Ian Dowman
University College London
London WC1E 6BT, United Kingdom

Model Deformation: An Interactive Demonstration

Height errors due to incorrect relative orientation are displayed on a CRT terminal.

Abstract: *A computer program which determines errors due to incorrect relative orientation is described. The program is written in APL for use on a CRT terminal; its purpose is to demonstrate the nature of model deformation to students.*

Introduction

The deformation of photogrammetric models due to errors in relative orientation is a phenomenon well known to practicing photogrammetrists. However it is difficult to demonstrate the full extent of the deformation to students when dealing with the subject in a theoretical way; this is partly because the deformation is three-dimensional. The problem is easier if the students can visualize a model expressed in mathematical terms but this in itself is difficult in three dimensions. If it is possible for the class to do practical work, the nature of the deformation may become clearer.

A good way to approach practical classes is to set up a model with no errors due to relative orientation and ask the student to investigate the effect on the heights of introducing small rotations and shifts to the projectors. The difficulty with this approach is that on most instruments y-parallax is also introduced, thus making the measurement of heights difficult. Exceptions to this are the Kern PG2 and the Cartographic Engineering CP1; on these instruments model deformations can easily be introduced because the x and y components of the ω and ϕ rotations can be changed separately.

It is possible to demonstrate three-dimensional deformation on the blackboard or with an overhead projector but the possible variations which can be drawn are limited. The use of a computer can help overcome these problems. It can be used in several ways. Perhaps the most accessible is to compute the deformation on a batch machine and to print the output in diagrammatic form on a graphic plotter or line printer. A much more versatile method is an interactive approach: if a Cathode Ray Tube display or a graphic plotter is available then an ideal solution is reached. A storage CRT allows one plot of a single deformation pattern to be produced at a time. A refresh CRT is potentially more powerful because the display can become dynamic and changing deformations can be demonstrated.

Computer Program

A software package has been developed at the University of New Brunswick using APL language implemented on a 4015-1 Tektronix Computer Display Terminal, which uses a storage CRT and is connected to an IBM 370/158J. A hardcopy of the image on the screen can be produced on the Tektronix 4631 Hardcopy unit. A flow diagram of the system is shown in Figure 1. The model dimensions can be chosen either to match the available instrumentation or to be at photograph scale; in either case, the effect of different principal distances or short bases can be demonstrated. In the program developed, the deformation is computed at 15 points in

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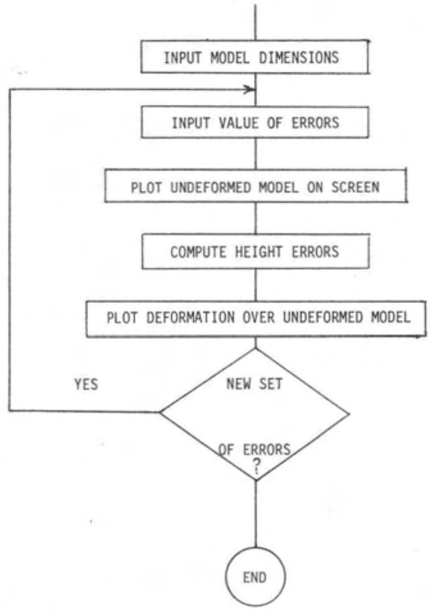

FIG. 1. Flow diagram of program DEFORM

the model as seen in Figure 2. The standard formula was used:

$$\Delta Z = (\Delta bx_2 - \Delta bx_1)\frac{Z}{B} + \frac{X}{B}\Delta bZ_1 - \frac{(X-B)}{B}\Delta bZ_2 + \frac{YZ}{B}(\Delta k_1 - \Delta k_2) + \frac{Z^2 + (X-B)^2}{B}\Delta\phi_2 - \frac{Z^2 + X^2}{B}\Delta\phi_1 + \frac{XY}{B}\Delta w_1 - (X-B)\frac{Y}{B}\Delta w_2 .$$

Plotting is done by using APL library functions. A printout of the driver function DEFORM and the function SUMERR, which computed the deformation, are shown in Figure 3. The screen coordinates of the undeformed model were taken off a drawing on graph paper which was constructed to be of the correct proportion; the coordinates of the errors are obtained by adding the height errors to the initial y coordinates.

The program can be cycled as many times as required to show deformations due to single rotations of either projector, for single translations, or for any combination of these.

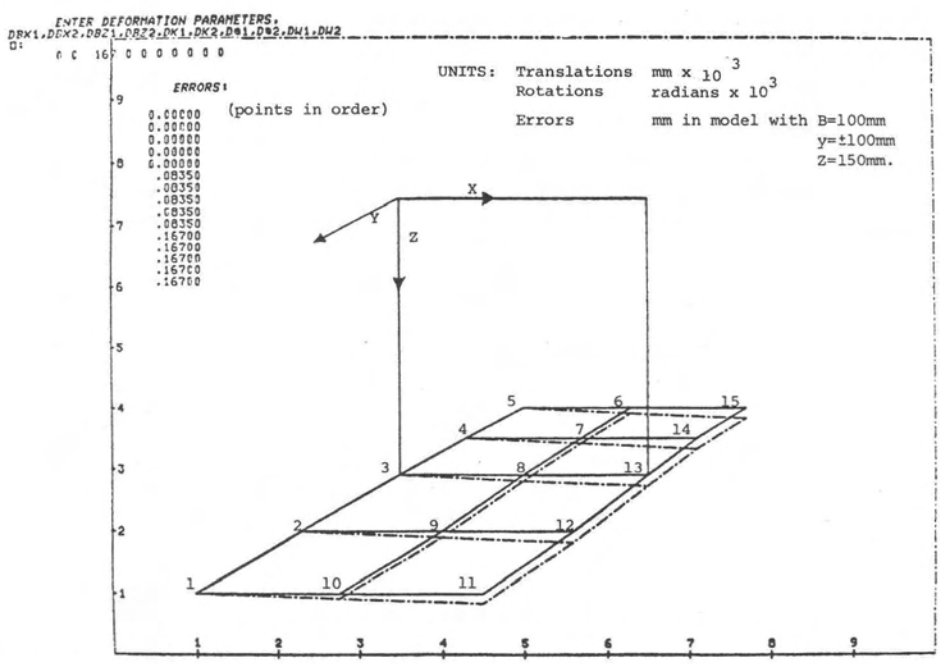

FIG. 2. Plot of errors caused by an error of 0.167 mm in Z_1. Units and point numbers have been added to the plot.

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    ∇DEFORM[⎕]∇
    ∇ DEFORM;CR
[1]   CR←⎕AV[157]
[2]   'ENTER MODEL DIMENSIONS, B Z Y:'
[3]   B←⎕
[4]   X1←B[1]
[5]   Y1←B[3]
[6]   Y2←B[3]÷2
[7]   X←(5⍴0),(5⍴X1÷2),(5⍴X1)
[8]   Y←Y1,Y2,0,(-Y2),(-Y1),(-Y1),(-Y2),0,Y2,Y1,Y1,Y2,0,(-Y2),(-Y1)
[9]   DATA←1,2.3,3.5,4.3,5,6.3,5.7,5,4,2.75,4.5,5.6,6.5,7.1,7.7,5,4.3,7.1,6.5,3.5,3.5,6.5,6.5,5.6,
      2.3,1,2.75,1,2,2.9,3.5,4,4,3.5,2.9,2,1,1,2,2.9,3.5,4,4,3.5,3.5,2.9,2.9,7.4,7.4,2.9,2,2,1,1
[10]  DATA←⍉ 2 27 ⍴DATA
[11]  PT← 15 2 ⍴0
[12]  PT[;1]←15⍴DATA[;1]
[13]  PLOT DATA
[14]  'ENTER DEFORMATION PARAMETERS,'
[15]  'DBX1,DBX2,DBZ1,DBZ2,DK1,DK2,D⍉1,D⍉2,DW1,DW2'
[16]  U←⎕
[17]  U←U÷1000
[18]  DZ←SUMERR U
[19]  DZ←DZ÷B[1]
[20]  CWRITE 'ERRORS:'
[21]  CWRITE(2⍴CR),,(25 5 ⍕ 15 1 ⍴DZ),CR
[22]  PT[;2]←(15 1 ⍴(15⍴DATA[;2]))- 15 1 ⍴DZ
[23]  6 PLT PT
[24]  6 PLT 2 2 ⍴PT[1;],PT[10;]
[25]  6 PLT 2 2 ⍴PT[2;],PT[9;]
[26]  6 PLT 2 2 ⍴PT[9;],PT[12;]
[27]  6 PLT 2 2 ⍴PT[3;],PT[8;]
[28]  6 PLT 2 2 ⍴PT[8;],PT[13;]
[29]  6 PLT 2 2 ⍴PT[4;],PT[7;]
[30]  6 PLT 2 2 ⍴PT[7;],PT[14;]
[31]  6 PLT 2 2 ⍴PT[6;],PT[15;]
[32]  WAIT 15
[33]  'DO YOU WISH TO ENTER NEW SET OF PARAMETERS?'
[34]  A←⎕
[35]  →0×⍳0=A
[36]  →11
    ∇
      ∇SUMERR[⎕]∇
    ∇ DZ←SUMERR U
[1]   C← 5 15 ⍴0
[2]   C[1;]←(U[2]-U[1])×B[2]
[3]   C[2;]←((X×U[3])-(X-B[1])×U[4])
[4]   C[3;]←(U[5]-U[6])×Y×B[2]
[5]   C[4;]←(((B[2]×B[2])+((X-B[1])*2))×U[8])-((B[2]×B[2])+(X×X))×U[7]
[6]   C[5;]←(X×Y×U[9])-(X-B[1])×Y×U[10]
[7]   DZ←(+⌿C)
```

FIG. 3. Computer listing of functions DEFORM and SUMERR. Other functions used are from APL library.

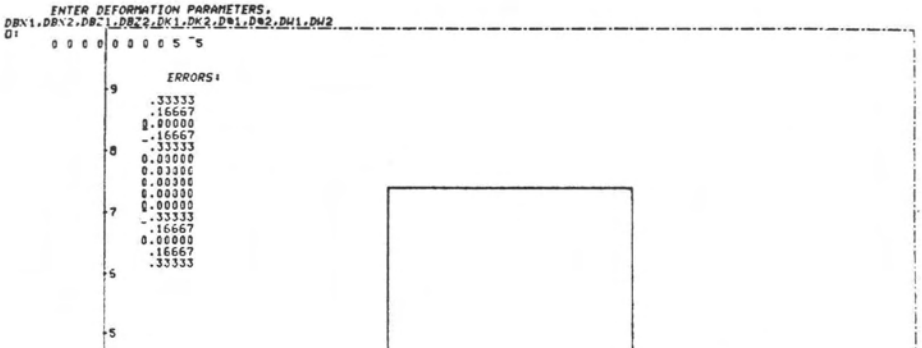

FIG. 4. Plot of errors caused by an error of 0.005 radians in ω_1 and an error of -0.005 radians in ω_2. An explanation of the units and point numbering are given on Figure 2.

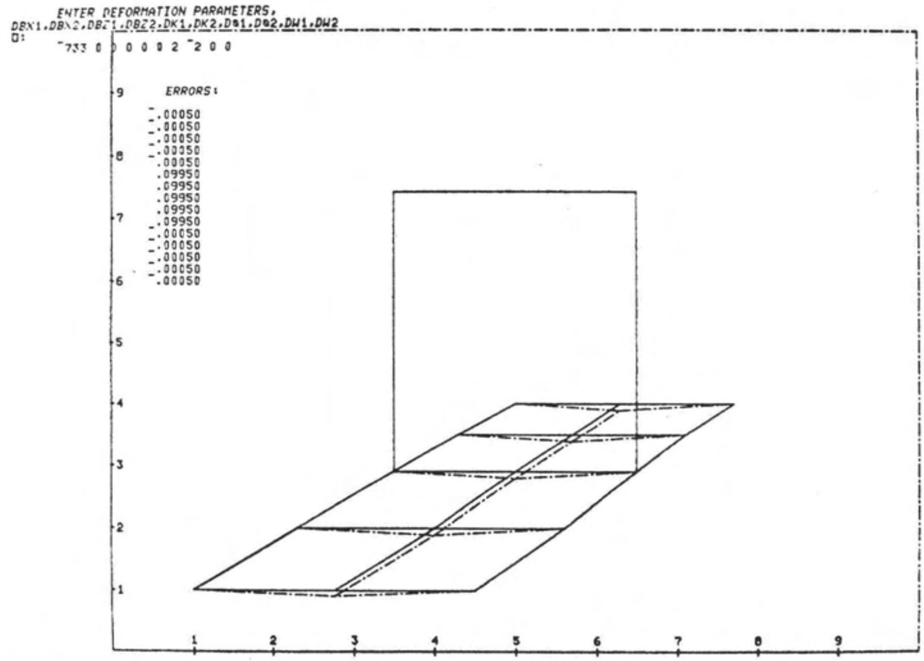

FIG. 5. Plot of residual errors after equal and opposite ϕ of 0.002 radians and ΔX of -0.733 mm. The ΔX removes the constant error caused by ϕ.

Sample outputs are shown in Figures 2 and 4. The program can be called again for different model dimensions.

The program can be used by individual students who wish to experiment with different error combinations or if class viewing facilities are available the deformations can be demonstrated to all.

Absolute Orientation

Some of the height errors caused by errors in the elements of relative orientation can be eliminated during absolute orientation. The rotations used for absolute orientation can be demonstrated by using equal and like phi with a bz movement for rotation about the Y axis, and by using equal and like omega for rotation about the X axis. A datum shift can be demonstrated by using a change, ΔX, in the base. A combination of $\Delta\phi_1$, $\Delta\phi_2$, and ΔX shows the residual error due to equal and opposite phi after absolute orientation; this is illustrated in Figure 5.

Residual errors due to variations in relief cannot be demonstrated on the model as it is constructed at present.

Conclusions

The use of an interactive computer terminal can aid the students to understand the nature of errors in relative orientation; this paper shows how this can be done for height errors. The idea could be extended to demonstrate the effect of orientation elements on y parallax in a model and on planimetric errors and has applications in demonstrating errors in any model which can be modeled mathematically.