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# An Analytical Approach to X-Ray Photogrammetry

## A calibration, employing the collinearity equations, and a laboratory process provided an accuracy of ±0.04 mm in distance and ±0.09° in rotation.

Abstract: *The metric capability of x-ray images has been reviewed. The most important systematic errors are discussed and their influences are evaluated. A calibration method has been established and the control field for the calibration has been described and illustrated. The mathematical model used for the calibration is the collinearity equation. This model is approximative in nature due to the finite size of the focal spot.*

*A laboratory process is described in order to compare the achieveable accuracy of the analytical x-ray process to the direct measurement with coordinatograph and modified transit. It was found that in distances the residual error is ± 0.04 mm and in rotation, ± 0.09 degrees. Based on these results, a routine type of operation has been employed for bio-medical purposes.*

### Introduction

X-ray photography records the anatomical or physiological data of the human body in pictorial form. Due to various types of distortion, considerable geometrical problems are encountered in obtaining three-dimensional data from x-ray photographs. Therefore, several other techniques such as kymography, tomography, x-ray scanner, etc., have been developed for the study of the interior part of a specimen. Many attempts at a solution have been made in the field of photogrammetry[1,2,3]. These methods were based on the classical concept of stereophotogrammetry. Thus the achieved accuracy was less, in many cases, than desirable. Most of these methods, however, were too complex to permit clinical applications. Desirable results were achieved in 1974[4] with semi-analytical x-ray photogrammetry: Semi-analytical photogrammetry using the analytical approach was employed in order to calibrate the x-ray apparatus and a stereoscopic concept was used for data collection. An analytical solution was developed only recently, in 1974[5]. However, the accuracy of this solution has not been reported.

The method to be described here is an analytical photogrammetric approach to determine three-dimensional displacement between skeletal structures and implant components. The term analytical photogrammetry should be interpreted here to be a numerical method for calibrating the x-ray



Photogrammetric Engineering and Remote Sensing,
Vol. 43, No. 12, December 1977, pp. 1503-1510.

equipment and a computational method for arriving at three-dimensional data of selected land-marks on the skeleton. These three-dimensional data are obtained from a geometry of convergent "photography" in order to increase the accuracy.

The expression "convergent photography" needs further explanation. The x-ray instrument used during this experiment was General Electric 1200 MA, Model MSI-1250. The instrument consists of two anodes which are movable up, down, and sideways, and are tiltable. Further, the radiation angle of each anode is changeable, separately, in both (rectangular) directions. Thus, the radiation can be concentrated on a particular portion of the object rather than covering an arc whose size is outside of medical interest. This provides a radiation safety. Because of this reason, the anodes were physically tilted to converge over the subject. As a consequence, an expression "photography with convergent radiation" would be a better one to use but, for the sake of simplicity and to use the traditional photogrammetric expression, it has been called "convergent photography."

## Metric Consideration of X-Ray Radiation

In conventional analytical photogrammetry, the mathematical model for the path of the ray from the object to the image, in most cases, is the collinearity equation. The same model can be used in x-ray photogrammetry; however, numerous parameters will influence this mathematical model. This influence is systematic or quasi-systematic in nature. A laboratory study was undertaken to determine whether or not this influence needed mathematical correction in the data reduction. Here, however, only the most important influences will be discussed.

- Deflection of the x-ray path due to the media to be imaged.
- Distortion caused by the x-ray film.
- Errors in the observation of the image coordinates caused by the observer and instrument.
- Deviation of the collinearity equation due to the physical construction of the x-ray machine.
- Quality of calibration of x-ray machines.

In the first category of problems, the refraction of x-rays should be mentioned first. According to Figure 1, the displacement due to refraction can be written as

$$d = w\left(\frac{\sin r}{\cos r} - \frac{\sin i}{\cos i}\right) .$$

If $u$ is the index of refraction, then

$$d = w \sin i \left[1/\sqrt{(u^2 - \sin^2 i)} - 1/\cos i\right]$$

For example, if one uses Plexiglass of angle $i = 45°$, then $d = 0.06$ micrometers and at $i = 75°$, $d = 1.18$ micrometers.

Using the definition of Compton[7], the refractive index may be modified to $q = 1 - u$. This quantity $(1-u)$ for a given wave length is approximately proportional to the density. [$(1-u) = k\rho$ $k = 1.984 \times 10^{-6}$ for x-ray with wave length of 1.279 A[6] where $\rho$ is the density of the material.]

If one uses the density of glass and Plexiglass at 2.52 and 1.18 respectively, then $u = 1-q = 1-5\times10^{-6}$ for glass and $1-2.3\times10^{-6}$ for Plexiglass. Using these values the above results are obtained.

These displacements are negligible from a practical point of view. Therefore, Plexiglass material is suitable for calibration purposes. The Plexiglass material, having a relatively low atomic-weight number, provides an image of hardly recognizable contrast and in consequence can be imaged along with a specimen without a disturbing effect.

The largest family of systematic errors investigated are in the second group, namely, distortion caused by x-ray film.

- Film Shrinkage
- Variations of Film Thickness
- Film Flatness
- Effect of Double Emulsion
- Effect of the Focal Spot

### Film Shrinkage

Film shrinkage as a source of systematic error has been studied extensively. One of the latest studies was published by Meier in 1972,[6] who used the equation

$$d_s = F_s + G_s \cdot t$$

where $t$ is the format size of the film and $F_s$ and $G_s$ are the film shrinkage parameters. A


Fig. 1. X-ray reflection and refraction.

test study of film shrinkage for x-ray photography indicates that $F_s = 1.5\ \mu m$ and $G_s = 0.15\ \mu m/cm$. Thus, considering the large format size of x-ray film (36 × 43 cm), $d_s = 7.9\ \mu m$. This is the largest systematic influence among the parameters affecting the accuracy.

### VARIATIONS OF FILM THICKNESS

X-ray film in general is acetate-based and its thickness varies from 120 to 250 $\mu m$. The measured variation of thickness is between 3 and 9 $\mu m$ and it was found that it is directly proportional to the thickness of the film within ± 3 percent. For polyester-based film whose thickness is only 80-140 $\mu m$, the variation is 3 to 4 $\mu m$.

### FILM FLATNESS

The flatness of the film is also a source of systematic error. The geometry of the problem is shown by Figure 2. This systematic error may be noted by its x and y components. Thus

$$dR_x = dR\ (x - x_o)/z_o$$
$$dR_y = dR\ (y - y_o)/z_o$$

where $z_o$ is the principal distance, $x_o$ and $y_o$ are the image coordinates of the principal point, and $x$ and $y$ are the correct image coordinates of a point (P) of interest. The same figure and notation may be used to describe the effect of double emulsion.

### EFFECT OF DOUBLE EMULSION

The geometry of the effect of double emulsion on the standard x-ray film is shown by Figure 2 from which the components of planimetric dislocation of the image can be defined as:

$$dS_x = dS\ (x - x_o)/z_o$$
$$dS_y = dS\ (y - y_o)/z_o$$

where $dS$ is the thickness of the film. These phenomena result in a blurred image and usually the middle point of the image $R$ and $S$ are measured. Consequently only one-half of these components affect the image.

The third group of errors occur from three sources; that is, the observer, the instruments, and errors in the observation of image coordinates. The errors of the observer and instrument are variable and are discussed in the literature. However, the observational error of the image, particularly the so-called Penumbra Effect, must be mentioned. The source of radiation is not a point but a finite size focal spot as shown by Figure 3. This fact magnifies the Penumbra Effect. As was indicated in the Introduction, the anodes are tilted; therefore, the physical size of the focal spot in Figure 3 is NM. The projection of this distance parallel to the plane of the film is $X$. The tilted anodes result in an unequal size of the penumbra and thus in a mathematical decentering error of $dx$. The mathematical problem is to determine the magnitude of the penumbra distance $x_1$ and $x_2$. The decentering error of the image measurement is

$$dx = |x_1 - x_2|$$

or

$$dx = S_1\bar{X}_1(1/D_1 - 1/D_2) + S_1\bar{X}_1 \cot u \left[E_1/D_1^2 - D_1\bar{X}_1 \cot u) - E_2/(D_2^2 - D_2\bar{X}_1 \cot u)\right]$$

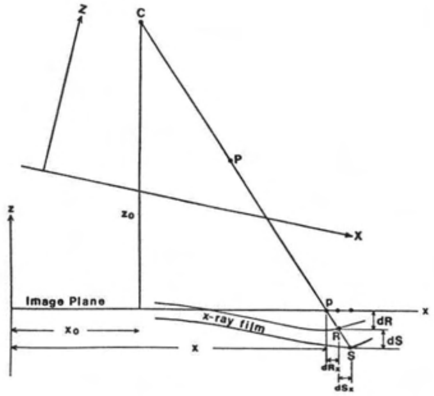


FIG. 2. Image displacement due to film flattening.

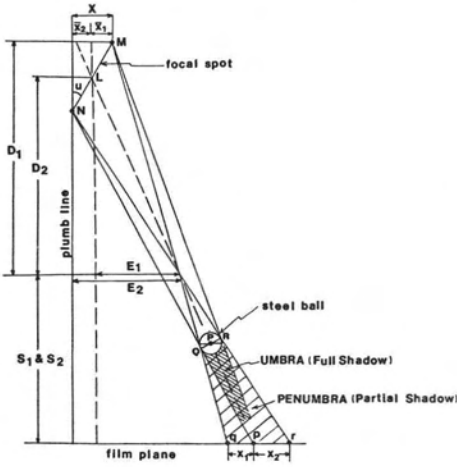


FIG. 3. Penumbra effects on x-ray image.

where $\overline{X_1}, \overline{X_2}$ are one-half of the effective size of the focal spot. The other quantities are readily visible from Figure 3.

In practice, the decentering error is about 0.2 $\mu$m, assuming convergent x-ray photographs.

#### EFFECT OF FOCAL SPOT

Finally, the effect of the focal spot can be discussed. The focal spot is of finite size. It is a surface large enough to generate only the certain amount of x-rays which penetrate the object and form an image. The interaction between the focal spot and the electrons produces heat. Manufacturers were able to design anodes with good heat dissipation, which allows a smaller focal spot size of about 0.1 to 1.5 mm in diameter. The size of the focal spot indirectly influences the results in that the mathematical models for reconstructing the geometry usually assume that the focal spot is a point.

In summary, it can be concluded that x-ray instruments are capable of providing images which can be used for quantitative measurement. The quantitative measurements should provide high accuracy if the major portion of the systematic errors are corrected.

One must take into account the fact that the x-ray image is larger than the specimen. Thus, the actual effect is quantitatively smaller on the specimen than on the photograph. On the other hand, however, the effect of a single anode only is discussed here and, in practice, two anodes were used simultaneously. Thus the error propagates and tends to decrease the accuracy of the results.

### CALIBRATION OF X-RAY EQUIPMENT

The calibration of the x-ray equipment as it was used in this study consisted of two parts. One was the utilization of a *reseau* plate and the second was the control field. The general geometry is shown by Figure 4.

The purpose of the *reseau* plate is to provide a reference system for the x-ray photographs. In addition, it is used as a base for the correction of image distortion.

The control field is needed to determine the spatial position of x-ray anodes. The *reseau* plate was manufactured from one-half-inch thick Plexiglass with *reseau* crosses at 10 × 10 cm intervals. The crosses have been filled with dental alloy. Each *reseau* cross was carefully calibrated with a coordinatograph of 0.01 mm least reading. The average standard error of the coordinates of *reseau* crosses was found to be ± 0.015 mm.

It is essential to provide a set of well-distributed points with known spatial coordinates in order to define the object space coordinates of the x-ray anodes position regarded as "projection centers." Further, the redundancy of control points allows evaluation of the system by comparing calibrated values and computed coordinates of control points.

As a practical criteria for the control field, the equipment should be portable, accurate, and must have proper geometrical shape. Considering these criteria, a triangular frame was manufactured from half-inch-thick Plexiglass. Small holes of 0.5 mm diameter were drilled in the Plexiglass sheets and filled with dental alloy. Their positions were measured and the standard error of the coordinates was found to be ± 0.015 mm.

For the mathematical model of the calibration, the collinearity equations were used. The input data for these equations are the object space coordinates of the control points and the image coordinates.

The general geometry for a single anode is shown by Figure 5.

The vector $d$ from the perspective center $c$ to an image point $p$ is regarded to be collinear with $D$ vector from the same exposure station to corresponding object point $P$. (Note the approximate nature of this mathematical model due to the size of focal spot or perspective center.) The two vectors differ only in length. They are expressed by the collinearity equations:

$$x - x_o = -z_o \frac{M_1\bar{X}}{M_3\bar{X}} \text{ and } y - y_o = -z_o \frac{M_2\bar{X}}{M_3\bar{X}}$$

$$\boldsymbol{M} = \begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix}$$

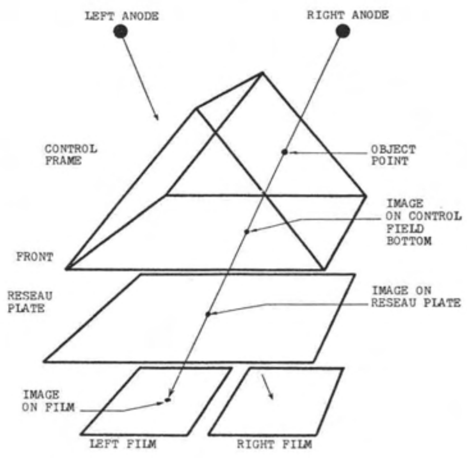


FIG. 4. Calibration of x-ray equipment.

$M$ is the well-known rotational matrix and $x$, $y$ are the measured photo-coordinates as defined by the "Photo System" established by the *reseau* plate and

$$\bar{X} = \begin{bmatrix} X - X_c \\ Y - Y_c \\ Z - Z_c \end{bmatrix}$$

The reference system is established by the control field noted in Figure 5 as "Q" system. The relation between Q system and the photo system is

$$\bar{X}_Q = \frac{1}{m} M^T \bar{X}$$

where

$$\bar{X}_Q = \begin{bmatrix} X_Q - X_c \\ Y_Q - Y_c \\ - Z_c \end{bmatrix} \text{and } \bar{x} = \begin{bmatrix} x - x_o \\ y - y_o \\ - z_o \end{bmatrix}$$

From these equations, the orientation and the coordinates of the anodes can be determined in the conventional manner. Due to the substantial amount of redundancy, the least-squares adjustment is used to arrive at the most probable values of coordinates.

Once the orientation matrix and the theoretical coordinates of the perspective center of the x-ray anodes are obtained, the object space coordinates of any point can be obtained by space intersection. The space intersection, a well-known procedure, is omitted here.

The number of transformations performed during the calibration are affine transformations. Such transformations, by their mathematical nature, partially compensate for the systematic and quasi-systematic errors.

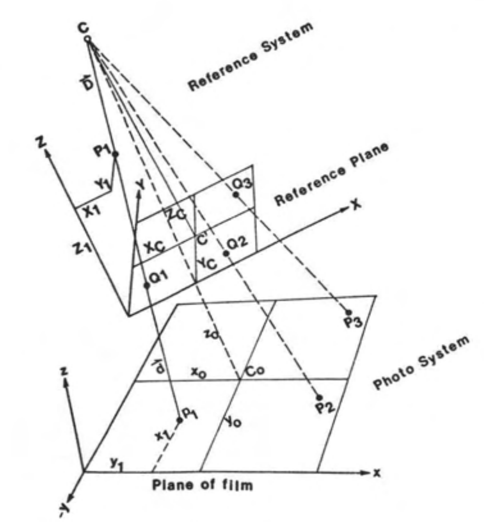


FIG. 5. Coordinate systems.

### TEST STUDY WITH A TRANSIT

The purposes of this experiment were (1) to determine the efficiency of x-ray photogrammetry in terms of achievable accuracy, (2) to provide numerical data for a mathematical model used to determine spacial motion, and (3) to formulate a practical solution for biomedical studies.

In order to fulfill these aims, distances and angular rotations were measured by means of x-ray analytical photogrammetry. At the same time, these distances and rotational angles also were measured with other physical means of high accuracy in order to substantiate the accuracy of the achieved results along with the standard errors obtained from redundant observations. For this purpose a transit with 10-seconds-of-arc least reading was modified. This transit is shown in Figure 6. The telescope was replaced with a Plexiglass plate on which 0.8 mm steel balls were attached. The rotation of the Plexiglass could be measured on the horizontal and vertical circles of the theodolite. The distances between the steel balls were determined on the AP/C analytical plotter of the University of Washington, which was used as a mono-comparator. A dry bone (patella) was placed at the center of the Plexiglass for further studies which are outside of this paper. The accuracy of these distances was found to be ± 0.006 mm.

FIG. 6. Modified transit.

The Plexiglass was imaged by x-rays at horizontal and vertical angles of 0, 15, and 20 degrees. These x-ray photographs were measured on the AP/C and, by using the space resection and intersection method described in the previous section, the distances were computed between the steel balls. This method allows the direct comparison of distances measured both directly and by x-ray photogrammetry.

The angular motion also should be determined. Therefore, a method was established which can be outlined as follows:

Assume that there are three points on a plane given by their co-ordinates $P_1$ $(X_1,Y_1,Z_1)$, $P_2$ $(X_2,Y_2,Z_2)$, and $P_3$ $(X_3,Y_3,Z_3)$, then the equation of the plane is

$$\begin{bmatrix} (X - X_1), (Y - Y_1), (Z - Z_1) \\ (X_2 - X_1), (Y_2 - Y_1), (Z_2 - Z_1) \\ (X_3 - X_1), (Y_3 - Y_1), (Z_3 - Z_1) \end{bmatrix} = 0$$

or in more general form:

$$AX + BY + CZ - D = 0$$

Geometrical identification of the coefficient in the above equation shows that they are direction cosines.

Thus:

$$\cos q = A/E, \cos r = B/E, \cos s = C/E$$

where

$$E = \sqrt{A^2 + B^2 + C^2}$$

The $q$, $r$, and $s$ are the angles between the $XYZ$ coordinate axes respectively and the vector normal to the plane from the origin of the coordinate system. By utilizing the direction cosines, the equation in form of function $(F)$ may be written as:

$$F = X_i \cos q + Y_i \cos r + Z_i \cos s - p = 0 \quad (1)$$

In this equation, $p$ is the distance between the origin of the coordinate system and the plane. In simplified form this equation may be shown as

$$F = Xe + Yh + Zq - p = 0 \quad (2)$$

For computational convenience in connection with least-squares adjustment, approximate values are introduced:

$$e = e' + \Delta e$$
$$h = h' + \Delta h$$
$$q = q' + \Delta q$$
$$p = p' + \Delta p$$

Further, realizing that the coordinates of points on the plane are not free from errors, the correction must be rendered to them. That is

$$X = X' + v_X$$
$$Y = Y' + v_Y$$
$$Z = Z' + v_Z$$

Equation 2 is not a linear equation; therefore, the Taylor series can be used for linearization:

TABLE 1. DIFFERENCES BETWEEN MEASURED DISTANCES AND COMPUTED DISTANCES TO THE PLEXIGLASS PLATE (IN MM)

| | Angle (in degree) | | #91/ 94 | #93/#96 | #91/#93 | #94/#96 |
|---|---|---|---|---|---|---|
| Film No. | $H$ | $V$ | $D$ = 50.90 | $D$ = 50.91 | $D$ = 40.74 | $D$ = 40.69 |
| 36 | 0 | 5 | −0.01 | −0.01 | −0.05 | −0.02 |
| 37 | 0 | 10 | −0.02 | −0.05 | −0.08 | −0.07 |
| 38 | 0 | 15 | −0.02 | −0.03 | −0.07 | −0.03 |
| 39 | 0 | 20 | −0.01 | −0.02 | −0.07 | −0.05 |
| 41 | 5 | 0 | −0.02 | −0.01 | −0.03 | +0.03 |
| 42 | 5 | 5 | −0.03 | 0.00 | −0.04 | +0.01 |
| 43 | 5 | 10 | −0.01 | 0.00 | −0.02 | +0.04 |
| 44 | 5 | 15 | −0.01 | −0.01 | −0.05 | 0.00 |
| 45 | 5 | 20 | −0.02 | −0.02 | −0.03 | −0.02 |
| 50 | 10 | 0 | +0.02 | +0.02 | 0.00 | +0.06 |
| 49 | 10 | 5 | −0.02 | +0.03 | +0.03 | +0.04 |
| 48 | 10 | 10 | +0.01 | −0.01 | +0.03 | +0.07 |
| 47 | 10 | 15 | −0.02 | +0.03 | 0.00 | +0.04 |
| 46 | 10 | 20 | +0.01 | 0.00 | +0.02 | +0.04 |
| 53 | 15 | 10 | −0.03 | +0.01 | −0.02 | −0.05 |
| 54 | 15 | 15 | −0.04 | −0.03 | +0.02 | +0.03 |
| 55 | 15 | 20 | −0.03 | 0.00 | +0.04 | +0.07 |
| 61 | 20 | 5 | −0.07 | −0.02 | +0.03 | +0.10 |
| 56 | 20 | 20 | −0.05 | 0.00 | +0.04 | +0.09 |

$\sigma = \pm$ 0.04 mm.

$$\frac{\partial F}{\partial X} v_x + \frac{\partial F}{\partial Y} v_y + \frac{\partial F}{\partial Z} v_z + \frac{\partial F}{\partial e} \Delta e + \frac{\partial F}{\partial h} \Delta h + \frac{\partial F}{\partial q} \Delta q + \frac{\partial F}{\partial p} \Delta p + F(X', Y', Z', e', h', q', p') = 0$$

where the $X', Y', Z'$ and $e', h', g', p'$ are approximate values. From this general equation the combined observation condition method of least-squares adjustment can be obtained which, in general form, is

$$\mathbf{BV} + \mathbf{AX} + \mathbf{W} = 0$$

The terms here are easily identifiable with the general equation. The solution of this least-squares adjustment problem can be found in several places in literature.[7,8]

For determining two positions of the plane as described above, the angle between those two positions can be computed by

$\cos t = \cos q_1 \cos q_2 + \cos r_1 \cos r_2 + \cos s_1 \cos s_2$ where $\cos q_1$, $\cos r_1$, and $\cos s_1$ are the direction cosines of one and $\cos q_2$, $\cos r_2$, and $\cos s_2$ are the direction cosines of the number two position of the plane.

## The Results

The modified transit was photographed at 0, 5, 10, 15, and 20 degrees vertical and horizontal angles on Kodak 36 × 43 cm x-ray film. Exposures were 60 KVP, 200 MAS, and 0.05 seconds.

The calibration of the anodes was performed and the space inter-sections of the points located on the Plexiglass were computed. The distances between points were computed from the points. The measured distances of these points were found to be 50.90, 50.91, 40.74, and 40.69 mm. These measured distances were regarded as fixed and the computed distances at various angles were compared to these and the deviations were recorded. The summary of results are given in Table 1. The standard residual error of the distances were found to be ± 0.04 mm.

Rotational angles of the Plexiglass target of the transit also were computed at 0, 15, and 20 degrees horizontal and 0, 5, 10, 15, and 20 degrees vertical angle intervals. The directional angles $q$, $r$, and $s$ were determined according to the method described in the previous section. The summary of these results is given in Table 2.

By using Equation 3, the angle and its residual errors at various positions of the plane were determined. The residual errors as compared to the measured ones are exhibited in Table 3. The average residual errors were found to be ± 0.09 degrees.

The results summarized above encouraged the investigators to implement the method for applied clinical research. This required that a "routine" method be established. Such a routine method was used for the determination of patellar motion at various angles of knee flexion. The practical routine type of work is summarized by the functional chart of the method used, given in Figure 7.

## Conclusion

X-ray photogrammetry can provide a high degree of accuracy in determining relative displacement between parts of the skeleton. The approximations introduced in the method tend to compensate for systematic and quasi-systematic errors to the degree that residuals remain negligibly small from the practical point of view.

This conclusion also is fortified by the fact that the method was used for tracking patellar motion of several patients. The results of this practical experiment have already been reported by Dr. F. G. Lippert[9]. The method is being further expanded for a larger number of patients and the methodology is

Table 2. Rotation Angles of the Plexiglass Plate (in Degrees)

| Film No. | $H$ | $V$ | $q$ | $r$ | $s$ |
|---|---|---|---|---|---|
| 35 | 0 | 0 | 90.2 | 90.1 | 180.2 |
| 36 | | 5 | 84.0 | 90.3 | 174.0 |
| 37 | | 10 | 79.1 | 90.3 | 169.1 |
| 38 | | 15 | 74.1 | 90.4 | 164.1 |
| 39 | | 20 | 69.1 | 90.5 | 159.1 |
| 63 | 15 | 0 | 89.2 | 75.2 | 165.2 |
| 52 | | 5 | 83.8 | 76.6 | 165.2 |
| 53 | | 10 | 78.8 | 75.7 | 161.7 |
| 54 | | 15 | 74.0 | 76.1 | 158.5 |
| 55 | | 20 | 68.9 | 76.6 | 154.7 |
| 60 | 20 | 0 | 53.6 | 74.9 | 139.6 |
| 61 | | 5 | 83.7 | 70.5 | 159.4 |
| 62 | | 10 | 79.1 | 70.8 | 157.7 |
| 57 | | 15 | 81.4 | 60.6 | 149.2 |
| 56 | | 20 | 68.9 | 71.9 | 151.6 |

Table 3. Residual Errors in the Angle Computations (in Degrees)
(unit: degree)

| $H$ \ $V$ | 5 | 10 | 15 |
|---|---|---|---|
| 0 | −.10 | +.04 | −.01 |
| 15 | +.09 | −.13 | +.03 |
| 20 | +.07 | +.10 | −.09 |

$\sigma = \pm 0.09$ degrees.

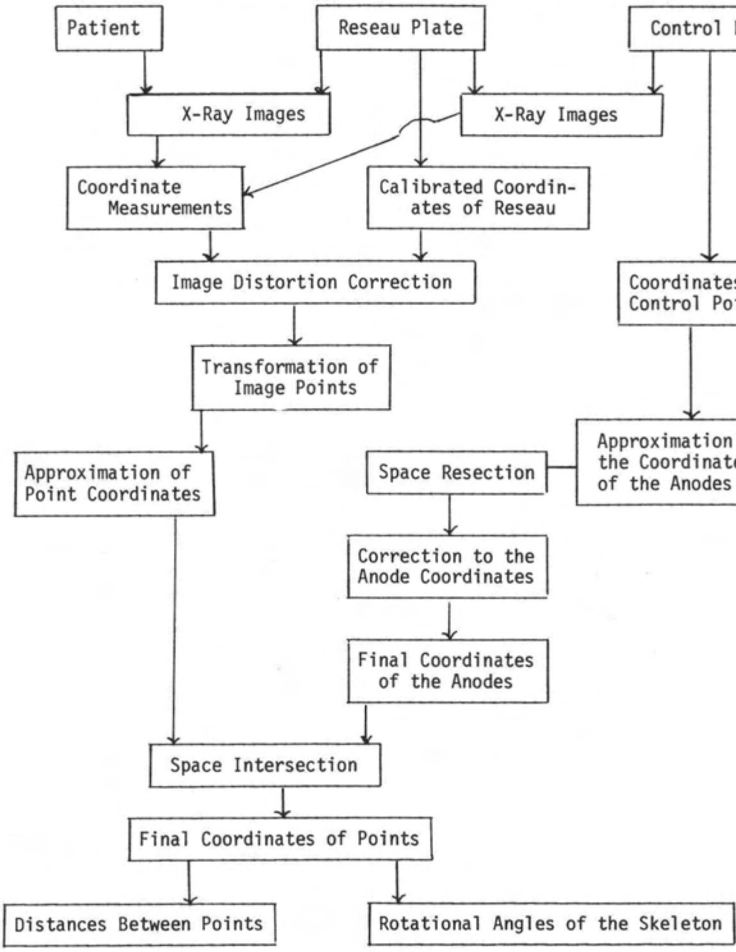


FIG. 7. Functional chart.

being modified in Seattle at the Veterans Administration Hospital.

The method described here is applicable for similar purposes in both bio-engineering and general engineering.


## ACKNOWLEDGMENT

The authors express their appreciation and sincere thanks to the National Science Foundation for sponsoring this project. The project extends into the field of close-range and x-ray photogrammetry; consequently this report is only part of the project.


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