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Limitations of the Narrow-Angle Convergent Pair

Equations are derived for Viking Orbiter photopairs.

The scheme of photography performed with Viking Orbiters I and II marks the first attempt to use narrow-angle photography for aerotriangulation and topographic mapping. In some respects the attempt has been remarkably successful, resulting in precise planimetric triangulations and mosaics. These follow rather automatically from the narrow-angle traits and the achieved precision of the telemetered data for spacecraft position and orientation. The mapping in three dimensions has been considerably less successful and it is useful to examine the reasons for this.

There is a certain inevitability in the successive choices and resulting characteristics of the pictures. An orbiting altitude of 1500 km was chosen in order to facilitate radio communication between the Viking Lander and the Viking Orbiter which serves the Lander as a relay link. Any lowering of this rather high altitude cuts into the time in which the link is available and thus increases the operational difficulties of the mission. But with a distance of 1500 km there is a very definite need for a large focal length in order to achieve a usable resolution. The nominal focal lengths for the Viking Orbiter cameras are 0.475 meters. Unfortunately, the Vidicon sensitive surfaces cannot be enlarged at the same time so that in the case of these

Abstract: *Spatial triangulations and topographies of the Martian surface derived from Viking Orbiter pictures depend on the use of symmetric narrow-angle convergent pairs. The overlap in each pair is close to 100 percent and the ground principal points virtually coincide. The analysis of this paper reveals a high degree of indeterminacy in such pairs and at least in part explains the rather disappointing precision of the associated spatial triangulations.*

orbiters the Vidicon cameras are virtually telescopes. Their angular fields are of about 1 degree.

These narrow-angle cameras lead inevitably to convergent pairs of pictures because there certainly would be no point in photographing the same piece of terrain twice from nearby air stations. Note that this does not apply to wide-angle pairs which are more often parallel, or "normal." There is one more inevitable feature which is quite important. In order to ensure that the convergent pairs achieve a useful degree of overlap, they were designed to be completely overlapping with virtually coincident ground principal points. Every photogrammetrist would anticipate difficulties in correspondence† settings with these coincident princi-

† *Author's note.* British photogrammetrists, following the usage of Fourcade, Hotine, and Thompson, utilize the terms correspondence, corresponding, correspendence setting, and in correspondence, for which there appear to be no satisfactory alternatives in either European or American photogrammetric usage. Corresponding images are those corresponding to the same ground point; corresponding rays are those through corresponding images; a correspondence setting is that in which the two photographs are placed so that all corresponding rays meet, and pictures in correspondence are so placed.

Editor's note. The Glossary in the *Manual of Photogrammetry*, 3rd cd., gives essentially the same definition for correspondence. Thus, though it may not be common American usage, it is the preferred form.

Photogrammetric Engineering and Remote Sensing,
Vol. 43, No. 12, December 1977, pp. 1511-1514.

pal points, because the simplest reasoning points to a confusion in the effects of the κ-rotations, that is, the rotations of the cameras about their own axes, for either κ aliases the negative of the other. However, the mission planners were concerned, quite correctly, with ensuring that there was *some* overlap and feared situations in which the pointing error would produce pairs with zero overlap. That this never happened is a tribute to the precision of the pointing processes. Indeed, these were somewhat too successful for the photogrammetry, because they produced pairs, one after the other, in which the principal points are very close on the ground.

The Viking Orbiter pairs considered in this paper have included angles of 25° between the principal rays. The spacecraft cameras are tilted forward when approaching the target area and then tilted back when leaving it. Camera slews permit targeting of areas to left and right of the trajectory.

Each pass produces a double strip of complete pairs and the spatial triangulations consist of systematic coverage by such pairs, adjacent pairs overlapping each other by up to 20 percent. This systematic coverage was performed with considerable success and gaps were relatively infrequent. Note, however, that the resemblance to a normal aerotriangulation is quite superficial. Each pair has its own two camera stations and does not share them with any other pair. The only links between adjacent pairs are common points on the ground. The only approach to a link in the air is that the space stations are given and come from a single coherent and consistent trajectory theory. Thus, the spatial triangulation starts with known camera stations and known camera orientations. This is just as well, because the correspondence conditions, which are so important for normal aerotriangulations, contribute relatively little in this case.

The 100 percent convergent symmetric narrow-angle pair is the basic brick of the Viking Orbiter spatial triangulations and is the essential data block in the plotting. Hence the study of this pair and its limitations is important for present and future applications in planetary triangulation and mapping.

There are of course numerous analytical schemes for photopairs, but none of them is particularly well-adapted to the purposes of this paper. The scheme given here has no practical application. It is entirely theoretical and designed to bring out the points of interest. We imagine the two pictures brought into correspondence by initial small rotations κ_1 and κ_2 about their principal axes, then by finite rotations α and $-\beta$ about the new y-axes to eliminate the large longitudinal tilts, and then by a final small rotation ω of the second photograph about the base joining the two camera stations. (See Figure 1.) All of these rotations are about the perspective centers of the pictures. The five elements of relative orientation are α, β, κ_1, κ_2, and ω. The rotated coordinates in the two parallel systems $x_1'y_1'z_1'$ and $x_2'y_2'z_2'$ are given by

$$\begin{pmatrix} x_1' \\ y_1' \\ z_1' \end{pmatrix} = \begin{pmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{pmatrix} \cdot \begin{pmatrix} 1 & -\kappa_1 & 0 \\ \kappa_1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix},$$

and

$$\begin{pmatrix} x_2' \\ y_2' \\ z_2' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -\omega \\ 0 & \omega & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{pmatrix} \cdot \begin{pmatrix} 1 & -\kappa_2 & 0 \\ \kappa_2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix},$$

in which (x_1,y_1) and (x_2,y_2) are the image coordinates of corresponding images and z_1,z_2 are the principal distances. Remembering that ω, κ_1, and κ_2 are small and neglecting their products we get

$$\begin{aligned} y_1' &= x_1\kappa_1 + y_1, \\ z_1 &= x_1\sin\alpha - y_1\kappa_1\sin\alpha + z_1\cos\alpha, \\ y_2' &= x_2(\kappa_2 + \omega\sin\beta) + y_2 - z_2\omega\cos\beta, \\ z_2' &= -x_2\sin\beta + y_2(\omega + \kappa_2\sin\beta) + z_2\cos\beta. \end{aligned}$$

The correspondence condition, that is, the condition that corresponding rays meet in space, is

$$y_1'/z_1' = y_2'/z_2',$$

or

$$y_1'z_2' - y_2'z_1' = 0. \tag{1}$$

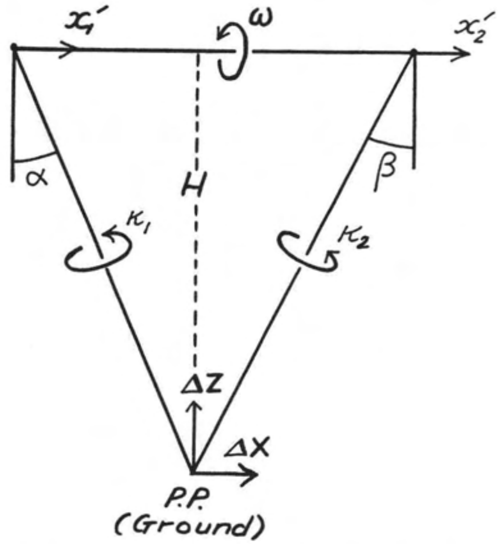

FIG. 1. The convergent narrow angle pair: notation.

Substituting in this and dropping all terms containing products of the type $\omega\kappa$, we get an equation which reduces to the narrow-angle case by further dropping all terms containing factors of the type ωxx, ωxy, κxx, etc., because these are all small compared to terms containing factors such as ωxz, ωyz, etc. The correspondence condition then takes the form

$$\begin{aligned} &x_1z_2\kappa_1\cos\beta - z_1x_2\kappa_2\cos\alpha \\ &+ \omega\,(x_1z_2\sin\alpha\cos\beta + z_1x_2\cos\alpha\sin\beta + z_1z_2\cos\alpha\cos\beta) \\ &= z_1y_2\cos\alpha - y_1z_2\cos\beta + x_1y_2\sin\alpha + y_1x_2\sin\beta. \end{aligned} \tag{2}$$

This is useful for that case in which the longitudinal tilts α and $-\beta$ are already known. In order to get the more general case substitute $\alpha + \delta\alpha$ for α and $\beta + \delta\beta$ for β in those terms not containing the small factors ω, κ_1, and κ_2. The resulting equation is

$$\begin{aligned} &z_1y_2\sin\alpha\delta\alpha - y_1z_2\sin\beta\delta\beta \\ &+x_1z_2\cos\beta\cdot\kappa_1 - z_1x_2\cos\alpha\cdot\kappa_2 \\ &+ \omega(x_1z_2\sin\alpha\cos\beta - z_1x_2\cos\alpha\sin\beta + z_1z_2\cos\alpha\cos\beta) \\ &= y_1x_2\sin\beta - y_1z_2\cos\beta + x_1y_2\sin\alpha + z_1y_2\cos\alpha. \end{aligned}$$

In the ω terms the z_1z_2 term is large compared to the other two coefficients which therefore can be dropped. Further, on the right the terms in y_1x_2 and x_1y_2 are much smaller than the others and can be dropped. Thus the general equation for the narrow-angle convergent pair is

$$\begin{aligned} &z_1y_2\sin\alpha\cdot\delta\alpha - y_1z_2\sin\beta\cdot\delta\beta \\ &+ x_1z_2\cos\beta\cdot\kappa_1 - z_1x_2\cos\alpha\cdot\kappa_2 \\ &+ \omega\cdot z_1z_2\cos\alpha\cos\beta = z_1y_2\cos\alpha - y_1z_2\cos\beta. \end{aligned} \tag{3}$$

This can be further simplified for the usual case of $z_1 = z_2 = z$, becoming

$$y_2\sin\alpha\cdot\delta\alpha - y_1\sin\beta\cdot\delta\beta + x_1\cos\beta\cdot\kappa_1 - x_2\cos\alpha\cdot\kappa_2 + \omega\cdot z\cos\alpha\cos\beta = y_2\cos\alpha - y_1\cos\beta. \tag{4}$$

Finally, for the symmetric narrow-angle convergent pair with equal principal distance we can write

$$\beta = \alpha,\ \delta\alpha = \delta\phi_1,\ \delta\beta = -\delta\phi_2$$

and the correspondence condition simplifies to

$$y_2\tan\alpha\cdot\delta\phi_1 + y_1\tan\alpha\cdot\delta\phi_2 + x_1\kappa_1 - x_2\kappa_2 + \omega\cdot z\cos\alpha = y_2 - y_1. \tag{5}$$

Now let ΔX, ΔY, and ΔZ be the deviations of the ground point from the common principal point. In the Viking Orbiter case ΔX and ΔY can range up to ± 20 km while ΔZ is usually limited to ±2 km. In general then we can neglect ΔZ; certainly we should not have to depend on large ΔZ values (large relief) to get a solution. Neglecting ΔZ we have

$$\begin{aligned} x_1 &\approx z_1 \Delta X \cos^2\alpha / H, \\ y_1 &\approx z_1 \Delta Y \cos\alpha / H, \\ x_2 &\approx z_2 \Delta X \cos^2\beta / H, \\ y_2 &\approx z_2 \Delta Y \cos\beta / H, \end{aligned}$$

where H is the altitude.

Substituting these in the general Equation 3 for the convergent narrow-angle pair, we then get the form

$$\begin{aligned} &(z_1 z_2/H)\Delta Y \sin\alpha\cos\beta\cdot\delta\alpha - (z_1 z_2/H)\Delta Y\cos\alpha\sin\beta\cdot\delta\beta \\ &+ (z_1 z_2/H)\Delta X\cos^2\alpha\cos\beta\cdot\kappa_1 - (z_1 z_2/H)\Delta X\cos\alpha\cos^2\beta\cdot\kappa_2 \\ &+ \omega z_1 z_2 \cos\alpha\cos\beta = z_1 y_2 \cos\alpha - y_1 z_2 \cos\beta. \end{aligned} \tag{6}$$

Inspection of this at once reveals the impossibility of using only the correspondence conditions for a solution, for the matrix of equations is doubly singular. First, the column of coefficients for the unknown $\delta\alpha$ stands in a constant ratio $-\sin\alpha\cos\beta:\sin\beta\cos\alpha$ to that for $\delta\beta$. Second, the column of coefficients for the unknown κ_1 stands in a constant ratio $-\cos\alpha:\cos\beta$ to that for κ_2. From either cause the determinant of the solution must vanish.

The above bears both on the triangulation and on the subsequent plotting of pairs. The analysis shows that the solution using the correspondence conditions for the narrow-angle convergent pair is doubly singular. In the Viking Orbiter spatial triangulations there is heavy dependence on external data, namely the telemetered camera orientations and the tracking values for the spacecraft coordinates. The analysis emphasizes just how strong this dependence is. The photogrammetry helps only in that it forces adjacent models into agreement and does relatively little within each model. The singularities are associated with the longitudinal tilts α and β and with the swings κ_1 and κ_2. The first was formally noted by this writer (Arthur, 1962) a decade ago, but the second only emerged with a closer examination of the problem in connection with the stereoplotting.

The lesson for the stereoplotting is that there is virtually no possibility of improving the ω, ϕ, and κ values coming from the triangulation. No doubt when these values are set in the plotter, y-parallaxes will sometimes be visible and on occasions appreciable. It is permissible to remove them locally and temporarily for convenience in plotting, but to attempt to improve the model must be a waste of time. The analysis shows that the correspondence conditions contribute very little. This may seem against the intuition of the plotter operator, but it must be accepted for economic working.

Acknowledgments

The work leading to this paper was supported by NASA Work Order L-55232.

References

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