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The Analytical Plotter in Close-Range Applications*

The problems of plotting in close-range photogrammetry, their solution employing the analytical plotter, and an example based on architectural photogrammetry are presented.

ABSTRACT: *The flexibility of the analytical plotter allows the use of different procedures to improve the accuracy and overcome most of the problems of close-range applications such as handling of incomplete stereo-models, plotting projections onto different planes, and distortion corrections. The paper summarizes some of the procedures and software developed for the analytical plotter and IBM 370/158J with emphasis on architectural applications. The techniques described are applicable to similar problems in close-range photogrammetry.*

INTRODUCTION

THUS FAR, in close-range photogrammetry, the restitution of photographs has been carried out mainly by following an analogue or analytical approach. (In special cases it is possible to use a single photograph and, consequently, no further treatment is necessary[1].) The analytical approach has the limitation, of course, in that plotting of detail from the restituted model is usually time-consuming and impractical. Whereas the analogue approach is free from this drawback, its limitations are due mainly to the limited range and accuracy of the photogrammetric parameters when represented physically.

The analytical plotter offers the flexibility of the analytical approach together with the continuous plotting feature of the analogue one. We present here some of the problems encountered with the treatment of close-range photographs and how they can be overcome by using the analytical plotter. An architectural application is used as an example.

PROBLEMS ENCOUNTERED

The following problems are often encountered. Although time-consuming, some of them may be solved by using a conventional plotter.

1. Problems with the orientation of the photographs.
2. Plotting of orthogonal projections onto different planes.
3. Difficulties with stereo-vision due to continuous change in relative magnification between the two photographs.
4. Use of non-metric cameras.

We deal now with each of these problems separately.

ORIENTATION OF THE PHOTOGRAPHS

The problems with the orientation of the photographs can be attributed mainly to an incomplete stereo-model. In the case of architectural photographs, part of the overlap

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area is usually of details too close to the camera station to be within the range of the "Z-movement" of photogrammetric instruments (the analogue type in particular) or to be viewed stereoscopically. Other parts of the overlap may contain the sky and cannot consequently be used in the orientation. This results in an ill-conditioned solution for the orientation.

A software program was developed to overcome this problem or, at least, to improve the determination of the orientation parameters. The program accepts photo-coordinates of points to be used in the orientation. It also accepts the photo-coordinates of control points together with any known orientation parameters, e.g., coordinates of camera stations and rotations. A simultaneous bundle adjustment is then carried out to solve for the exterior orientation parameters of the two cameras. We may note that points in the "difficult" areas mentioned above can also be selected. The observation is done stereoscopically by eliminating the parallaxes in a small area at a time, or monocularly if stereo-vision is not at all possible. The program is too large to fit into the memory of the present control computer of the analytical plotter as it requires about 64K words. The solution is therefore carried out on the IBM 370/158J, and the orientations are fed back into the analytical plotter. (This procedure does not introduce loss in precision since the orientations are represented digitally in the plotter.)

It may be argued that the same procedure could be followed with an analogue instrument. The situation would not be so simple for two reasons:

(1) Accurate calibration of the instrument would have to be carried out to measure the photo-coordinates accurately, or an additional comparator would have to be used.

(2) The orientations cannot usually be fed into the analogue instrument without the introduction of errors.

Also related to the difficulties with the orientation is the utilization of photographs with large tilts. The layout of the site may require such photographs to be taken. Contrary to most analogue instruments, the range of tilts which can be accommodated in the analytical plotter is limited only by the available plotter software.

PLOTTING OF ORTHOGONAL PROJECTIONS ONTO DIFFERENT PLANES

To illustrate this problem we take as an example the building shown in Figure 1 in which some of the walls are not in the same plane. In a number of architectural applications it is required that an orthogonal projection of one of the walls onto a plane parallel to its face be produced. This may be achieved by taking a stereo-pair of photographs with a base approximately parallel to each wall in question. (It is not always possible to do this in practice due to the presence of obstacles.) However, fewer photographs would be needed if it were possible to plot other walls of the building from the same pair. In other words, it would be advantageous if it were possible to produce orthogonal projections of both walls HEBD and BCGE in Figure 1 from a stereo-pair of photographs taken from points O_1 and O_2. In the case of analogue instruments, this might be achieved by performing absolute orientation once so that the *x-y* plane in the model becomes parallel to wall BCGE and the other time so that it becomes parallel to wall HEBD. The limited range of rotations and the repetition of the orientation are drawbacks to this approach.

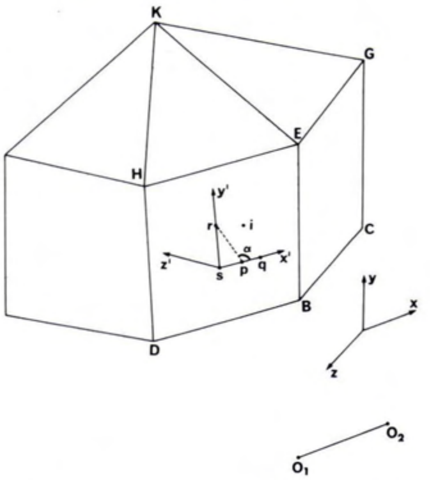

FIG. 1. Coordinate system definition for architectural applications.

We now introduce a general solution to this problem which can be applied in the case of the analytical plotter. Take as an example wall *HEBD* of the building. Assume that absolute orientation has been established and that the model coordinate system is the *xyz* system. Assume that points *r*, *p*, and *q* lie in the plane of the wall and can be accurately observed. It is assumed also that the three points are selected so that a set of perpendicular axes x' and y' can be drawn through the points as shown. (This can be done easily in practice.) Accordingly, the points will define the $x'y'z'$ Cartesian system with its x'-y' plane coincident with the plane of the wall. The real-time program of the instrument must then perform two additional functions so that plotting of the x' and y' coordinates can be achieved: (1) the movement of the measuring mark is constrained to be in the plane $x'y'$; and (2) plotting of the coordinates x' and y' is carried out.

We deal now with the two functions in some detail. Constraint of the measuring mark movement is desirable so that detail plotting in the plane of the wall can be carried out efficiently and accurately. The function is implemented by observing and storing in memory the coordinates of the three selected points r, p, and q in the model coordinate system xyz. The equation of the plane containing the three points is expressed by the determinant

$$\begin{vmatrix} x & y & z & 1 \\ x_r & y_r & z_r & 1 \\ x_p & y_p & z_p & 1 \\ x_q & y_q & z_q & 1 \end{vmatrix} = 0 \qquad (1)$$

It follows that the z-coordinate of any point i in the plane $x'y'$ can be evaluated from its x and y coordinates from the formula

$$z_i = \frac{Ax_i + By_i + D}{C} \qquad (2)$$

where

$$A = -\begin{vmatrix} y_r & z_r & 1 \\ y_p & z_p & 1 \\ y_q & z_q & 1 \end{vmatrix}$$

$$B = \begin{vmatrix} x_r & z_r & 1 \\ x_p & z_p & 1 \\ x_q & z_q & 1 \end{vmatrix}$$

$$D = \begin{vmatrix} x_r & y_r & z_r \\ x_p & y_p & z_p \\ x_q & y_q & z_q \end{vmatrix}$$

$$\text{and } C = \begin{vmatrix} x_r & y_r & 1 \\ x_p & y_p & 1 \\ x_q & y_q & 1 \end{vmatrix}$$

After observation of the three points, the real-time program branches to a routine which computes the coefficients A, B, C, and D above. The program is then "self-modified" so that the input from the Z-movement of the instrument is ignored and the Z coordinate is computed from the x- and y-coordinates using Equation 2.

In order to deal with the plotting of x' and y' coordinates we must first transform the coordinates of any point i from the model coordinate system xyz to the system $x'y'z'$. The transformation parameters between the two systems are established by using the three observed points r, p, and q as follows: the coordinates of the origin s of the system $x'y'z'$ in the system xyz are determined by computing the spatial angle between the line pq and pr. This angle is given by

$$\alpha = \cos^{-1} \frac{u_1u_2 + v_1v_2 + w_1w_2}{\sqrt{u_1^2 + v_1^2 + w_1^2} \cdot \sqrt{u_2^2 + v_2^2 + w_2^2}}$$

where (u_1v_1, w_1) and (u_2, v_2, w_2) are the components of the vectors pq and pr respectively in the system x, y, z.

The angle α lies in the plane $x'y'$ and can be used to determine the distance sr and sp from the right-angle triangle rsp (knowing rp and α). It follows that the coordinates of the points r, p, and q in the system $x'y'z'$ will be

$$\begin{pmatrix} 0 \\ rs \\ 0 \end{pmatrix}, \begin{pmatrix} sp \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} sq \\ 0 \\ 0 \end{pmatrix}.$$

The transformation parameters relating the two systems can then be computed using the equation

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \boldsymbol{R} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} x_o \\ y_o \\ z_o \end{pmatrix}. \qquad (3)$$

where (x', y', z') and (x, y, z) are now known for the three points, with the transformation parameters being the three rotations of the orthogonal matrix $\boldsymbol{R}$ and the three shifts (x_o, y_o, z_o).

Once the transformation parameters are established, Equation 3 is used to compute, in real time, the coordinates x' and y' of any point i. These coordinates are plotted by the plotting table routine of the system.

The computations of the transformation parameters are carried out in a routine to which the real-time program branches after observation of the points r, p, and q.

The procedure performed by the operator is

(i) Select three well positioned points on the face of the wall to be plotted.
(ii) Observe each of the points selected and store its coordinates by pressing one of the viewer switches.
(iii) Initiate the computations of the routines described by pressing another viewer switch.

The last routine will modify the real-time program so that the functions are achieved. A simplified flow chart of the modified program is shown in Figure 2. Normal flow of the program is restored by pressing another viewer switch.

The above procedure was tested using a stereo pair of photographs, one of which is shown in Figure 3. Three walls with different orientations were plotted. Figure 4 shows the plot obtained for the wall labelled A on the photograph. This wall had an inclination of about 89° with respect to the *x-y* plane of the stereo model. The other two walls had inclinations of 75° and 11° respectively.

RELATIVE MAGNIFICATION

Generally, magnification changes between the two photographs are a function of the tilts of the photographs and the distance between object point and camera.

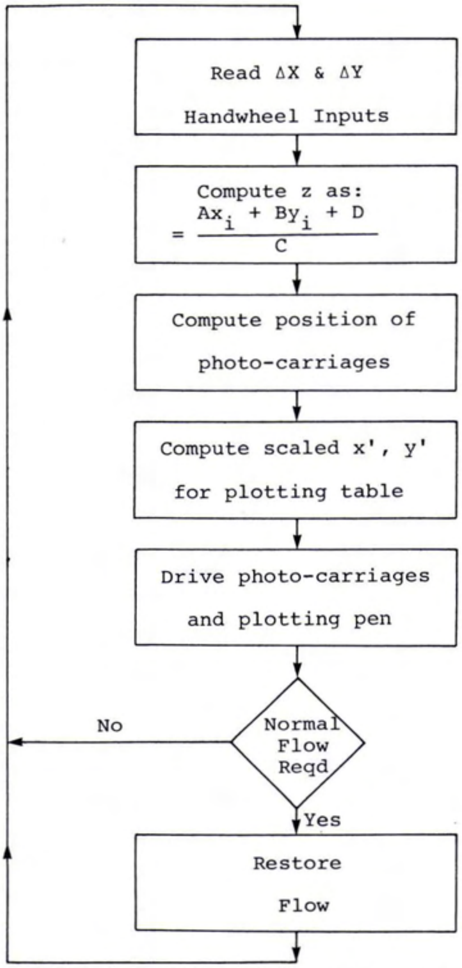

FIG. 2. A simplified flow chart of the real time program to plot different orthogonal projections.

We define the relative magnification between photo points i_1 and i_2 in Figure 5 as

$$r_i = \frac{d_1/D_1}{d_2/D_2}$$

where d_1, D_1, d_2, and D_2 are as shown in Figure 5.

Accordingly, if the two points are to be viewed as equal in size, the ratios between the magnifications of the two optical trains in the instrument should be equal to

$$\frac{1}{r_i}.$$

A simple procedure to establish and maintain the correct magnification is as follows: An initial magnification is established manually at two corresponding image points. Preferably, two corresponding areas of the photographs at which the magnifications are approximately equal are selected. The values of the magnification for each optical train are read off the viewer and fed into the real-time program. Suppose these are m_1 and m_2 for the left and right photos respectively, then the value of r_i can be readily evaluated by the real-time program at any other points i_1 and i_2 since the corresponding photo and model coordinates are known to it. An increment δ, by which m_1 and m_2 are to be altered, is computed using the equation

$$\frac{m_1 + \delta}{m_2 - \delta} = \frac{1}{r_i}$$

from which

$$\delta = \frac{m_2 - m_1 r_i}{1 + r_i}.$$

The change δ is used to update m_1 and m_2, and to adjust the zoom magnification in the optical train. The adjustment is carried out by step motors under computer control. Related to the above procedure we may note that since r_i is usually a slow changing function, the rate of updating m_1 and m_2 is not critical. A rate of 5 times per second or less should be sufficient.

USE OF NON-METRIC CAMERAS

Their relatively low cost, and general availability in varying sizes and focusing ranges make non-metric cameras attractive for use in close-range applications. Obvi-

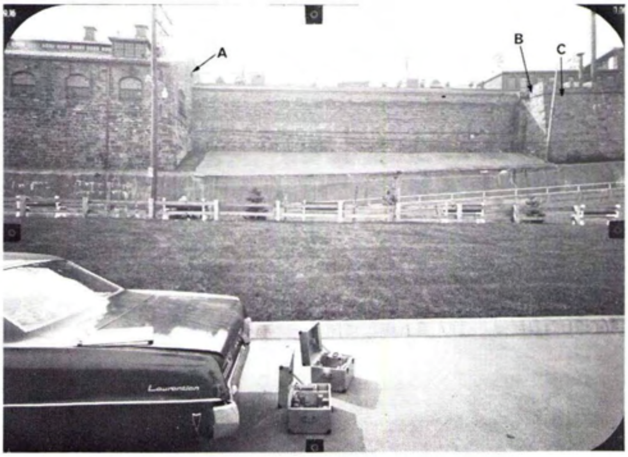

FIG. 3. One of a stereo pair of photographs of the historic structure, Nouvelle Casernes, Artillery Park, Quebec City. Orthogonal projections of the walls A, B, and C were plotted. The walls were inclined to the x-y plane of the model by approximately 89°, 75°, and 11° respectively.

ously, for precise evaluation, the interior orientation prarameters must be accurately determined for each photograph. The evaluation of non-metric photographs by using the analytical plotter is carried out as follows: Using the plotter, photo-coordinates of each stereopair of photographs are measured in relation to two convenient sets of axes in the plotter because usually no fiducial marks are provided with non-metric cameras. The photo-coordinates together with the available control are used in the computation of interior and exterior orientation parameters. For this purpose, a comprehensive mathematical model was developed and programmed in FORTRAN IV. It is based on the two fundamental conditions of analytical photogrammetry, namely the collinearity and coplanarity conditions, both of which have been expanded to include distortion parameters. The coplanarity condition is used to perform both calibration of each photograph and relative orientation of the stereo model, while the collinearity equa-

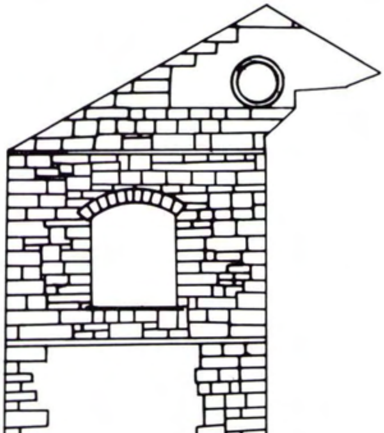

FIG. 4. Orthogonal projection of wall A shown in Figure 3. The wall's inclination to the x-y plane was approximately 89°.

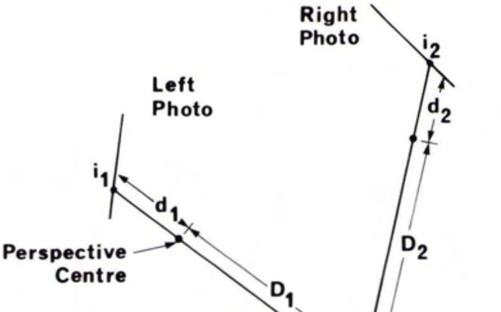

FIG. 5. Parameters affecting relative magnification of two photographs.

tions are used to relate the exterior orientation to the object coordinate system as defined by control points. The program is written to permit multiple stereo-models as well as photogrammetric blocks. The working unit is always the individual photograph. (More detail about the program can be found in Faig.[2])

The interior orientation parameters consist of the position of the principal point with respect to the instrument axes, the principal distance, and the coefficients of two polynomials to describe radial and tangential distortions. These parameters together with the ones for exterior orientation are then utilized by the real-time program to establish and evaluate each model.

As reported by Faig[2], the procedure was tested with a small object photographed with a Nikomat-FT equipped with a 50 mm Nikkor lens at a photo scale of 1:15. A contour plan in object scale (1:1) with half-centimetre contours was easily obtained while using 10 full control points. Residual coordinate errors of less than 2 μm in the photo scale for all 32 check points proved that non-metric photography was plotted without any loss in practical accuracy because the maximum error is much less than drafting accuracy.

CONCLUSIONS

Some of the problems usually encountered in close-range applications can be dealt with successfully, by using the analytical plotter. In future plotter models, it is expected that the control computer will have larger storage (in the form of core and disc), which would allow some of the software described to be accommodated in the control computer and, consequently, would make the plotter even more flexible.

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REFERENCES

1. Jachimski, J. J. "Problems of photogrammetric applications to architectural and urbanistic problems", *Proceedings ASP Annual Meeting,* St. Louis, March, 1974.
2. Faig, W. "Precision plotting of non-metric photography," *Proceedings of the Symposium of Commission V, ISP,* Washington, D.C., 1974.

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