per period. We also assume some "overtime" inspections can be performed, each at a variable cost C'_m where by hypothesis $C'_m > C_m$. Let ψ denote the ratio of maximum possible overtime inspections to regular time inspections. Given these definitions, if z manned inspections are required of a work force of M inspectors, the manned inspection costs are

$$\begin{array}{ll} C_m rM & \text{if } 0 \leq z \leq rM \\ C_m rM + C'_m(z-rM) & \text{if } rM \leq z \leq rM(1+\psi), \end{array}$$

for values of z greater than the maximum number of regular time and overtime inspections $rM(1+\psi)$, sites must go uninspected and penalty costs are incurred on those sites which are problem areas. Assume first that the availability, f, is known and a constant. To a first approximation* the analog to Equation 3 that incorporates the above fixed cost aspects of manned inspection is

$$\begin{aligned} C(M|f) = {} & C_s + C_m\, rM + \min\,(C'_m\, \psi\, Mr,\ \max\,(0,\ C'_m\,(z-Mr))) \\ & + \max\,(0,\ (z-(1+\psi)\,Mr)\frac{N_1}{N}\,C_p) + fN_1\,\beta_s\,C_p \\ & - \min\,(\max\,(0,\ rM-z),\ N-z)\,C_p\,N_1\,\beta_s/(N_1\,\beta_s + (N-N_1)\,(1-\alpha_s)) \end{aligned} \tag{4}$$

where $z = f(N_1\,(1-\beta_s) + (N-N_1)\,\alpha_s) + (1-f)N$, the expected number of inspections required, and $C(M|f)$ is used to explicitly denote the cost as a function of the number of inspectors for a given availability, f, and is a function *inter alia* of the decision variable M. The first term in Equation 4 is, as before, the fixed cost of satellite inspection. The second term, $C_m rM$, is the fixed cost of a force of M inspectors. The third term represents the cost of overtime inspections. The fourth term is the expected penalty cost associated with any sites that go uninspected, *viz* if $z > (1+\psi)Mr$, then $z - (1+\psi)Mr$ sites must go unexpected. The expectation fraction of these which are "bad," is N_1/N. Hence, $C_p(z - (1+\psi)Mr)\,N_1/N$ is the penalty cost of uninspected sites. The fifth term, $fN_1\,\beta_s\,C_p$, is the penalty costs associated with satellite false negatives. The sixth term needs some explanation. It represents some cost recoveries that can be obtained in circumstances when the required number of inspections, z, is less than those available, rM. In this event, inspectors could be assigned to inspect sites which have been classified as non-problem areas and correct some of the satellite false negatives. There are $\min(rM-z,\ N-z)$ such possible inspections to be made (assuming $z<rM$). The expected fraction of these which are problem areas (given satellite classification as non-problem areas) is easily obtained from Bayes' theorem as $N_1\,\beta_s\,/\,(N_1\,\beta_s + (N-N_1)\,(1-\alpha_s))$, from which the last term follows.

For fixed availability, Equation 4 can be used to determine the optimal number of inspectors. Careful examination of Equation 4 reveals that the optimal number of inspectors, M^*, is

$$M^* = \begin{cases} N/r & \text{if } C_m(N-z) < fN_1\,\beta_s\,C_p + C_s \\ z/r & \text{if } C_m(N-z) > fN_1\,\beta_s\,C_p + C_s. \end{cases} \tag{5}$$

The economic interpretation of Equation 5 is intuitively appealing *viz*, the work force should be sized to meed the expected inspection demand from satellite classification if the savings in inspection cost if a satellite is employed, $C_m(N-z)$ are greater than the sum of those costs occasioned by use of the satellite, $C_s + fN_1\,\beta_s\,C_p$. To illustrate the above, suppose that $r = 30$, $C'_m = 75$, $\psi = 0.25$, $f = 0.5$, and other parameters are as shown in run 5 of Figure 4. In this case $C_m(N-z) = \$21{,}500$ which is greater than $fN_1\,\beta_s\,C_p + C_s = \$5{,}200$, so from Equation 5 $M^* = z/r$ or 19 inspectors are optimal. The expected period cost of this policy from Equation 4 is \$33,700. (A computation treating z as a random variable, as suggested in the previous footnote, yields a true cost which differs by less than 0.4 percent from that computed from Equation 4.) Figure 4 shows a family of optimal solutions to the above example as f varies from zero to one. The important question is not, of course, what is the optimal number of inspectors for a fixed

* Strictly speaking, $C(M|f)$ is not a conditional expectation because the random variables x = number of satellite true positions and y = number of satellite false positions have been replaced by their expectations $fN_1(1-\beta_s)$ and $f(N-N_1)\alpha_s$ respectively, in Equation 4. x and y are independent binomially distributed for fixed N_1 and N and the proper form of Equation 4 follows directly. In practice, for parameter values, as illustrated in the examples throughout, the error introduced by the simplified form of Equation 4 is negligible.

value of f, but rather what is the optimal policy for a given distribution of f. That is, what is the value of M which minimizes the expected cost, or

$$\min_{M} \sum_{i} C(M|f_i)\, p\,(f_i)$$

if f is assumed to be a discrete distribution. Cloud cover data are collected and reported in such a form for numerous locations throughout the world, so the inputs to the above

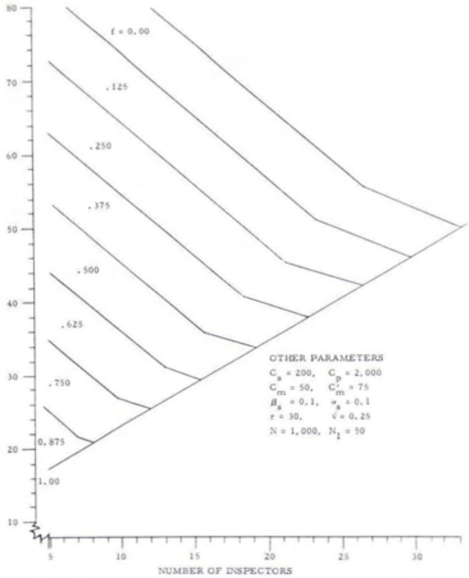


FIG. 4. Optimization of number of inspectors as a function of a satellite availability.

TABLE 3. ILLUSTRATIVE OPTIMIZATION OF NUMBER OF INSPECTORS GIVEN DISTRIBUTION OF SATELLITE AVAILABILITY DUE TO COUD COVER.

Number of Inspectors	Cost if Fraction of Sites Observable is									Expected Cost
	0	1/8	2/8	3/8	4/8	5/8	6/8	7/8	8/8	
5	91,763	82,263	72,623	63,263	53,763	44,263	34,763	25,263	17,584	53,357
7	88,388	78,888	69,388	59,888	50,388	40,888	31,388	22,263	19,886	50,923
9	85,013	75,513	66,013	56,513	47,013	37,513	28,013	22,188	22,188	49,059
11	81,638	72,138	62,638	53,138	43,638	34,138	26,075	24,491	24,491	47,815
13	78,263	68,763	59,262	49,763	40,263	31,388	26,793	26,793	26,793	46,868
15	74,888	65,388	55,888	46,388	36,888	29,888	29,095	29,095	29,095	46,141
17	71,513	62,013	52,513	43,013	35,200	31,398	31,398	31,398	31,398	45,616
19	68,138	58,638	49,138	40,513	33,700	33,700	33,700	33,700	33,700	45,180
21	64,763	55,263	45,825	39,012	36,002	36,002	36,002	36,002	36,002	44,913
23	61,388	51,888	44,325	38,305	38,305	38,305	38,305	38,305	38,305	44,869 Optimal Point
25	58,013	49,638	42,825	40,607	40,607	40,607	40,607	40,607	40,607	45,143
27	54,950	48,138	42,909	42,909	42,909	42,909	42,909	42,909	42,909	45,740
29	53,450	46,638	45,212	45,212	45,212	45,212	45,212	45,212	45,212	46,826
31	51,950	47,514	47,514	47,514	47,514	47,514	47,514	47,514	47,514	48,268
33	50,450	49,816	49,816	49,816	49,816	49,816	49,816	49,816	49,816	49,924
35	52,700	52,700	52,700	52,700	52,700	52,700	52,700	52,700	52,700	52,700
	0.17	0.15	0.10	0.05	0.03	0.05	0.10	0.20	0.15	0.51
	Assumed Distribution of Cloud Cover									Expected Cloud Cover

equation are generally easy to obtain. Minimization of this equation is easily accomplished by enumeration as shown in Table 3 for an assumed cloud cover distribution. The mean cloud cover fraction of the distribution is about 0.5. However, the distribution of f is such that the minimum expected cost is about \$45,000 (corresponding to M^* of 23); much closer to the man-only policy. Thus, if the fixed cost representation of manned inspection is appropriate to the actual application and if appreciable probabilities of high cloud cover exist, the relative cost advantage of satellite man systems narrows sharply. The above model enables explicit determination of optimal policies in these circumstances. (A simple extension enables N_1 to be treated as a random variable.)

We return now to the variable cost representation of manned inspection (i.e., C_m per site) for further analysis.

Some Sensitivity Analyses

Table 4 contains the result of a sensitivity analysis for run 2 of Table 2 to explore the parameter ranges over which policy P_3 is optimal. For each parameter it shows the lower and upper limits and the policies which become optimal beyond the intervals. For example, the ground inspection cost can vary over a wide range from \$35 to \$203. Policy P_2 requires more ground inspections and, consequently, benefits more from a lower inspection cost. Conversely, policy P_4 requires fewer men and suffers less from an increased cost. The satellite cost presents a different situation. Reducing the cost helps P_2 and P_4 but, since at most only \$200 can be saved, it is not sufficient to make either of these policies optimal. P_1 and P_3 are not dependent on the satellite cost so there is no change in their relative status and we see that P_3 is optimal over the full range of C_s. A similar review can be made for each of the other parameters, showing when and why each range limit and policy shift occurs.

Perhaps the most interesting entries in Table 4 are those associated with the satellite and aircraft α and β errors. Note that for this example a small increase in the aircraft β error from 0.15 to 0.171 changes the optimal policy from aircraft/ground to satellite/ground. A decrease in satellite β error from 0.25 to 0.228 produces an equivalent result. In practice this will act to mask the optimal policy since it is unlikely that these parameters are likely to be known with such precision.

Variations in Errors

As in all problems of statistical hypothesis testing, there exists a choice of decision rules, sample size, processing technology, and the like which affects the magnitude of the α and β errors. For a fixed technology and sample size it is generally the case that the $\alpha(\beta)$ error can be decreased only at the expense of increasing the $\beta(\alpha)$ error, i.e., there is a tradeoff between α and β errors. Since in this case it is possible to explicitly determine the cost consequences of these respective errors, it is possible to determine optimal values for α and β. This problem is explored in what follows.

The "power law" equation below is capable of representing a wide variety of possible relations between the α and β errors;

$$\alpha^{\gamma}\,\beta^{\omega} = T^{\gamma+\omega} \tag{6}$$

where γ and ω are exponents which characterize the rate at which α and β errors vary. Note that for this model,

$$\frac{(d\alpha/\alpha)}{(d\beta/\beta)} = \frac{\omega}{\gamma}$$

for T equal to a constant which characterizes the "technology" or scales the α and β errors. Figure 5 illustrates the variety of possible α,β tradeoffs possible with this relationship for various values of γ,ω, and T. For the case where $\gamma = \omega = 1$ Equation 6 is a hyperbolic curve with the property that reducing one probability by a factor of 50 percent results in a doubling of the other probability. This particular choice of parameters is for illustrative purposes only; empirical tradeoff curves must be determined for each application.

This information combined with the earlier derived cost equations allows us to determine the optimal values of α and β to be used and consequently how to establish optimal acceptance criteria for the aircraft and satellite inspections. As an example, consider the cost expression for the satellite and man inspection system:

TABLE 4. ILLUSTRATIVE SENSITIVITY ANALYSIS.

Base Case (Run 2):

DIRECT COMMON COST FACTORS

Ground Cost per Site (men)	Satellite Cost	Aircraft cost Per Site	Cost of Misclassification per site by β type error
$C_m = 50$	$C_s = 200$	$C_a = 15$	$C_p = 2000$

Incidence Factors

Sites	Defective Sites
$N = 1000$	$N_1 = 50$

Misclassification Factors

Satellite		Aircraft	
$\alpha_s = 0.20$	$\beta_s = 0.25$	$\alpha_a = 0.05$	$\beta_a = 0.15$

Sensitivity Analysis:

Variable perturbed		Original	Range over which Policy P_3 is optimal		Optimal Policy at end of range	
Symbol	Name	Value	Lower	Upper	Lower	Upper
C_m	Ground Cost	50	34.9	202.8	P_2	P_4
C_s	Satellite Cost	200	0	∞	—	—
C_a	Aircraft Cost	15	0	17.1	—	P_2
C_p	Misclassification Cost	2000	1585	4066.7	P_2	P_1
N_1	Defective Sites	50	38.9	95.6	P_2	P_1
α_s	Satellite α − error	0.20	0.094	∞	P_2	—
β_s	Satellite β − error	0.25	0.228	∞	P_2	—
α_a	Aircraft α − error	0.05	0	0.094	—	P_2
β_a	Aircraft β − error	0.15	0	0.171	—	P_2

$$C_s + C_p N_1 \beta_s + C_m \left[(N-N_1)\alpha_s + N_1 (1-\beta_s)\right]. \tag{7}$$

Substituting Equation 6 into Equation 7 and rearranging terms, this expression becomes

$$(C_s+C_m N_1) + (C_p-C_m) N_1 \beta_s + C_m \beta_s^{-\frac{\omega}{\gamma}} \; T^{(1+\frac{\omega}{\gamma})} (N-N_1) \; . \tag{8}$$

The optimal value of β_s is obtained by setting to zero the derivative of this expression with respect to β_s:

$$(C_p-C_m) N_1 - \frac{\omega}{\gamma} C_m T^{(1+\frac{\omega}{\gamma})} (N-N_1) \beta_s^{-(1+\frac{\omega}{\gamma})} = 0 \; . \tag{9}$$

Solving for β_s yields the optimal value, denoted β^*_s:

$$\beta^*_s = T \left[\frac{\omega C_m (N-N_1)}{\gamma N_1 (C_p-C_m)}\right]^{(\frac{\gamma}{\gamma+\omega})} . \tag{10}$$

By a similar procedure we obtain the optimal value of α_s, denoted α^*_s:

$$\alpha^*_s = T \left[\frac{\gamma (C_p-C_m) N_1}{\omega C_m \, (N-N_1)}\right]^{(\frac{\omega}{\gamma+\omega})} . \tag{11}$$

A check shows the second derivatives to be positive in each case as required to prove that these are minimizing values. If these expressions result in either α^*_s or β^*_s being greater than one, then the correct solution is obtained by setting that probability to 1.0 and calculat-

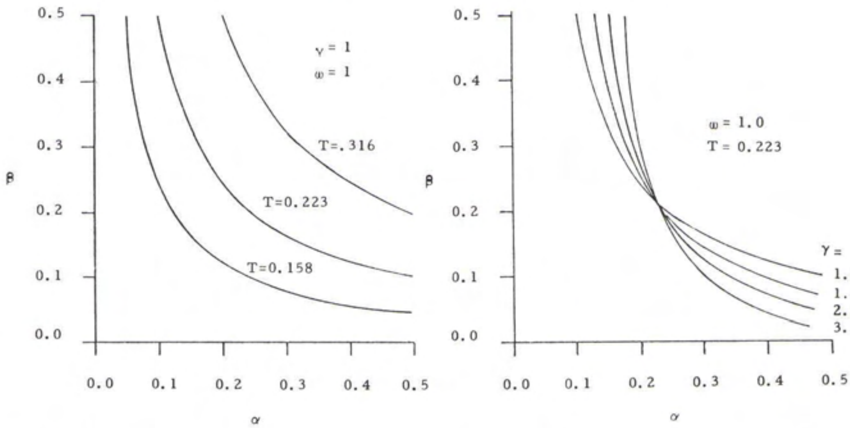


FIG. 5. Various forms of the α,β trade-off relation given by Equation 6, $\alpha^{\gamma}\beta^{\omega} = T^{\gamma+\omega}$.

ing the other from Equation 6, though such a circumstance can only be viewed as a pathological case. Moreover, as can be calculated from the decision model in the appendix, other decision rules are appropriate.

If C_p is less than C_m, the expressions for α^* and β^* become imaginary. This occurs if the penalty cost is less than the cost of manual inspection, another extreme situation which warrants reexamination of the problem as a policy of no inspection becomes rational.

A corresponding result for the aircraft and ground system can be obtained and is identical to Equations 10 and 11.

Using the values of the parameters given in Table 2, the optimal values for α^* and β^* for both satellite/ground and aircraft/ground systems are shown in Table 5.

A similar analysis can be conducted for the three-tier system. In this case a pair of simultaneous nonlinear equations is obtained which can be reduced to a single fourth-order equation. The various cases resulting from the several roots of the equation and the interactions with the boundary conditions are too complex for presentation here but are obtained in a straightforward manner.

Generally, the value of T can be decreased by the expenditure of more money. Increasing the time per aircraft inspection, for example, might produce such an improvement. Note that for the two tier system the change in α^* and β^* is proportional to T and the ratio of α^* to β^* is independent of T. It has the simple expression:

$$\frac{\alpha^*}{\beta^*} = \frac{\gamma}{\omega} \cdot \frac{(C_p - C_m)N_1}{C_m\,(N - N_1)} \tag{12}$$

For the data presented in Table 5, for example, the ratio is $0.039N_1$, as can be readily verified. The cost of decreasing T generally rises nonlinearly as T approaches zero. Hypothetical cost curves for T are shown in Figure 6. Because changes in T often result from improvements in the technology used, these are known as technological cost curves. The impact of technological improvement is thus seen as a change (reduction) in the α,β errors of the system. Technological improvements might also alter the system availability by allowing classification when sites are cloud covered, for example, but this effect is not considered here.

The cost of the satellite ground inspection system, which was given in slightly different form in Equation 7, is

$$(C_s + C_m N_1) + (C_p - C_m)\,N_1\,\beta_s + C_m\,(N - N_1)\,\alpha_s \tag{13}$$

where the optimal values of β_s and α_s are given by (for $\gamma = \omega = 1$)

$$\beta_s^* = T_s\,(C_m\,(N - N_1))^{1/2}\,((C_p - C_m)\,N_1)^{-1/2} \tag{14}$$

$$\alpha_s^* = T_s\,((C_p - C_m)\,N_1)^{1/2}\,(C_m\,(N - N_1))^{-1/2}\;. \tag{15}$$

Substituting into Equation 13 and combining like terms yields the expression

$$(C_s + C_m N_1) + 2T_s\,(C_m\,(C_p - C_m)\,(N - N_1)\,N_1)^{1/2} \tag{16}$$

TABLE 5. OPTIMAL α AND β ERRORS.

Costs:	Ground (men) $C_m = 50$	Satellite $C_s = 200$	Aircraft $C_a = 15$	Cost of Misclassification by β – type Errors $C_p = 2000$
α, β Tradeoff Parameters	$\gamma = 1$ $\omega = 1$			

Run	Input Factors				Optimal Error Factor (Rounded to 2 Decimal Pts.)				Corresponding Cost of Survey ($1,000)			Compared w/ Table 2 Reduction in Cost (%)	
					Satellite/Grnd.		Aircraft/Grnd.						
No.	N	N_1	T_s^*	T_a^*	α_s	β_s	α_a	β_a	P_1	P_2	P_3	P_2	P_3
1	1000	10	0.224	0.087	0.14	0.36	0.05	0.14	50	14.6	20.9	6.8	0.1
2	1000	50	0.224	0.087	0.32	0.16	0.12	0.06	50	33.1	29.3	9.5	15.1
3	1000	100	0.224	0.087	0.47	0.11	0.18	0.04	50	47.1	36.2	25.1	29.5
4	1000	10	0.1	0.05	0.06	0.16	0.03	0.08	50	6.9	18.6	9.0	1.6
5	1000	50	0.1	0.05	0.14	0.07	0.07	0.03	50	16.3	24.3	5.2	1.6
6	1000	100	0.1	0.05	0.21	0.05	0.10	0.01	50	23.9	29.4	18.0	7.9

* These values of T_s and T_a correspond to those implicit in run 1 through 6 of Table 2.

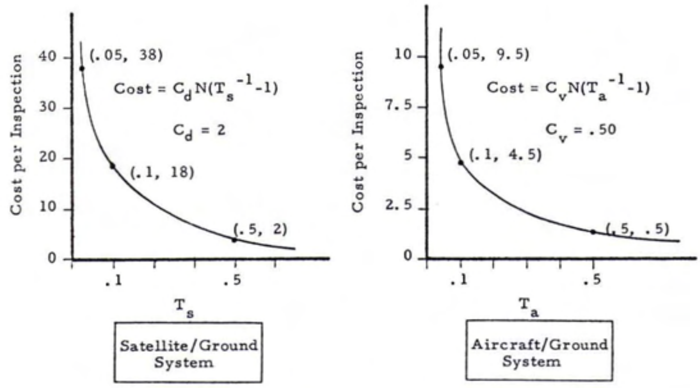


FIG. 6. Hypothetical technological cost curves.

In order to find the optimal value of T, we add the cost of technological improvement from Figure 6:

$$C_d\, N(T_s^{-1}-1) \ . \tag{17}$$

The sum of these two terms is then differentiated with respect to T_s and set equal to zero, yielding

$$2(C_m\,(C_p-C_m)\,(N-N_1)\,N_1)^{1/2} = C_d\,N\,T_s^{-2}. \tag{18}$$

A check indicates that the second derivative is positive. The optimal value of T_s, denoted T^*_s, is thus

$$T_s^* = (C_d\,N)^{1/2}\,(4C_m\,(C_p-C_m)\,(N-N_1)\,N_1)^{-1/4}. \tag{19}$$

A similar analysis for the aircraft/ground system yields

$$T_a^* = (C_v N)^{1/2}\,(4C_m\,(C_p-C_m)\,(N-N_1)\,N_1)^{-1/4}. \tag{20}$$

The same approach may be used for the three-tier system but is too complex for presentation here.

TABLE 6. OPTIMAL TECHNOLOGICAL AND ERROR PARAMETERS.

$\gamma = \omega = 1$			Common Factors		
Ground (men)	Satellite	Aircraft	Cost of Misclassification by β type error	Satellite Technological Cost	Aircraft Technological Cost
$C_m = 50$	$C_s = 200$	$C_a = 15$	$C_p = 2000$	$C_d = 2$	$C_r = 0.50$

Run	Input Factors		Optimal Parameters						Cost of Survey			Reductions in Cost/Table*			
			Satellite/Ground			Aircraft/Ground			($1,000)			Runs 1, 2, 3		Runs 4, 5, 6	
No.	N	N_1	T_s	α_s	β_s	T_a	α_a	β_a	P_1	P_2	P_3	P_2	P_3	P_2	P_3
A	1000	10	0.179	0.112	0.285	0.089	0.056	0.142	50	11.8	21.1	44.9%	12.4%	52.4%	25.0%
B	1000	50	0.121	0.173	0.084	0.060	0.086	0.042	50	19.2	25.7	52.1%	25.5%	44.0%	23.8%
C	1000	100	0.103	0.215	0.049	0.051	0.107	0.024	50	24.6	29.7	54.6%	28.5%	41.4%	23.6%

* After adjustments for technological development cost not included there.

Using the data presented in Table 2, the following selections of optimal T, α, and β values can be derived as shown in Table 6 for the two two-tier systems.

CONCLUSIONS

It is anticipated that a model such as we have described can be very useful in determining the optimal strategy for alternative remote sensing systems since it incorporates cost, technology characteristics, econometric estimation, and public policy. The description given is for a general model, and individual specifications, of course, must be tailored to the application or case study to be investigated. As can be seen, the model is simple yet powerful. The alpha and beta risks are technical questions and, therefore, allow us to parameterize the quality or accuracy of alternative remote sensing systems. In addition, the model allows us to parameterize the operational availability achieved by the remote sensing systems and examine the cost impact of this important system characteristic.

POSSIBLE EXTENSIONS

There are several relevant directions for further model development that are readily apparent:

- introduction of a larger set of classification outcomes (i.e., "fuzzy results"),
- multiple inspection objectives and violation types,
- more realistic cost functions for inspection techniques (e.g., fixed cost aspects),
- dependence of alpha and beta errors upon the magnitude of a problem area,
- more realistic tradeoffs between α and β errors,
- budget constraints on inspection policies, and
- more complex inspection policies (e.g., using random inspection of sites classified as no problem).

The potential of each of these factors to sharpen the analysis of, and thereby enhance, the study results, may be determined by extending this model.

ACKNOWLEDGMENTS

The authors wish to thank the referees of *Photogrammetric Engineering and Remote Sensing* for their constructive comments.

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Appendix

Decision Analysis of Three-Tier Systems

Figure 2 and supporting text defines a three-tier satellite/aircraft/ man policy as one in which aircraft inspection is conditioned on satellite classification, and manned inspection occurs only for those sites classified "bad" by aircraft inspection. There are, of course, other alternative policies. In this appendix we illustrate a systematic approach to determination of the optimal two-tier policy for comparison with other policy alternatives.

The device used to structure analysis is termed a decision tree. A decision tree is a stylized representation of decision alternatives and chance outcomes. Figure A1 shows an illustrative decision tree for a family of two-tier inspection policies. It is conditioned on the event {Satellite Classes Site as Potential Problem}. In this event the choices are

(i) aircraft follow-up inspection,
(ii) manned follow-up inspection, or
(iii) no follow-up inspection.

These choices are represented schematically by the three branches emanating from node A in Figure A1. Node A is termed a *decision node* and is represented by a square symbol.

The next type of node, represented by a triangle, is a terminal node. A terminal node is used to represent some end event and delimits analysis. Associated with each terminal node is a terminal value or cost. Terminal node C for example involves a manned inspection following satellite classification. The cost associated with this node is C_m, the cost of manned inspection plus the effective per site cost of satellite inspection. Since all terminal nodes involve the satellite cost in this representation, we will omit this cost element and add it in at the end of the analysis.

The final type of node is called a chance node and is used to represent a situation where there are two or more possible outcomes which can be described in probabilistic terms. If, in the example, no follow-up action is taken on satellite positives, there are two possible outcomes represented by chance node D, *viz.*, the site is a problem area (terminal node D with cost penalty C_p) or the site is not a problem area (terminal node F with cost equal to zero).

The algorithm for determination of the optimal sequence of decisions in a decision tree is called the rollback technique. It operates by starting at the terminal nodes and working backwards to chance nodes. Chance nodes are evaluated by the expected value. (For example, suppose the problem parameters are as given in run 5 of Table 2 in the text.) Chance node N, for example, has an expected value given by

$$E[N] = p(0)\,C(0) + p(p)\,C(p)$$

that is, the sum of the product of the cost of each of the succeeding nodes, $C(0)$ and $C(p)$ for node N, with the probability that this event occurs. This expectation is shown to be \$1815 in the next section. Now consider the evaluation of decision node L. If LN is selected, the expected cost is \$1815, if LM is selected the expected cost is \$65; thus LM is the optimal decision given L. The cost associated with a decision node is the minimum of the costs associated with each action, in this case \$65. By repeated application of this approach the optimal actions at each node can be identified. The section below presents the detailed equations and illustrates these with an example.

Detailed Equations

The expected value of chance node N is given by

$$\frac{N_1\,(1-\beta_s)\,(1-\beta_a)}{N_1\,(1-\beta_s)\,(1-\beta_a)+(N-N_1)\,\alpha_s\alpha_a}\,(C_p+C_a)+\frac{(N-N_1)\alpha_s\,\alpha_a\,C_a}{N_1\,(1-\beta_s)\,(1-\beta_a)+(N-N_1)\,\alpha_s\,\alpha_a} \quad \text{(A1)}$$

which for the appendix example data is equal to \$1815. The value of terminal node M is $C_m + B_a$ or \$65. Thus, the optimal decision at L is to choose M and the expected cost at L is \$65.

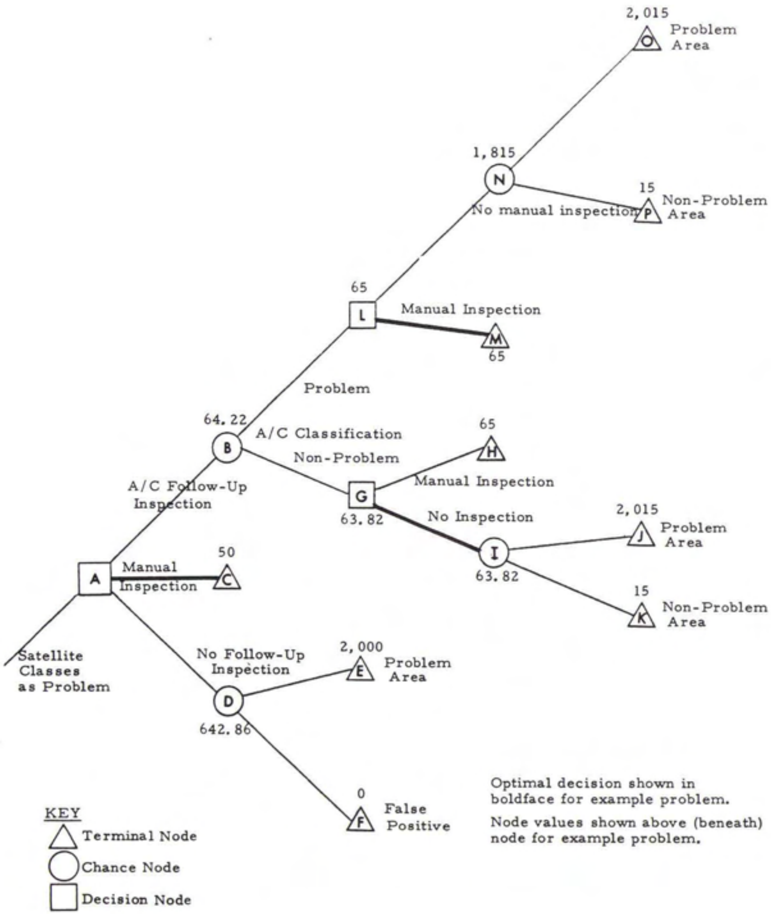

FIG. A1. Decision tree analysis of three-tier inspection scheme given satellite positive.

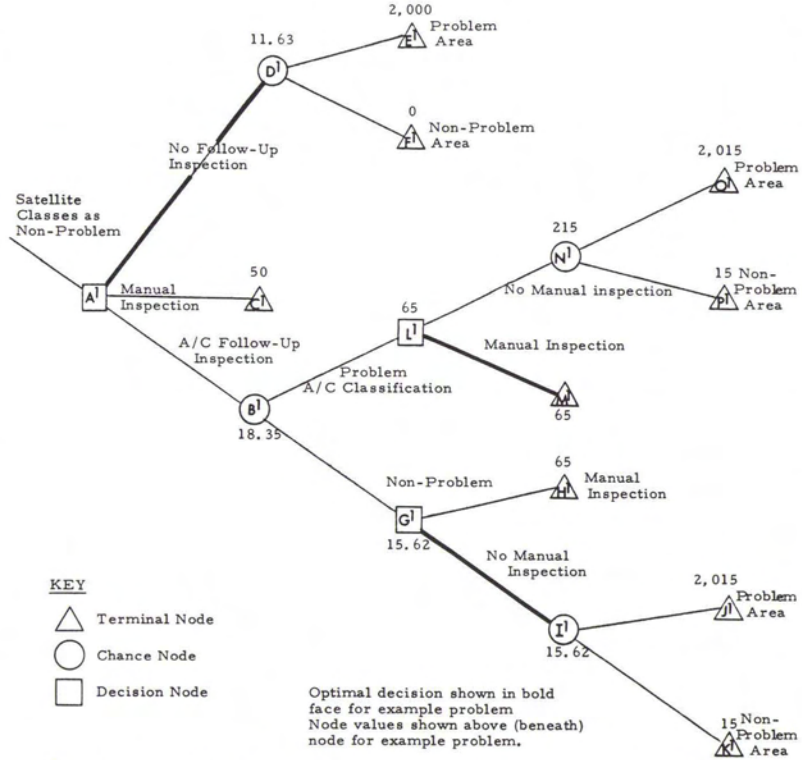

FIG. A2. Decision tree analysis of three-tier inspection scheme given satellite negative.

The expected value of chance node I is given by,

$$\frac{N_1 (1-\beta_s) \beta_a (C_p + C_a)}{N_1 (1-\beta_s) \beta_a + (N-N_1) \alpha_s (1-\alpha_a)} + \frac{(N-N_1) \alpha_s (1-\alpha_a) C_a}{N_1 (1-\beta_s) \beta_a + (N-N_1) \alpha_s (1-\alpha_a)} \cdot \tag{A2}$$

which for the appendix example data is equal to \$63.82. The value of terminal node H is $C_m + C_a$ or \$65. Thus, the optimal decision given aircraft classification of the site as non problem, node G, is not to inspect, node I, and the expected cost at node G is \$63.82.

The difference between \$63.82 and \$65 is small. Had this comparison gone the other way and manned inspection were the optimal choice, it is clear that aircraft classification could not possibly be optimal since the decision would be to have manned inspection regardless of the current classification. As it stands, the best possible cost for chance node B would be \$63.82, given the computations thus far, a figure higher than C_m associated with terminal node C, so that for these parameters aircraft inspection is not cost/effective.

The expected cost of chance node B is given by

$$\frac{((N-N_1) \alpha_s \alpha_a + N_1 (1-\beta_s)(1-\beta_a)) E[L]}{(N-N_1) \alpha_s + N_1 (1-\beta_s)} + \frac{((N-N_1) \alpha_s (1-\alpha_a) + N_1 (1-\beta_s)(\beta_a)) E[G]}{(N-N_1) \alpha_s + N_1 (1-\beta_s)} \tag{A3}$$

where $E[L]$ and $E[G]$ are the expected costs at nodes L and G respectively. For the example the expected value at node B from Equation A3 is \$64.22.

The value of terminal node C is C_m or \$50.

The expected value at terminal node D is given by

$$\frac{N_1 (1-\beta_s) C_p}{N_1 (1-\beta_s) + (N-N_1) \alpha_s} \tag{A4}$$

which, for the example, is \$642.86.

Thus, the optimal decision at node A is to have manned inspection, node C (i.e., no subsequent aircraft inspection should be scheduled). However, if a subsequent aircraft inspection is employed, the optimal action is to inspect if the site is classified as a problem and not to inspect otherwise.

Figure A2 shows the decision tree conditioned on the event that the satellite site classification is negative. It is a mirror image of that shown in Figure A1. Equivalent nodes are given the same letter with a prime superscript. The analysis follows.

The expected value of chance node N' is given by,

$$\frac{N_1 \beta_s (1-\beta_a) (C_p+C_a)}{N_1 \beta_s (1-\beta_a) + (N-N_1)(1-\alpha_s)(\alpha_a)} + \frac{(N-N_1)(1-\alpha_s) \alpha_a C_a}{N_1 \beta_s (1-\beta_a) + (N-N_1)(1-\alpha_s)(\alpha_a)} \tag{A5}$$

which for the example is equal to \$215. The value of terminal node M' is $C_m + C_a$ or \$65. Thus, the optimal decision at node L' is to choose node M' and the expected cost at node L' is \$65.

The expected value of chance node I' is given by,

$$\frac{N_1 \beta_s \beta_a (C_p + C_a)}{N_1 \beta_s \beta_a + (N-N_1)(1-\alpha_s)(1-\alpha_a)} + \frac{(N-N_1)(1-\alpha_s)(1-\alpha_a) C_a}{N_1 \beta_s \beta_a + (N-N_1)(1-\alpha_s)(1-\alpha_a)}, \tag{A6}$$

which for the example is equal to \$15.62. The value of terminal node H' is $C_m + C_a$ or \$65. Thus, the optimal decision given aircraft site classification as a non problem, node G', the optimal decision is not to inspect, node I', and the expected cost at node G' is \$15.62.

The expected cost of chance node B' is given by

$$\frac{(N_1\beta_s (1-\beta_a) + (N-N_1)(1-\alpha_s) \alpha_a)E[L']}{N_1\beta_s + (N-N_1)(1-\alpha_s)} + \frac{(N_1 \beta_s \beta_a + (N-N_1)(1-\alpha_s)(1-\alpha_a)) E[G']}{N_1 \beta_s + (N-N_1)(1-\alpha_s)} \tag{A7}$$

where $E[L']$ and $E[G']$ are the expected costs at nodes L' and G' respectively. For the example the expected value at node B' from A7 is \$18.35.

The value of terminal node C' is C_m or \$50.

The expected value at terminal node D' is given by

$$\frac{N_1\,\beta_s\,C_p}{N_1\,\beta_s + (N-N_1)\,(1-\alpha_s)} \tag{A8}$$

which for the example is \$11.63.

A comparison of the values associated with nodes B', C', and D' indicates that the optimal decision at node A' is not to have any follow-up inspection (either aerial or manned) for sites classed as non problems by satellites, the policy assumed in Figure 2.

Given these computations it is easy to evaluate the expected cost of any two-tier policy. For example, the expected cost corresponding to the satellite/aircraft/man system depicted in Figure 2 of the text is

$$\left\{\begin{matrix}\text{Probability satellites classification}\\ \text{yields apparent problem}\end{matrix}\right\} \times \begin{matrix}\text{Expected cost with}\\ \text{aircraft inspection}\end{matrix}$$

$$+\left\{\begin{matrix}\text{Probability satellite classification}\\ \text{indicates no problem}\end{matrix}\right\} \times \begin{matrix}\text{Expected cost with}\\ \text{no follow-up inspection}\end{matrix}$$

or

$$\left(\frac{N_1}{N}(1-\beta_s) + \frac{(N-N_1)}{N}\,\alpha_s\right)(64.22) + \left(\frac{N_1}{N}\,\beta_s + \frac{(N-N_1)}{N}\,(1-\alpha_s)\right)(11.63)$$

or \$18.90 per site times 1000 sites, a cost of \$18,900 plus the satellite cost of \$200 or \$19,100 as shown in Table 2.

Articles for Next Month

Harry C. Andrews, An Educational Digital Image Processing Facility.

G. I. Ballew and *R. J. P. Lyon*, The Display of Landsat Data at Large Scales by Matrix Printer.

Maurice G. Brumm and *James G. Waters*, Photodensity Control System for Orthophoto Products.

Dr. Alden P. Colvocoresses, Proposed Parameters for an Operational Landsat.

T. L. Cox, Integration of Land-Use Data and Soil Survey Data.

H. W. Gausman, D. E. Escobar, and *E. B. Knipling*, Relation of *Peperomia obtusifolia's* Anomalous Leaf Reflectance to its Leaf Anatomy.

John F. Kenefick, Applications of Photogrammetry in Shipbuilding.

J. C. Leachtenauer, Optical Power Spectrum Analysis: Scale and Resolution Effects.

Dr. J. L. van Genderen and *B. F. Lock*, Testing Land-Use Map Accuracy.

A. N. Williamson, Corrected Landsat Images Using a Small Computer.