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Planimetric Martian Triangulations

A planimetric triangulation of rectified Viking Orbiter photography was performed in order to provide control for the mapping of the prime Martian landing site.

Introduction

The modern cartography of Mars depends on the use of mosaics of rectified pictures taken by American spacecraft in 1969, 1971, and 1976 (Batson, 1973, 1976). For identification of the geologic unit(s) containing the landing site of explorers, triangulating in the map plane with these pictures is necessary. This paper is concerned with the theory and practice, mainly practice, of such triangulation.

Abstract: *Narrow-angle photographs, which have severe drawbacks for stereophotogrammetry, have advantages for simple plane triangulations. Rectified narrow-angle pictures corrected for map projection effects can be combined in the map plane in relatively accurate planimetric triangulations. Provided the strict precepts of least squares are not followed, these triangulations can incorporate considerable overdetermination without increase in the labor of solving the equations. These plane triangulations have been used successfully in the cartography of Mars and are illustrated here by a triangulation of the environs of the prime Martian landing site.*

The interpretation of the term rectification is unambiguous for areas that are small enough to render negligible the effects of curvature of the planetary surface. For the Mariner '69 and '71 pictures, however, the coverage is often very extensive and never really small enough to neglect curvature. Hence the Mars rectified pictures are generally computed as parts of maps at definite scales and on definite map projections. In principle, the picture element (pixel) position (x,y) is computed to a planetary surface position (λ,ϕ) and then to a map position (X,Y). In practice, the computation proceeds in reverse, as the position (λ,ϕ) is computed to an image position (x,y) and an appropriate brightness interpolated from the four surrounding pixels. The resulting picture is a valid portion of a map provided that the relief has negligible effects and the chosen spheroid of reference approximates the real surface. Since most of the Mariner '71 pictures are near-verticals with relatively narrow angle (14°), the relief distortions are rather small.

The Viking pictures are very narrow angle (1°) and most are tilted through 12.5°. Appreciable relief distortions can be anticipated here and may have to be taken into account in the more precise work with these images. They have been ignored in the triangulations described in this paper. Since the Viking mapping pictures cover very small areas (25 by 25 km), they are not rectified to map projections but are instead computed as orthographic projections of the landscape on planes tangent at the centers of the pictures. Their relatively small area permits the treatment of these pictures as fragments of maps on orthomorphic (conformal) projections. Indeed, in these triangulations the Viking orthographic pictures have been treated as pieces of maps on both the Mercator and the transverse Mercator projections. However, there are in fact small nonlinear differences between the orthographic pictures and the orthomorphic rectifications so that the tertiary triangulation, based on Viking Orbiter orthographic pictures, is slightly affected by system. A typical orthographic rectification of a Viking Orbiter picture is shown at Figure 1.

Triangulations of the Martian Surface

The primary triangulation of the surface of Mars (Davies and Arthur, 1973) depends on Mariner '71 pictures, which are corrected for distortion but are not rectified. The approach is that of a normal aerotriangulation except that the exposure stations were determined from the radio-tracking rather than the photogrammetry. Another abnormal feature was the assignment of the altitude or radius of the surface point from the result of radio-occultation rather than from the photogrammetry. This deviation from the norm is produced by the rather narrow angle of the Mariner A-cameras (about 14°). The precision of this triangulation is extremely variable, being a few kilometers in the Mars northern hemisphere and very often worse than ± 20 km in the southern. The poor quality in the south is a product of the smaller-scale coverage and, in the early part of the Mariner 9 '71 mission, the obscuration of surface by a veil of dust.

The primary triangulation was supplemented by plane secondary triangulations in difficult areas. These were not as successful as anticipated and were not taken through to completion except in those areas where the primary net was either too sparse or too inaccurate to control the mosaics. The main problems here were poor imagery and skimpy overlapping. The results, however, indicated that the plane triangulation method was essentially sound. There were lessons for future work of this type. In the Viking triangulations, the overlap was increased and all preparation and measurement restricted to professional photogrammetrists. Thus, the data problems associated with the Mariner '71 triangulations are absent from the Viking work, which has been easy to edit and clean up.

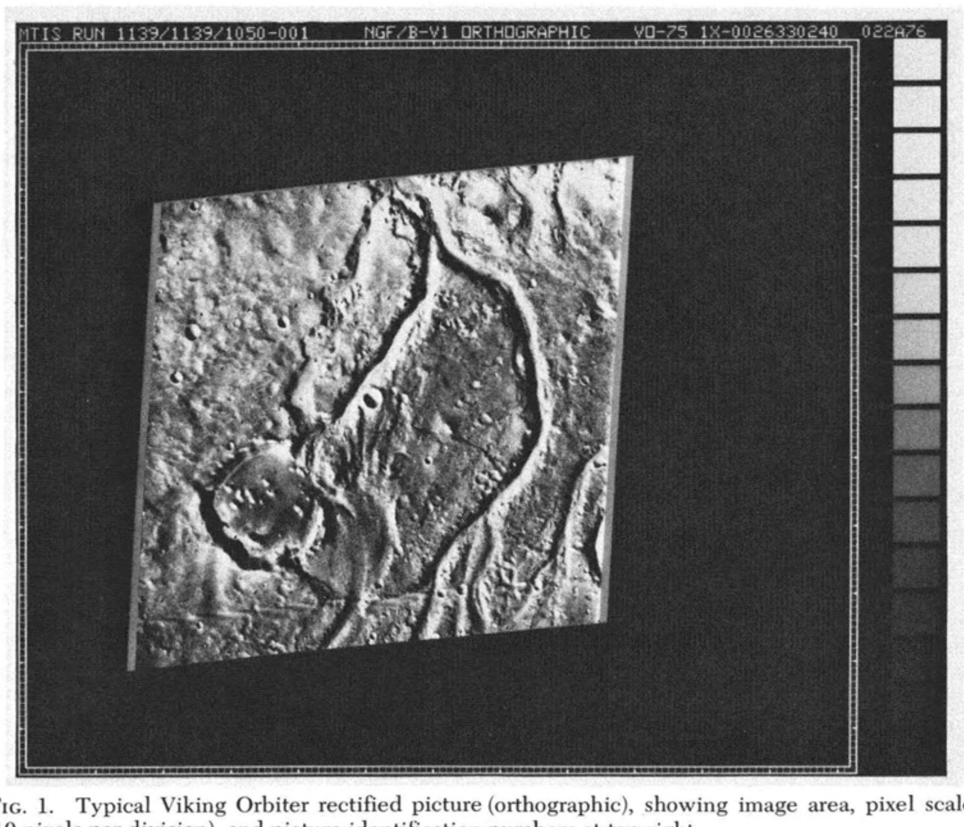

Fig. 1. Typical Viking Orbiter rectified picture (orthographic), showing image area, pixel scales (10 pixels per division), and picture identification numbers at top right.

THE TRIANGULATION MATERIALS, PREPARATIONS, AND MEASUREMENTS

The basic materials for the plane triangulations are rectified transparencies on stable-base film and corresponding prints, either rectified or unrectified. The hard copy images are produced in two ways, either by the so-called Dicomed process or by the Optronics Photowrite. The first is no more than a coaxial arrangement of cathode ray tube and copying camera in which the television type image is converted into a negative. In the second, the taped image is converted into its photographic equivalent in an electromechanical scanning arrangement with good dimensional stability. The photosensitive material is wrapped around a uniformly rotating drum. The tape controls the brightness of a diode which moves parallel to the axis of the drum. Each revolution of the drum thus provides one line of the image and the diode moves in uniform steps between lines.

The two systems have different distortion characteristics. Optronic pictures generally are subject to linear errors amounting to a top-to-bottom shear of a few pixels. These distortions are easily controlled and eliminated by measurements of four corner pixels whose coordinates are known in terms of lines and counts along lines. Controls of this kind are readily available in the pixel scales along the four sides of the pictures generated in the rectification programs. The correction routines are no more than linear double interpolations and need not be described here.

The dicomed distortions are rather more troublesome and their estimations require the measurements at midpoints of the scales measured above. Thus, in addition to the linear corrections made for Optronic pictures, the Dicomed pictures require quadratic corrections of the type

$$\delta x = ax + bx^2$$
$$\delta y = a'y + b'y^2$$

in which the coefficients are such that δx, δy vanish in the picture corners.

The prints are used in preparation for measurement, each point being marked on each print on which it occurs with a unique number. The practice here follows normal aerotriangulation procedure quite closely. The best points are small craters, but in the Mariner '71 secondary nets we also used small hills, corners of shadows, or whatever was available. The superior resolution of the Viking pictures generally provides many times the number of points needed in each overlap. Larger craters were measured by boxing them in with tangents, placing the measuring mark (cross) above and to the right, then below and to the left. The mean corresponds to the center of the crater.

THE TRIANGULATION THEORY

Let (X,Y) be plane rectangular coordinates in the map and (x,y) be measurements on the pictures. From the viewpoint of least squares, the appropriate observation equations are

$$\begin{aligned} \mu(X \cos\theta + Y \sin\theta) + a &= x \\ \mu(Y \cos\theta - X \sin\theta) + b &= y \end{aligned} \tag{1}$$

where μ is a scale factor, θ a rotation, and a,b the unknown shifts. With the obvious substitutions

$$\mu \cos\theta = p,\ \mu \sin\theta = q,$$

these can be written in more convenient form

$$\begin{aligned} pX + qY + a &= x \\ pY - qX + b &= y \end{aligned} \tag{2}$$

These are not linear in the unknowns, as they contain their products. Further, for m pictures and n points, we have $4m + 2n$ unknowns. In any reasonable arrangement, the addition of each new picture implies the inclusion of at least two new points; for m pictures, then, we have about $8\,m$ unknowns. This is distinctly more unfavorable than the primary method (Davies and Arthur, 1973) and hence has not been given serious attention here. The inverse of Equations 2 is the pair

$$\begin{aligned} Px - Qy + A &= X \\ Py + Qx + B &= Y \end{aligned} \tag{3}$$

These are easily applied to map control, but there are some ambiguities for the ties. For the tiepoint common to pictures i and k, we have

$$X_i - X_k = 0,$$
$$Y_i - Y_k = 0,$$

that is,

$$(P_i x_i - Q_i y_i + A_i) - (P_k x_k - Q_k y_k + A_k) = 0,$$

with a similar equation in Y. The subtractive nature of the tiepoint equation is thereby somewhat different from the direct nature of the control-point equation. When the tie falls on three or four pictures, as it often does, an arbitrary choice must be made of the available equations. For example, when a point falls on three pictures, i, j, k, the obvious selection appears to be

$$X_i - X_j = 0, X_j - X_k = 0,$$

or, if on four pictures, i, j, k, l,

$$X_i - X_j = 0, X_j - X_k = 0, X_k - X_l = 0.$$

The only feature we object to here is the lack of symmetry in the treatment of the various images. In the last, i and l occur once only and are treated differently from j and k. For this reason, we prefer to write our tiepoint equations as

$$X_i - \overline{X} = 0, X_j - \overline{X} = 0, X_k - \overline{X} = 0, \ldots$$

in which $\overline{X}$, $\overline{Y}$, are the means. In the case of three pictures, our X_i equation is

$$\tfrac{2}{3}(P_i x_i - Q_i y_i + A_i) - \tfrac{1}{3}(P_j x_j - Q_j y_j + A_j) - \tfrac{1}{3}(P_k x_k - Q_k y_k + A_k) = 0.$$

There are theoretical drawbacks here, of course, as the residuals are now conditioned to sum to zero. It must be remembered, however, that the entire method is not rigorous from the viewpoint of least squares. A more serious problem is the assignment of weights, particularly between tiepoint equations and control equations. Our treatment here is frankly approximate and heuristic. Weights are assigned roughly on the basis of the known precision of the control and the measurements but have been deliberately altered to avoid excessively large residuals in the primary control. In many cases we can largely evade the weight problem by performing separate internal and external adjustments.

Despite its theoretical drawbacks, the non-rigorous method based on Equations 3 is bound to be attractive. Since the number of unknowns is four per picture and is independent of the number of points, the redundancy can be increased to any desired level, provided points can be found in the overlaps. Our results clearly indicate that this redundancy produces plane triangulations that are more precise than the corresponding rigorous triangulations with fewer ties could produce.

The Adjustment Calculations

Our adjustment computations always use the same equations, namely, Equations 3 and tiepoint equations of the type

$$X - \overline{X} = 0, Y - \overline{Y} = 0,$$

but the various programs use different data sets. First, note that a direct one-shot solution applied to a block of 30 to 40 pictures is unlikely to succeed with raw data. We attempted this in the case of the Mariner '71 secondary triangulations with the result that in some situations we had large residuals but no indication of the location of the excessive errors.

To meet this, the block was divided into 'bricks' of six to ten pictures and each brick assembled separately into one figure using two controls. These could be either real controls with known X,Y values or merely two points for which we roughly estimated these values. In either case, the program BRICK converted the pictures of the brick into a single figure coordinated approximately in the map system, in kilometers. The discrepancies were printed out as the deviations

$$\delta X = X_i - \overline{X}, \delta Y = Y_i - \overline{Y}$$

for each image, the means being those of the several images referring to the same ground point. Thus, BRICK diagnoses the existence of errors and misidentifications of the picture-picture ties used in constructing the brick.

The next step is to unite all the bricks of the block in one free figure using the ties along the common edges of the bricks, and again, two real or fictitious controls. This is performed by the program EXTEND, which is identical to BRICK, but applied to different data, i.e., the brick-to-brick ties. This program completes the examination of the internal consistency of the block.

At this point, the adjustment sequence can branch. In the treatment of the Mariner '71 secondary sets we used MORTAR, which repeats EXTEND but incorporates the external controls by adding Equations 3 for each control point. This certainly brings in weighting problems and, we found, difficulties in assessing the validity of certain control points.

We now favor the simpler program SCALE, which adjusts the EXTEND figure to the external control as a rigid figure free only to change size, orientation, and position (but not shape). Equations 3 are used in the form

$$\begin{aligned} PX_E - QY_E + A &= X \\ PY_E + QX_E + B &= Y \end{aligned} \qquad (4)$$

In these X_E, Y_E are the extension values of the controls while P, Q, A, and B are the single set of unknowns for the SCALE adjustment. (In practice, we also applied SCALE to the output of MORTAR to take care of a slight scale effect detected in BRICK and EXTEND. See below.) In the Mariner '71 adjustments, we noted that the brick-to-brick discrepancies tended to be larger than the photo-to-photo discrepancies of BRICK. In order to suppress these seam effects, each brick was subjected to a final cosmetic adjustment LOCAL identical to MORTAR but using the meaned brick ties as controls. This step was quite successful in bringing the discrepancies to a uniform level.

It had been intended to subject the cleaned-up block to a single one-shot solution in which the pictures are simultaneously adjusted to each other and to the external controls, but time and cost considerations ruled this out.

The superposition of a relatively precise secondary block on imprecise primary control brought out some unexpected problems, produced in part by the departure of Equations 3 from complete rigor. It was noted in each BRICK solution that there were outstanding residuals at the two arbitrary controls and that these were always equivalent to vectors directed *inward* along the joint of the two controls. In principle, there should be no residuals, as the control configuration is barely sufficient for a solution; there is no redundancy. We assume that this 'shrinkage' of the figure arises from the fact that we minimize the residuals expressed in the computed, or map, system. This implies that the residuals can be reduced not only by appropriately adjusting each P, Q, A, and B but also *by reducing the scale of the figure slightly*. The effect is most marked when the controls are few and imprecise while the ties are numerous and precise. In recent work, we have evaded this problem and used SCALE in preference to MORTAR. This is theoretically defective but nevertheless both simpler and more satisfying.

The Viking P-20 Triangulations

The most interesting plane triangulations performed to date relate to the site of the first successful landing on Mars. This area was first covered by the so-called P-20 photography, that is, images acquired in the twentieth pass over the target site. The problem was to provide a relatively precise control net for the mosaic constructed with the P-20 pictures. There was no primary control in the area, which is distinctly monotonous and featureless. In order to acquire control we first selected nine Mariner '71 pictures taking in 12 primary control points. We thus performed a plane secondary triangulation yielding six points that could be identified in the Viking Orbiter P-20 pictures. These were used in a tertiary plane triangulation of 25 Viking pictures to provide a net of control around the landing site.

In the secondary triangulation (see Figure 2), there is then only one brick of nine pictures. The largest deviations of ties from their means are 0.6 and 0.9 km in X and Y, respectively. The rms value in both coordinates is close to one quarter of a kilometer. The secondary figure was fitted to 12 Davies' primary points (Davies, 1974) using SCALE. Thus the adjustment determined only four unknowns: P, Q, A, and B. Two of the Davies' points, numbers 478 and 483, showed excessive residuals, 17 km and 19 km, both in Y, and were rejected. The adjustment was repeated without these and the maximum residual dropped to 2.1 km. The rms value of the residuals at the ten remaining primary control points is about ±0.8 km in either coordinate.

The tertiary triangulation of 25 Viking picture (see Figure 3) was divided into three bricks of similar sizes. The interior precision of the bricks is shown in Table 1.

Despite their derivation from Dicomed images with relatively large nonlinear distortions, these bricks have excellent internal

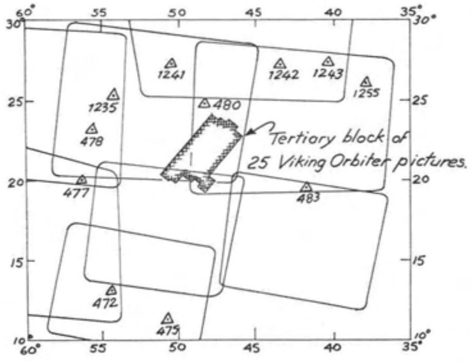

Fig. 2. The Mariner 9 secondary block showing Davies' primary points and incidence of tertiary block.

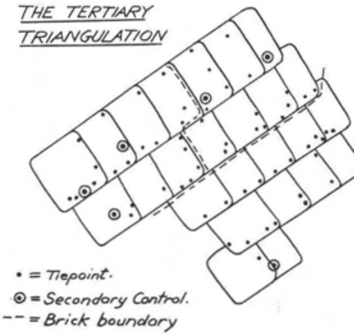

Fig. 3. The Viking Orbiter tertiary block showing ties and secondary controls.

TABLE 1. INTERIOR PRECISION OF THE BRICKS.

	Maximum meters		rms meters	
	δX	δY	δX	δY
Brick 1	59.0	56.1	±21.7	±25.8
Brick 2	98.5	62.2	±47.4	±26.6
Brick 3	43.2	33.4	±16.3	±13.5

precision. The BRICK adjustments were followed by a single EXTEND adjustment to form a single tertiary figure. For the brick-to-brick deviations from means, we have

	Maximum meters		rms meters	
	δX	δY	δX	δY
Brick-to-Brick	97.9	169.7	53.9	84.7

From this, the overall coherency of the tertiary figure is about 100 meters. This figure covers a relatively small area on the Martian surface and includes only six points common to the secondary net. These were used in another SCALE adjustment. The results are shown in Table 2.

The largest residual is 1.8 km and the rms value is about ±0.9 km. Since the nonrigorous approach represented by Equations 3 does not lead to an inverse normal matrix involving the coordinates, we have no direct estimates of the final precision of the tertiary net. It appears from the above, however, that the positions are consistent with each other to 100 m and cannot be in error with respect to the Mars fundamental circles by more than 2 km.

CONCLUSIONS

For the plane triangulations discussed in this paper, the ideal materials are ultra-narrow-angle pictures that are rectified versions of near-verticals. The hard copy should be generated by photomechanical devices such as the Optronics Photowrite rather than the less expensive cathode ray tube display with coaxial camera. The Mariner '71 images used in this paper were produced by a Photowrite, whereas the Viking Orbiter images were generated by the CRT-camera method, which produced large nonlinear errors. All coordinates were determined by measurement on a Mann 422-D monocomparator with 5× eye pieces. Even lower powers would be preferable.

The measurements indicated that in-focus images, in which each pixel can be seen as a discrete panel, are undesirable. Better measurements are made on slightly out-of-focus images in which the separate pixels cannot be distinguished. The picture then appears more as a rather grainy photograph.

Stereoscopy no doubt has a useful role, but we never found it really necessary on the Mariner '71 pictures for the purposes of triangulation. There was even less need in the Viking Orbiter imagery, with its great number of well-defined pass points.

The Mariner 9 imagery, which had extremely small overlap (usually < 2 percent), is affected by frequent misidentification of common points that hampers the reductions with an abundance of blunders. This rather serious data problem was eventually solved by the bit-by-bit treatment in 'bricks'. This approach accorded with the rather limited computer facilities available and with the economic constraints that apply to this work.

Despite these constraints and limitations, the non-rigorous approach in which the equations are written the wrong way round, has real advantages. Since the only unknowns are the constants P, Q, A, and B, for each picture or brick, the redundancy in points can be increased to the limits of available points without increase in the labor of solution. This feature will always be attractive. Indeed, there is definite evidence that these plane triangulations are more precise than conventional rigorous primary nets.

The Viking Orbiter triangulation was performed with the materials at hand, namely Dicomed hard copy of orthographic rectifications of tilted pictures. These are definitely inferior to Photowrite versions of true orthomorphic rectifications of near-verticals. Yet even with these cost-limited materials, we have been able to perform a Martian surface triangulation with an estimated *local* precision of 100 m. Who would have believed a decade ago that we would be achieving these precisions on the surface of a distant planet?

TABLE 2. RESULTS OF THE TERTIARY NET.

	Mercator Coordinates (km)		Residuals (km)	
Point	X	Y	δY	δY
90	849.0	146.3	−1.8	−0.6
307	642.8	154.9	+1.3	−1.6
308	805.4	216.2	+0.1	−0.0
340	765.1	288.4	+0.4	+1.0
341	715.3	320.1	−0.2	−0.0
342	694.9	298.1	+0.1	+1.2

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