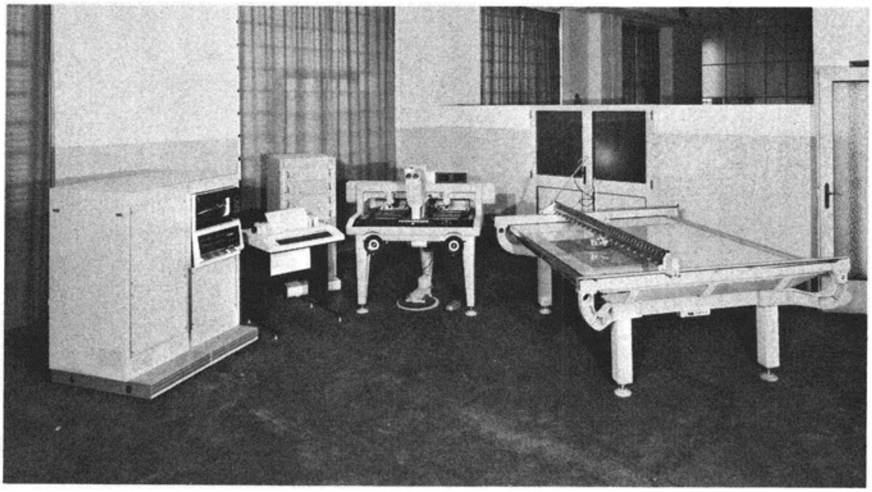

FRONTISPIECE. The Galileo Digital Stereocartograph.

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The D.S.-Type Galileo Analytical Plotters

The Galileo Digital Stereocartographs feature software with simplified differential formulas.

INTRODUCTION

THE ANALYTICAL PLOTTERS built by Galileo are essentially stereocomparators interfaced with an electronic computer (Frontispiece): the photo coordinates x_1, y_1, x_2, and y_2 measured by the stereocomparator are the input of a real-time program which determines as an output, 50 times per second, the ground coordinates X_T, Y_T, and Z_T. Of course, the orientation parameters of the stereopair must be determined by preliminary operations. The computations, which are very simple, are based on the formulas given in Figure 1, which are self-explanatory (see also Inghilleri, 1973).

It must be understood that an analytical plotter, as described above, could not be easily operated because of the necessity of having four manual controls for the movements x_1, y_1, x_2, and y_2. Furthermore, the operator should tentatively find the contour lines. In fact, it is necessary to have a feedback from the computer to the stereocomparator in order to eliminate the y-parallax automatically. In addition, when a contour line must be plotted, the x-parallax corresponding to a stereoscopic collimation on a horizontal plane of predetermined elevation H, must be automatically fed into the stereocomparator. This problem was dealt with in the development of the prototype of a Digital Stereocartograph (D.S.)-type analytical plotter, presented at the XII International Congress for Photogrammetry in Ottawa in 1972. Another solution was displayed at the XIII International Congress for Photogrammetry in Helsinki in 1976 (see DeMichelis, 1976).

In the meantime, another D.S.-type analytical plotter, the P-C-1 (Photogrammetric Compiler), has been developed and built. Due to different settings of the moving carriages of the stereocomparator, a new solution based on differential formulas had to be found.

In this connection it was realized that differential formulas apply very well also to the

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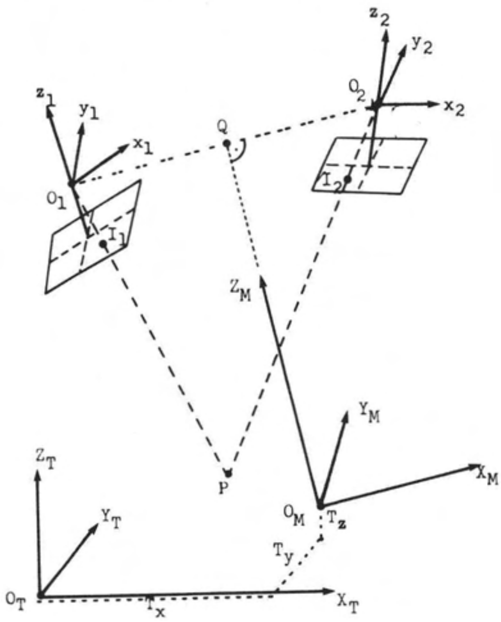

DEFINITIONS

O_M-$X_MY_MZ_M$ is the model coord. reference system having X_M parallel to O_1O_2 and Z_M perpend. to O_1O_2 in the plane $O_1O_2z_1$; $O_MQ = p$ and $O_1Q = p/4$

Coord. of $O_1 = -p/4,\ 0\ ,\ p$ p=focal length
Coord. of $O_2 = p/4,\ 0\ ,\ p$
Coord. of $I_1 = X_1 - p/4,\ Y_1, Z_1 + p$
Coord. of $I_2 = X_2 + p/4,\ Y_2, Z_2 + p$

The direction tangents of the rays O_1P and O_2P are denoted as: t_{x1}, t_{y1}, t_{x2}, t_{y2}

O_T-$X_TY_TZ_T$ is the ground coord. reference system having X_T approx. parallel to O_1O_2

T_x, T_y, T_z are the ground coord. of O_M and λ is the scale factor

COMPUTATIONS FOR DETERMINING THE GROUND COORD. OF A POINT P

$$X_1 = x_1\cos(x_1X_M) + y_1\cos(y_1X_M) - p\cos(z_1X_M)$$
$$Y_1 = x_1\cos(x_1Y_M) + y_1\cos(y_1Y_M) - p\cos(z_1Y_M)$$
$$Z_1 = x_1\cos(x_1Z_M) + y_1\cos(y_1Z_M) - p\cos(z_1Z_M)$$

$$X_2 = x_2\cos(x_2X_M) + y_2\cos(y_2X_M) - p\cos(z_2X_M)$$
$$Y_2 = x_2\cos(x_2Y_M) + y_2\cos(y_2Y_M) - p\cos(z_2Y_M)$$
$$Z_2 = x_2\cos(x_2Z_M) + y_2\cos(y_2Z_M) - p\cos(z_2Z_M)$$

$$t_{x1} = X_1/Z_1 \quad t_{y1} = Y_1/Z_1 \quad t_{x2} = X_2/Z_2 \quad t_{y2} = Y_2/Z_2$$

$$D_z = p/2(t_{x1} - t_{x2}) \quad X_M = D_z t_{x1} - p/4 \quad Y_M = D_z t_{y1} \quad Z_M = D_z + p \qquad (D_z = Z_P - Z_{O_1})$$

$$X_T = \lambda\left[X_M\cos(X_MX_T) + Y_M\cos(Y_MX_T) + Z_M\cos(Z_MX_T)\right] + T_x$$
$$Y_T = \lambda\left[X_M\cos(X_MY_T) + Y_M\cos(Y_MY_T) + Z_M\cos(Z_MY_T)\right] + T_y$$
$$Z_T = \lambda\left[X_M\cos(X_MZ_T) + Y_M\cos(Y_MZ_T) + Z_M\cos(Z_MZ_T)\right] + T_z$$

FIG. 1. Formulas for the numerical determination of the ground coordinates of a point.

software of the standard D.S. (Frontispiece). Furthermore, if *simplified* differential formulas are used, the same real-time program, with some slight variations, could be used for the three different types of Galileo analytical plotters now available.

Due to these simple formulas, the real-time program, originally written in assembler language, could be written in FORTRAN language without decreasing the cycling speed.

It is worth mentioning that differential formulas cope very well with the D.S.-type analytical plotters because of the simplicity of their mechanical and electronic components. This simplicity depends essentially on the fact that small corrections, Δx, Δy, to both photocoordinates must be computed and fed to the hand moved carriages. When a profiling mode of operation is required, motors controlled by the computer operate the carriage movement.

Mechanical Description of the D.S.-Type Analytical Plotters

In the D.S.-type analytical plotters, two plate coordinates among the four (x_1, y_1, x_2, y_2) have to be considered as independent (those manually or motor operated), and two plate coordinates have to be considered as dependent (those controlled by the computer). In the following formulas the dependent plate coordinates are denoted with an asterisk.

Of course, this distinction is meaningless when the interior, relative, and absolute orientations are performed. In these cases, all the plate coordinates have to be considered as independent coordinates. When single points must be determined, only one plate coordinate is controlled by the computer for clearing the y-parallax. For the sake of simplicity, in the following discussion two dependent plate coordinates are considered.

In the P-C-1 analytical plotter, a main carriage moving in the x and y directions holds the plates. One differential motion is fed to the left optical system and the other differential motion is fed to the right one.

With the possibility of two different photograph positionings, two different mechanical arrangements must be considered. In the first arrangement (Figure 2a) the left optical system moves in the y direction and the right one moves in the x direction so that $y_1^* = y_2 + \Delta y$ and $x_2^* = x_1 + \Delta x$ (x_1, y_2 = indep. coord.; y_1^*, x_2^* = depend. coord.). In the second arrangement (Figure 2b) the left optical system moves in the x direction and the right one in the y direction so that $x_1^* = x_2 + \Delta x$ and $y_1^* = y_2 + \Delta y$ (x_2, y_1, = indep. coord.; x_1^*, y_2^* = depend. coord.). In the standard D.S., both the plateholders move in the y direction and both the optical systems move in the x direction and differential gears allow the differential movements, Δx and Δy. In order to use the same arrangement of the P-C-1, we can imagine that the photos are put on the main carriage, M, and that the right optical system has differential movements, Δx and Δy, so that $x_2^* = x_1 + \Delta x$ and $y_2^* = y_1 + \Delta y$ (x_1, y_1 = indep. coord.; x_2^*, y_2^* = dep. coord.).

Condition Equations for the Plate Coordinates

Using the notations of Figure 1, where a stereoscopic y-parallax free collimation on a given horizontal plane (elevation $= H$) must be maintained, the photocoordinates must satisfy the following two condition equations (see Inghilleri, 1973):

$$\begin{aligned} t_{y_1} - t_{y_2} &= 0 \\ k_3 t_{x_1} + k_4 t_{y_1} + k_5 + k_6 t_{x_2} &= 0 \end{aligned} \tag{1}$$

where

$$\begin{aligned} k_2 &= 2(H - T_Z)/\lambda p \\ k_3 &= \tfrac{1}{2}\cos(X_M Z_T) + 2\cos(Z_M Z_T) - k_2 \\ k_4 &= \cos(Y_M Z_T) \\ k_5 &= \cos(Z_M Z_T) \\ k_6 &= \tfrac{1}{2}\cos(X_M Z_T) - 2\cos(Z_M Z_T) + k_2 \end{aligned} \tag{2}$$

(only the first equation need be taken into consideration when it is not necessary to keep track of a contour line). These two equations can be denoted as:

$$\begin{aligned} f(x_1, y_1, x_2, y_2) &= 0 \\ g(x_1, y_1, x_2, y_2) &= 0 \end{aligned} \tag{3}$$

because the only variables to be taken into consideration are the plate coordinates, x_1, y_1, x_2, y_2. When dealing with the P-C-1 arrangement, a system of two 2nd degree equations having as unknowns the two dependent plate coordinates is obtained by manipulating Equations 1. For the solution of the problem, a linearization is needed and an iterative procedure must be performed.

With the arrangement of the standard D.S. one obtains a system of two linear equations having as unknowns the dependent coordinates x_2^*, y_2^*. Also, in this case, the differential formulas are more convenient than the finite ones.

We will see later on that the same formulas can be used to deal with all the arrangements indicated above. We can therefore take into consideration only the first arrangement (Figure 2a) in which Δy applies to the left optical system and Δx applies to the right. When the main carriage is stationary, the four plate coordinates, x_1, y_1^*, x_2^*, y_2, satisfy the Equations 1. When

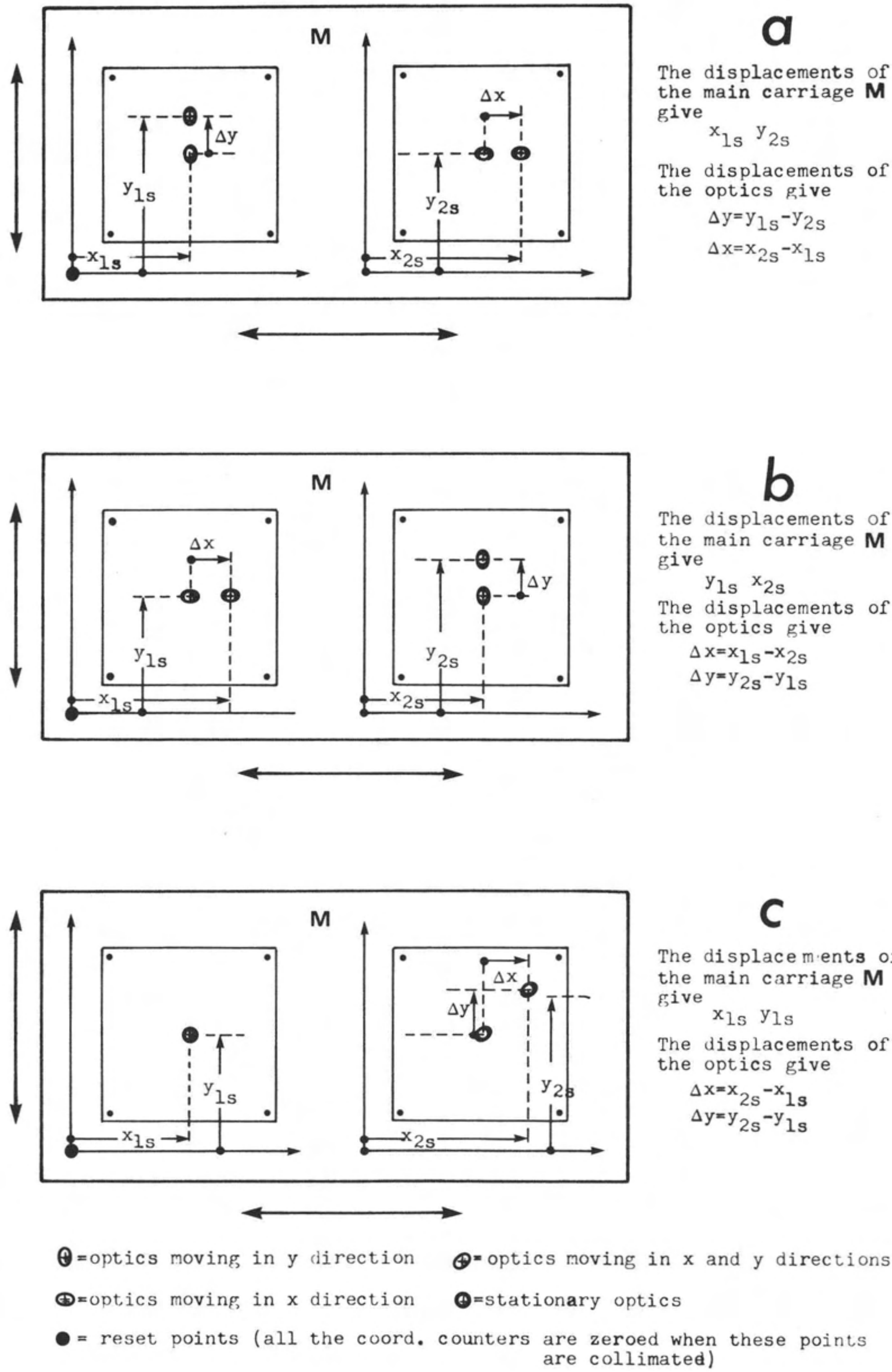

FIG. 2. Mechanical schemes of the P-C-1 and standard D.S.-type analytical plotter.

the main carriage is moved, the independent plate coordinates x_1, y_2 change by a certain amount and the dependent coordinates, y_1^*, x_2^*, change by the same amount. Consequently the dependent coordinates assume values $\bar{y}_1^*$, $\bar{x}_2^*$ which do not satisfy the Equations 1 because the y-parallax and the x-parallax (on a given horizontal plane) vary according to the position on the plates. The values $\Delta y = y_1^* - \bar{y}_1^*$ and $\Delta x = x_2^* - \bar{x}_2^*$ must be computed by the real-time program and fed as displacements of the optical system so that

$$\begin{aligned} f(x_1, \bar{y}_1^* + \Delta y, \bar{x}_2^* + \Delta x, y_2) &= 0 \\ g(x_1, \bar{y}_1^* + \Delta y, \bar{x}_2^* + \Delta x, y_2) &= 0 \end{aligned} \tag{4}$$

By expanding these equations in series and disregarding the squares and higher powers of Δy and Δx, we obtain

$$\begin{aligned} \left(\frac{\partial f}{\partial y_1^*}\right)_0 \Delta y + \left(\frac{\partial f}{\partial x_2^*}\right)_0 \Delta x + f(x_1, \bar{y}_1^*, \bar{x}_2^*, y_2) &= 0 \\ \left(\frac{\partial g}{\partial y_1^*}\right)_0 \Delta y + \left(\frac{\partial g}{\partial x_2^*}\right)_0 \Delta x + g(x_1, \bar{y}_1^*, \bar{x}_2^*, y_2) &= 0 \end{aligned} \tag{5}$$

The derivatives and the known terms are of course computed using the known values x_1, $\bar{y}_1^*$, $\bar{x}_2^*$, y_2. By deriving and manipulating, we obtain

$$\begin{aligned} \left(\frac{\partial f}{\partial y_1^*}\right)_0 &= a_1 = -\frac{1}{Z_1}\left[\cos(y_1 Y_M) - t_{y_1}\cos(y_1 Z_M)\right] \\ \left(\frac{\partial f}{\partial x_2^*}\right)_0 &= a_2 = \frac{1}{Z_2}\left[\cos(x_2 Y_M) - t_{y_2}\cos(x_2 Z_M)\right] \\ f(x_1, \bar{y}_1^*, \bar{x}_2^*, y_2) &= N_1 = t_{y_1} - t_{y_2} \\ \left(\frac{\partial g}{\partial y_1^*}\right)_0 &= b_1 = \frac{k_3}{Z_1}\left[\cos(y_1 X_M) - t_{x_1}\cos(y_1 Z_M)\right] + \frac{k_4}{Z_1}\left[\cos(y_2 Y_M) - t_{y_1}\cos(y_2 Z_M)\right] \\ \left(\frac{\partial g}{\partial x_2^*}\right)_0 &= b_2 = \frac{k_6}{Z_2}\left[\cos(x_2 X_M) - t_{x_2}\cos(x_2 Z_M)\right] \\ g(x_1, \bar{y}_1^*, \bar{x}_2^*, y_2) &= N_2 = k_3 t_{x_1} + k_4 t_{y_1} + k_5 + k_6 t_{x_2} \end{aligned} \tag{6}$$

Note that k_3, k_4, k_5, and k_6 have constant values because they change only when the elevation, H, of the contour line is changed. Also, Z_1, Z_2, t_{x_1}, t_{y_1}, t_{x_2}, and t_{y_2} must be computed in any case for the determination of the ground coordinates (see Figure 1).

The corrections, Δy and Δx, are so obtained by solving the linear system:

$$\begin{aligned} a_1 \Delta y + a_2 \Delta x + N_1 &= 0 \\ b_1 \Delta y + b_2 \Delta x + N_2 &= 0 \end{aligned} \tag{7}$$

the improved values $\hat{y}_1^* = \bar{y}_1^* + \Delta y$ and $\hat{x}_2^* = \bar{x}_2 + \Delta x$ can be used for the computation of the improved values of t_{x_1}, t_{y_1}, t_{x_2}, and t_{y_2} (see flow chart in Figure 3) and a new iteration for the computation of new corrections $\Delta y'$ and $\Delta x'$ can be made. One iteration should be more than sufficient. In fact, assuming that the speeds of the independent plate coordinate displacements during the plotting (operated manually or by motors) do not exceed 2 millimeters per second, and assuming that the variations of y-parallax and x-parallax do not exceed 10 percent of the displacements, the speeds for the Δx and Δy corrections should be 200 micrometers per second. Since the real time program runs 50 times per second, at the first computation (Equations 7) the values Δx and Δy should not exceed 4 micrometers. For this reason, even if the first computation leaves errors of 25 percent in Δx and Δy, the errors in dependent plate coordinates should not exceed 1 micrometer. This could be disregarded and no iteration should be necessary. The errors could not cumulate because the real-time routine provides a kind of iterative computation. However, an iterative procedure must be used in the real-time routine for outstandingly tilted photographs, which may occur.

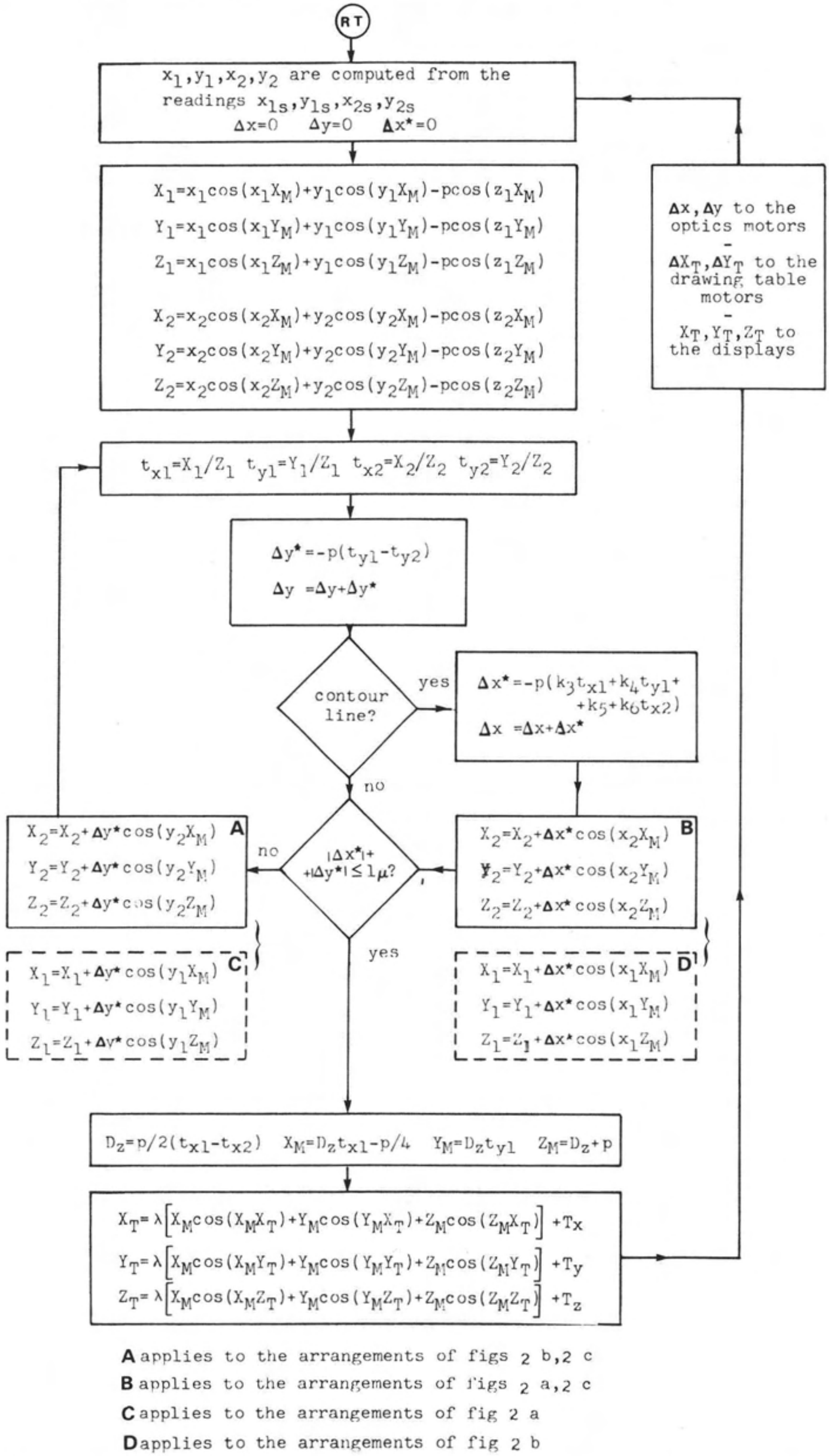

FIG. 3. Flow chart of the real time program for the various D.S.-type analytical plotters.

Differential Simplified Formulas

The reported computations, even if so simple, are time-consuming and must be performed by using a large program written in the assembler language of the computer interfaced with the stereocomparator. The computations, however, would be highly reduced if the coefficients a_1, a_2, b_1, and b_2 could be computed for the theoretical case of $\phi_1 = \kappa_1 = \phi_2 = \omega_2 = \kappa_2 = 0$, that is, $a_1 = 1/p$, $a_2 = 0$, $b_1 = 0$, and $b_2 = -k_6/p$ where p is the calibrated focal length.

The corrections could be computed with the following formulas:

$$\Delta y = p(t_{y_1} - t_{y_2})$$
$$\Delta x = -\frac{p}{k_1}(k_3 t_{x_1} + k_4 t_{y_1} + k_5 + k_6 t_{x_2}) \tag{8}$$

which are not time consuming. In fact, t_{x_1}, t_{y_1}, t_{x_2}, and t_{y_2} are needed in any case for the computation of the model coordinates X_M, Y_M, and Z_M. Furthermore, there is no need to solve a system of equations. It is necessary, however, to demonstrate that these simplified formulas may actually be used. First, let us determine under which conditions the linear system (Equations 7) can be solved with an iterative procedure, established by using $a_2 = 0$ and $b_1 = 0$ and the current values of a_1, b_2, N_1, and N_2.

The linear system (Equations 7) can be written

$$\Delta y = m_1 \Delta x + q_1$$
$$\Delta x = m_2 \Delta y + q_2 \tag{9}$$

where $m_1 = -a_2/a_1$, $m_2 = -b_1/b_2$, $q_1 = -N_1/a_1$, and $q_2 = -N_2/b_2$.
Assuming $a_2 = 0$ and $b_1 = 0$, one obtains the first solutions,

$$\Delta y' = q_1 \text{ and } \Delta x' = q_2.$$

Using these values as approximate solutions and putting

$$\Delta y = \Delta y' + \Delta y'' \text{ and } \Delta x = \Delta x' + \Delta x'',$$

one obtains the system

$$\Delta y'' = m_1 \Delta x'' + m_1 q_2$$
$$\Delta x'' = m_2 \Delta y'' + m_2 q_1$$

and using the above mentioned method

$$\Delta y'' = m_1 q_2 \text{ and } \Delta x'' = m_2 q_1.$$

Again putting

$$\Delta y = \Delta y' + \Delta y'' + \Delta y''' \text{ and } \Delta x = \Delta x' + \Delta x'' + \Delta x'''$$

one obtains

$$\Delta y''' = m_1 \Delta x''' + m_1 m_2 q_1$$
$$\Delta x''' = m_2 \Delta y''' + m_1 m_2 q_2$$

and

$$\Delta y''' = m_1 m_2 q_1$$
$$\Delta x''' = m_1 m_2 q_2 .$$

It is easy to realize that the known terms of the equations derive from those of the previous ones multiplied by m_1 or m_2. The conditions for the convergence of the series

$$\Delta y = \Delta y' + \Delta y'' + \Delta y''' + \ldots \qquad \Delta x = \Delta x' + \Delta x'' + \Delta x''' + \ldots$$

are consequently,

$$|m_1| < 1 \qquad |m_2| < 1 . \tag{10}$$

If we show Equations 9 in a graphic form (Figure 4), we can observe that the straight lines s_1 and s_2 cannot be tilted more than 45° with respect to Δx and Δy axes in order to obtain an iterative method, based on $a_2 = b_1 = 0$, which converges. Obviously, the smaller m_1, m_2 are, the quicker is the convergence.

We can easily establish that for all the photos taken with vertical cameras the conditions (Equations 10) are always satisfied.

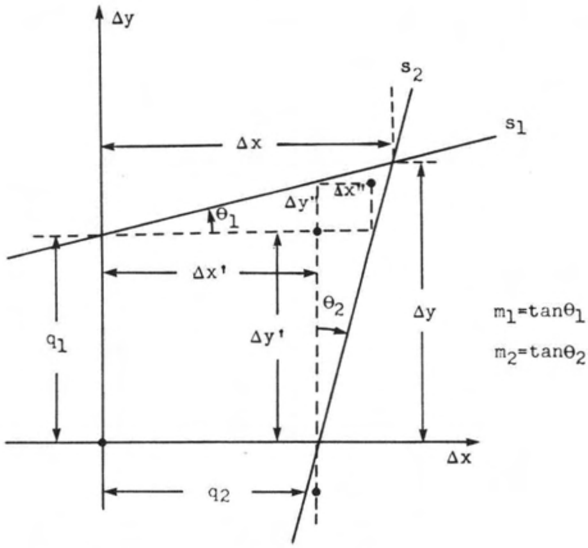

FIG. 4. Graphical representation of the linear condition equations.

In fact, as far as $m_1 = -a_2/a_1$ is concerned, we can establish that $|m_1| < 1$ observing that $Z_1 \cong Z_2 \cong -p$, t_{y_1}, t_{y_2} do not exceed the value 1, that $\cos(y_1 Y_M) \cong 1$, and that the other involved direction cosines have values not far from zero.

The same applies to $m_2 = -b_1/b_2$ since also t_{x_1}, t_{x_2} do not exceed the value 1, and (see Equations 2) $k_4 = 0$, $k_3 = -k_6$, k_3 multiplies a cosine that is near to zero, while k_6 multiplies a cosine that is near to one.

It is now necessary to take into account that in using the Equations 8 we do not use the correct values $q_1 = -N_1/a_1$ and $q_2 = -N_2/a_2$ but the approximate values $q_1^{(0)} = pN_1$ and $q_2^{(0)} = -pN_2/k_6$ Comparing the value of a_1 with $1/p$ and that of b_2 with $-k_6/p$, it is easy to establish that the differences between the actual values q_1, q_2 and the approximate values $q_1^{(0)}$, $q_2^{(0)}$ cannot exceed 20 percent of their values and that consequently the corrections $\Delta y'$, $\Delta y''$ . . . and $\Delta x'$, $\Delta x''$ . . . are found with relative errors which cannot exceed 20 percent.

This kind of error cannot be accepted because it makes the iteration procedure useless. In fact the errors in the first result $\Delta y'$, $\Delta x'$ would not be eliminated by the following iterative computation.

It is possible to eliminate this kind of error by just computing again after each iteration the values N_1 and N_2 with the improved values of Δy and Δx. After each iteration the values t_{x_1}, t_{y_1}, t_{x_2}, and t_{y_2} must be computed again. This is not time consuming because in any case the final values of t_{x_1}, t_{y_1}, t_{x_2}, and t_{y_2} have to be determined by taking into account the corrections Δy and Δx in order to obtain correct values for the model coordinates X_M, Y_M, and Z_M. In fact, the first computation gives $\Delta y' = q_1^{(0)}$ and $\Delta x' = q_2^{(0)}$ and the errors $e_y = q_1^{(0)} - q_1$, $e_x = q_2^{(0)} - q_2$ are made but, since the values N_1 and N_2 are computed again, the known terms in the second iteration shall be

$$m_1 q_2 + q_1 - q_1^{(0)}$$
$$m_2 q_1 + q_2 - q_2^{(0)} .$$

(Actually N_1 and N_2 are slightly different because they are computed with the non-linear formulas, but the differences are perfectly negligible.) It is obvious that the residual errors after the second iteration do not exceed the 20 percent of the 20 percent of the previous errors, e_y and e_x. Therefore, only one iteration is needed for the practical elimination of the errors of Δy and Δx. It is worth remembering in this connection that even if the iterations should not be performed the errors could not cumulate because the cycling of the real-time program operates like a kind of iterative computation.

Real-Time Program Flow Chart

The differences in the real-time programs of the various D.S.-type analytical plotters can be easily shown on a flow chart. In determining the linear system of Equation 5 for the other two arrangements shown for the D.S.-type analytical plotters (Figure 2b, 2c), it can be easily recognized that the coefficients should change according to the arrangement previously mentioned. (The changes apply mainly to direction cosines.) It can be immediately deduced that the simplified formulas (Equations 8) have no formal change dealing with the different arrangements. The same program can consequently be used with some slight differences in the corrections of the values t_{x_1}, t_{y_1}, t_{x_2}, and t_{y_2} as is shown in the flow chart of Figure 3.

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