Dr. A. Maarek
Terra Surveys Limited
Ottawa, Ontario K1G 3P5, Canada

Practical Numerical Photogrammetry

Procedures of orientation, aerial triangulation, and digitization are discussed, and results are compared with other authors.

Introduction

The field of practical numerical photogrammetry deals with three basic areas: orientation of stereo models, aerial triangulation, and digitization. The numerical procedures utilized in various production projects is described. The various projects and subjects are divided into the following sections:

Abstract: *This article contains results and comparisons in the field of numerical photogrammetry. It deals with three areas: orientation of models, aerial triangulation, and digitization.*

In the orientation area, relative orientation of incomplete models is discussed. A procedure of absolute orientation for the stereo model is developed, giving a close approximation of the stereo model base, the absolute longitudinal tilt, and the absolute lateral tilt to their final values. The required time for absolute orientation has been halved by using this procedure.

In the aerial triangulation area, two large samples of strips and blocks have been adjusted and analyzed. The first contains 320 models of large-scale mapping, while the second contains 3800 models of small-scale mapping. The results of those adjusted strips and blocks were compared with other investigations. Also, a simple procedure for the use of the statoscope is shown. Independent model triangulation on the Wild A10 is the procedure adopted for aerial triangulation. An essential part in this type of aerial triangulation is the determination of the perspective center coordinates and their accuracy. This is given in a separate section.

In the digitization area, a simple operational system is shown together with the mathematical transformations and computer programs used.

- relative orientation of incomplete models,
- absolute orientation of stereo models from numerical adjustment outputs,
- large scale aerial triangulations,
- stability of perspective center coordinates,
- practical results of photogrammetric blocks for mapping,
- digitization, and
- conclusions.

Relative Orientation (RO) of Incomplete Models

Incomplete models occur in coastal areas near lakes and shores and in cases with partial cloud cover. Because these models are partly covered either by water or cloud, the over

Photogrammetric Egineering and Remote Sensing,
Vol. 43, No. 10, October 1977, pp. 1295-1313.

correction factor (OCF) for the lateral tilt (ω) during RO is abnormal and could create quite a problem for the operators. For this reason, graphs shown in Figure 2 have been drawn for the OCF of ω on point 7 (Figure 1) for the swing-swing method of RO (American Society of Photogrammetry, 1966) for two cases:

(1) Wide-angle photography with height-base ratio (Z/d) equals 1.5.
(2) Super-wide-angle photography with Z/d equals 1. The formula used in computing the OCF is:

$$\text{OCF} = \frac{Z^2 + de}{e^2 - ed} \tag{1}$$

where e, d are shown in Figure 1.

As a final remark, the accuracy of incomplete models has been treated by Gagnon (1972).

Absolute Orientation of Stereo Models from Numerical Block Adjustment Outputs

For absolute orientation (AO), it is necessary to determine seven orientation parameters (three shifts, three rotations, and one scale) for relatively oriented models. In practical work, empirical RO is well established because it is fast, convenient, and economical for most stereoplotters. The question, therefore, arose for developing a numerical AO method from which the computed parameters could be introduced in the plotter in order to (a) reduce the required model-setting time for AO, and (b) check the control and triangulation qualities and hence the adjustment procedure.

Since, in most plotters, the absolute swing (K) is not available, the three shift parameters can be easily introduced and the scale (λ) can be replaced by the base of the model. The AO parameters required in map production are, therefore, the base (B), the absolute longitudinal tilt (Φ), and the absolute lateral tilt (Ω). The mathematical formulation is based on:

(1) Equating the two orthogonal matrices given by Equations 2 and 3a or 3b.

$$\boldsymbol{R} = \begin{bmatrix} d^2 + a^2 - b^2 - c^2 & 2ab - 2cd & 2ac + 2bd \\ 2ab + 2cd & d^2 - a^2 + b^2 - c^2 & 2ab - 2ad \\ 2ac - 2bd & 2bc + 2ad & d^2 - a^2 - b^2 + c^2 \end{bmatrix} \tag{2}$$

a, b, c; or a, b, d are the parameters of the orthogonal matrix (Schut, 1967)

$$\boldsymbol{R} = \begin{bmatrix} \text{C}.\Phi\ \text{C}.K & \begin{matrix}\text{S}.\Omega\ \text{S}.K \\ +\text{S}.\Omega\ \text{S}.\Phi\ \text{C}.K\end{matrix} & \begin{matrix}\text{S}.\Omega\ \text{S}.K \\ -\text{C}.\Omega\ \text{S}.\Phi\ \text{C}.K\end{matrix} \\ -\text{C}.\Phi\ \text{S}.K & \begin{matrix}\text{C}.\Omega\ \text{C}.K \\ -\text{S}.\Omega\ \text{S}.\Phi\ \text{S}.K\end{matrix} & \begin{matrix}\text{S}.\Omega\ \text{C}.K \\ +\text{C}.\Omega\ \text{S}.\Phi\ \text{S}.K\end{matrix} \\ \text{S}.\Phi & -\text{S}.\Omega\ \text{C}.\Phi & \text{C}.\Omega\ \text{C}.\Phi \end{bmatrix} \tag{3a}$$

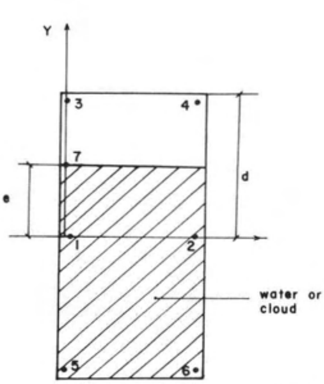

FIG. 1. Case of incomplete model (e/d = 1/2).

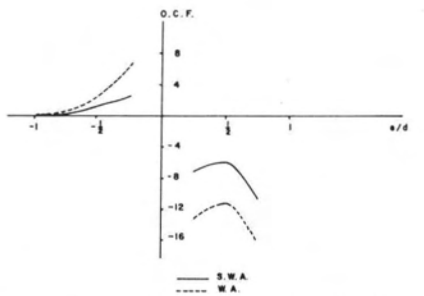

FIG. 2. Over correction factor (OCF).

S. = Sine; C. = Cosine; Equation 3a is used if Ω and $\Phi > 3°$.

$$R = \begin{bmatrix} 1 & DK & -D\Phi \\ -DK & 1 & D\Omega \\ D\Phi & -D\Omega & 1 \end{bmatrix} \quad (3b)$$

D = small change of the orientation angles in radians, Equation 3b is used if Ω and $\Phi < 3°$.

(2) Computing B from the scale ($\hat{\lambda}$), where $\hat{\lambda}$ is calculated on the assumption that the length of the vectors from the point 1 (Figure 3) in the model to the other measured points (e.g., 2 to 10, Figure 3) in either the model coordinate system or the ground coordinate system should be the same.

Because all the numerical block adjustment procedures include the coordinates of tie points in their outputs; and since those points are usually in a good geometrical location for solving B, Ω, and Φ; the tie points are used only for the solution of the numerical absolute orientation (NAO). The procedure avoids the use of control points (these appear as check points) for the solution and also gives the photogrammetric residuals for the tie points. The magnitude of these residuals immediately reflects the quality of the aerial triangulation and the photography.

PRACTICAL USE OF THE METHOD

The procedure for introducing B, Ω, and Φ in a stereoplotter (Wild A10, A8, or B8) is—

(1) RO by the swing-swing method, keeping the lateral tilt for the left projector (ω_L) of the plotter in zero position;
(2) Introducing B;
(3) Indexing the plotter on point 1;
(4) Introducing Φ and Ω (by using the lateral tilt for left and right projector simultaneously);
(5) If the plotter does not have Φ (e.g., the A10), then the combination φ_L, φ_R, and b_Z should be used instead of Φ, where b_Z is given by

$$b_Z \text{ (mm)} = \frac{B \text{ (mm)} \times \Phi}{63.68}$$

where b_Z is the Z-component of the base in the stereoplotter in millimeters and φ_L, φ_R are the longitudinal tilt angles for the left and right projectors, respectively; and
(6) Checking the residuals on the tie and control points in the model and making the final setting for B, Ω, and Φ.

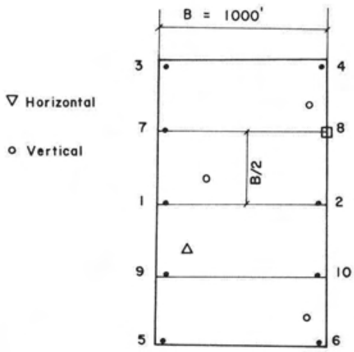

FIG. 3. Case of large-scale mapping model (PT. −8).

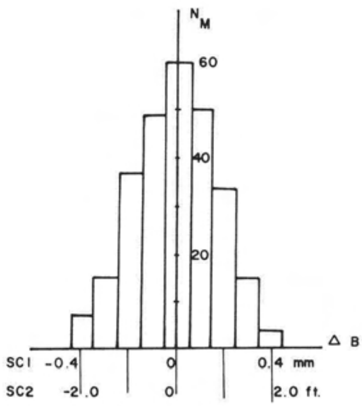

FIG. 4. ΔB
SC1: Model scale
SC2: Ground scale
N_M: Number of models

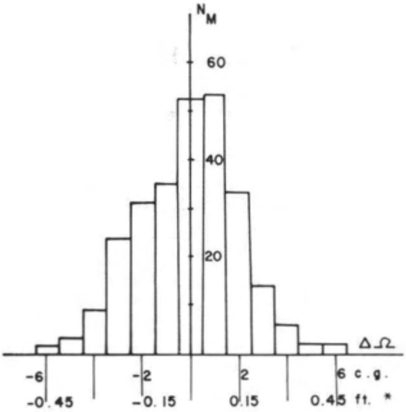

FIG. 5. $\Delta\Omega$
* Error in ground pt. 8.

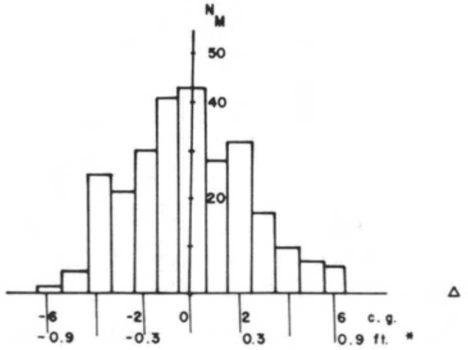

FIG. 6. $\Delta\Phi$

Some developments in this connection have been made (Erio, 1974; Jeyapalan, 1975), but these fundamentally differ from the content of this section.

The above procedures have been applied to a great extent in large-scale projects listed in Tables 1, 2, and 3. The differences (Δ) between the computed and the final values for B, Ω, and Φ for a sample of about 270 models (Table 2), plotted on two Wild A8 stereoplotters are shown in Figures 4, 5, and 6. These figures are constructed from data similar to that in Table 1. Table 1 is given as an example in order to show the values of ΔB, $\Delta\Omega$, and $\Delta\Phi$ for two strips. Table 2 is constructed from Figures 4, 5, and 6 and gives the percentage proximity of B, Ω, and Φ to their final values.

ANALYSES OF THE RESULTS

From the results obtained in Table 2, it is easy to observe that

- 59 percent of the bases are correct to within ± 0.5 ft., and a further 26 percent are correct to within ± 1.0 ft. This implies that only 15 percent of the bases need a minor adjustment. Similarly, 9 percent and 20 percent of Ω and Φ respectively need some minor corrections.
- The time required for AO has been halved compared with the empirical method for AO using control points without introducing the AO elements.
- The procedure provides two powerful statistical estimators, namely σ_{OH} and σ_{AH}.

TABLE 1. ΔB, $\Delta\Omega$, and $\Delta\Phi$ (AREA VANIER—OTTAWA REGION)
WIDE ANGLE PHOTOGRAPHY—A-8 PLOTTING

Strip	Ph. Sc. Mod. Sc. PL. Sc.	Model Number	Δ NAO B	ΔB (mm)	NAO Ω	$\Delta\Omega$ (g)	NAO Φ	$\Delta\Phi$ (g)
	1:3000	88-87	171.06	−0.16	98.94	0.00	101.16	−0.04
16	1:1600	87-86	171.66	−0.19	100.39	0.02	100.46	−0.03
	1:480	86-85	164.06	−0.04	101.27	0.01	99.58	−0.02
		85-84	166.51	0.03	100.52	−0.02	100.07	−0.04
		100-99	212.77	−0.04	99.91	+0.06	95.52	0.04
		99-98	212.27	−0.08	100.77	0.02	100.30	−0.03
	1:2400	98-97	204.83	−0.10	100.56	0.01	100.65	−0.01
15	1:1440	97-96	206.24	−0.24	101.32	0.02	100.90	0.06
	1:480	96-95	187.80	−0.13	97.82	−0.03	99.28	−0.08
		95-94	196.62	0.02	101.91	0.11	100.08	0.00
		94-93	198.77	−0.15	102.31	0.01	98.57	−0.04
		93-92	196.18	−0.15	102.01	0.01	98.52	−0.08

Ph. = Photo; PL. = Plotting; Sc. = Scale; Δ = Differences between NAO Value and Final Value; g = Grad

TABLE 2. PERCENTAGE PROXIMITY OF B, Ω, Φ

	ΔB (ft.)				$\Delta\Omega$ (C.g.)					$\Delta\Phi$ (C.g.)				
	±0.5	±1	±1.5	±2.0	±1	±2	±3	±4	±5	±1	±2	±3	±4	±5
N_M	159	71	30	11	140	65	38	15	9	112	62	38	35	19
%	59	26	11	4	53	24	14	6	3	43	23	14	13	7
Total	271				267					266				

N_M = number of models; % = percentage; C.g. = Centigrade

LARGE SCALE AERIAL TRIANGULATION

Most production of large-scale mapping consists of one or more strips (forming single lines of photography), for highway engineering projects. The two principal scales employed are shown in Table 3. They are

(1) HW-A, 1:480 with 1 foot (ft.) contour interval (C.I.). This is a detailed topographic map of a narrow corridor (300 or 500 feet wide) along a selected prepared route for design purposes in urban areas.
(2) HW-B, 1:1200 with 2 ft. C.I. This is a detailed topographic map for a corridor similar to the above for design purposes in rural areas.

Forty-eight strips of black-and-white photography (Tables 3 and 4) will be considered as our sample in this section.

AERIAL TRIANGULATION (AT) PHASE

In preparing the AT phase, the operator familiarizes himself with the photography, identifies all given control points, and marks five pass points in the overlap area between the models by employing a Wild PUG-4 (point transfer device). The average preparation time per model is 20 to 25 minutes.

The AT phase is carried out by the well-known procedure of forming independent models on the Wild A10 stereoplotter and using the swing-swing method of RO. The average time for reading the pass points and control points and for RO is 40 to 50 minutes.

TABLE 3. REQUIRED MAPPING AND CONTROL ACCURACY

Type Lens PL.Sc. Number Ph.Sc. Number C.I.	Mapping Accuracy		Reduced by			Control Accuracy	
	σ_{MP}	σ_{MH} (ft.)	σ_O (um)	σ_I (um)	σ_{CL} (ft.)	σ_{CP} (ft.)	σ_{CH} (ft.)
HW-A W.A. 480 2400 – 3000 1 ft.	0.18 m 0.60 ft	0.30	10	15	0.22	0.60	Full*
HW-B Wide Angle 1,200 3000 – 3600 2 ft.	0.44 m 1.44 ft	0.60	10	15	0.22	1.42	0.56
Ghana Super W.A. 50,000 45 – 55,000 50 ft	15.0 m 49.2 ft	15.00	30	50	4.3	48.0	14.4

W.A. = Wide Angle; ft. = feet; m = meter; μm = micrometer.
For definitions, see section on specifications for Aerial Triangulation.
* Full control in the height adjustment.

The perspective center determination will be dealt with later. The measurements obtained on the A10 are assembled as an input to the strip formation program (Schut, 1967).

CONTROL DISTRIBUTION AND SPACING

A minimum of one horizontal control point every two models on the central corridor of the route, together with two well-spaced horizontal control points every five or six models, is used as the horizontal control layout. This layout is best suited to routine traverse work. The horizontal control points must be targeted, and can be considered as third-order control which satisfies the following requirements:

(1) The angular misclosure in arc seconds should be less than $3n$ and/or $10\sqrt{n}$, where n is the number of angles
(2) The linear misclosure should not be greater than 1:20,000 or 1:10,000 for closed traverses fixed respectively on one or two terminals (a base contains two second-order horizontal control points).

For vertical control, third-order levelling control points obtained by a closed loop are used. Full vertical models are required in type HW-A mapping (Table 3). In HW-B mapping, however, bands of vertical control points are used every third or fourth model in the strip, together with other vertical points on the center-line of the required route.

STRIP FORMATION AND BLOCK ADJUSTMENT PROCEDURE

During the strip formation phase, the residual limits from the mean in directions of Y and Z (R_Y, R_Z) of the pass points, and their computed standard deviations (σ_Y and σ_Z) are on the order shown in Table 5. σ_Y and σ_Z are reflections of the quality of the photography and the AT procedure.

In the block adjustment phase, the polynomial block adjustment based on Schut (1966), is adopted. The following abbreviations will be used:

122 POLY—First-degree polynomial adjustment in planimetry (P) and second-degree in height (H).
222 POLY—Second-degree polynomial (POLY) in both P and H.
211 POLY—Second-degree POLY in P and first-degree in H.
211R POLY—The same as 211 POLY, but applying earth curvature correction.

In the polynomial adjustment phase, each line of photography (more than one strip) or single strip is subjected to three different polynomial adjustments. They are

- 122 POLY, using only the well-placed horizontal control points to detect the errors in the horizontal control;
- 211 POLY, using the vertical control in bands with bridging distance of three or four models, to detect any errors in the vertical control values or in their machine values; and
- 222 POLY, as the final adjustment, with control fitting residual limits of approximately ± 0.6 ft. in the height adjustment.
- Every single model is subjected to the numerical absolute orientation described earlier.

SPECIFICATIONS FOR AERIAL TRIANGULATION (AT)

For the evaluation of the results of AT projects, one has to deal with mapping accuracy specifications expressed by standard deviations in planimetry and height (σ_{MP} and σ_{MH}). σ_{MP} and σ_{MH} have to be reduced by the effect of σ_O, σ_I, and σ_{CL} in order to produce σ_{CP} and σ_{CH}. Those standard deviations have the following relationships:

$$\sigma_{MP}{}^2 = \sigma_O{}^2 + \sigma_I{}^2 + \sigma_{CP}{}^2 \tag{4a}$$

$$\sigma_{MH}{}^2 = \sigma_{CL}{}^2 + \sigma_{CH}{}^2 \tag{4b}$$

TABLE 4. σ FOR LARGE-SCALE MAPPING (UNITS ARE μm)

Type	N_S	N_M	σ_{OH}	σ_{FH}	σ_{AH}	σ_{ZO}
WH-A	22	117	7	20	22	20
WH-B	26	162				

For details and definitions (Tables 7, 8, 9, 12).

TABLE 5. THE LIMIT FOR R_Y, R_Z, σ_Y, σ_Z IN STRIP FORMATION IN LARGE-SCALE MAPPING.

Photo Scale Number	Limits in μm				Remarks
	R_Y	R_Z	σ_Y	σ_Z	
2400 to 3600	± 25	± 35	± 8	± 12	In image scale

TABLE 6. σ_{FP} (ft.)

N_S / n/strip	7 / 6	3 / 7	4 / 8	5 / 9	1 / 12	1 / 14	1 / 17
σ_{FP} (ft)	0.14 to 0.31	0.14 to 0.53	0.14 to 0.31	0.22 to 0.64	0.58	0.72	0.67

n, number of control used in the adjustment.

where

σ_O is the standard deviation (σ) for RO and AO for stereo-models,

σ_I is σ for identification purposes,

σ_{CL} is σ for contour lines tracing (in this paper, σ_{CL} is considered as a function of the C-factor (e.g., for the B8, 900; for the A8, 1600)), and

σ_{CP}, σ_{CH} are standard deviations for control requirements in planimetry (P) and height (H) respectively.

Table 3 is constructed using Equations 4a and 4b, and σ_{MP} and σ_{MH} are computed using the well-known U.S.A. specification (American Society of Photogrammetry, 1966, pp. 1182–1184).

An important point in the evaluation of a photogrammetric block adjustment is that it should satisfy the following:

$$\sigma_{AP} \leqslant \sigma_{CP} \tag{4c}$$

$$\sigma_{AH} \leqslant \sigma_{CH} \tag{4d}$$

where σ_{AP}, σ_{AH} are σ for the absolute accuracy in P and H (using check points), respectively.

Because σ_{AP} and σ_{AH} necessitate check points and they hardly exist in practice, the task is to relate σ_{AP} and σ_{AH} with the fitting accuracy of the control points used (σ_{FP} and σ_{FH}) in P and H respectively.

ANALYSES OF THE RESULTS

σ_{FP} is computed for strips which use more than five horizontal control points for planimetric adjustment as listed in Table 6. Those strips are for WH-B mapping projects. σ_{FP} ranges from 0.14 ft. to 0.72 ft., which is barely half the σ_{CP} value (1.42 ft.). In computing σ_{AP} (by relaxing half of the horizontal control points, i.e., check points), its value was on the order of 1.3 to 1.5 σ_{FP}. Our experience regarding the horizontal accuracy requirement in large-scale mapping is an agreement with Derenyi and Maarek (1974) in which (1) with target horizontal control every two models, σ_{CP} is fully satisfied; and (2) the planimetric accuracy did not prove critical for the two map scales in question.

Because the height accuracy is the governing factor in large-sacle map production, great attention will be paid to the statistical estimators in the height direction, which are given in Tables 3, 7, 8, and 9. It is evident that:

- From Table 7, the standard deviations for the model coordinates in the Z-direction (σ_{OH}) computed from 193 models were on the order of 7 μm in the image scale.
- From Table 3, full vertical control models for WH-A mapping are essential, because σ_{MH} and σ_{CL} are 0.3 and 0.22 ft., respectively.
- From Table 8, σ_{FH} is (on the average) 0.18 ft.
- From Table 9, the ratio between σ_{AH} and σ_{FH} is in the order of 1.15.

TABLE 7. σ_{OH} (μm)

Sample	σ_{OH} for individual models in μm: 2	3 / 4	5 / 6	7 / 8	9 / 10	11 / 12	13 / 14	15 / 16	16	Average σ_{OH}	Total N_M
1, N_M	9	17	29	24	12	8	8	3	1	7	112
2, N_M	2	20	20	21	10	7	0	1	0	6	81

TABLE 8. σ_{FH}

Sample	N_S	N_M	n	σ_{FH} μm	σ_{FH} ft	Range in ft
1	22	112	381	20	0.19	±0.10 to ±0.28
2	13	81	365	19	0.17	±0.10 to ±0.23

TABLE 9. RELATIONSHIP σ_{AH}/σ_{FH}

Strip	3	4	5	6	7	11-A	11-B
N_M	5	6	5	4	5	6	6
n	23	26	20	17	19	36	23
σ_{FH} (ft)	0.19	0.19	0.17	0.15	0.16	0.14	0.16
σ_{AH} (ft)	0.18	0.21	0.20	0.18	0.20	0.16	0.17
$\frac{\sigma_{AH}}{\sigma_{FH}}$	0.95	1.1	1.2	1.2	1.3	1.1	1.1

ACCURACY COMPARISONS

Work had previously been carried out in the field of large-scale aerial triangulation in various research and government organizations (Derenyi and Maarek, 1972; Derenyi and Maarek, 1974; Hou, 1974; Karara and Marks, 1969; Katibah, 1968; Wong, 1969). The instruments used for the AT were analytical plotters (Derenyi and Maarek, 1972, 1974), comparators (Derenyi and Maarek, 1972, 1974; Karara and Marks, 1969; Wong, 1969), the A7 stereoplotter (Hou, 1974), and the A10 stereoplotter (Derenyi and Maarek, 1972, 1974). The results obtained for σ_{AH} from those organizations are listed in Table 10 together with the Terra results. From Table 10, the σ_{AH} obtained is similar to that in Hou (1974) and Katibah (1968) and slightly better than that given in Derenyi and Maarek (1972, 1974) and Karara and Marks (1969). This suggests that the results in Wong (1969) should not be used as a basis for comparison.

STABILITY OF THE PERSPECTIVE CENTER (PC) COORDINATES

Independent model AT is a well-established procedure. An essential part of this kind of AT is the determination of the PC coordinates and their accuracy and stability. Earlier results can be found in Altenhofen (1970), Ligternik (1970), Maarek (1971), Maarek and Konecny (1973), Stewardson (1972), and Togliatti (1968). Table 11 is a summary of the work in Maarek (1971), Maarek and Konecny (1973), and Stewardson (1972).

The experiment in Maarek (1971) and Maarék and Konecny (1973) was based on ten sets of monocular grid measurement (nine points each), and a further nine sets (25 points each) with ω_O, φ_O, and K_O equal zero. The geometrical model is the space resection models given by:

TABLE 10. COMPARISONS OF LARGE-SCALE AERIAL TRIANGULATION WITH OTHER ORGANIZATIONS

Publications	This Report	Hou, 1974	Derenyi and Maarek, 1972, 1974	Karara and Marks, 1969	Katibah, 1968	Wong, 1969
Photo Scale	1:2,400 1:3,000 1:3,600	1:3,000	1:2,400	1:3,000	1:4,000	1:3,000
σ_{AH} (ft)	0.22	0.18	0.31	0.30	0.28	0.70
N_S	35	10	9	3		

$$V_x = dX_o + \frac{X}{Z}dZ_o + \frac{XY}{Z}d\omega_o - \frac{X^2 + Z^2}{Z}d\varphi_o - YdK_o - dX \tag{5}$$

$$V_y = dY_o + \frac{Y}{Z}dZ_o + \frac{Y^2 + Z^2}{Z}d\omega_o - \frac{XY}{Z}d\varphi_o + XdK_o - dY$$

where dX_o, dY_o, dZ_o are corrections to the initial assumed values of PC coordinates X_o, Y_o, Z_o;
$d\omega_o$, $d\varphi_o$, dK_o are corrections to the assumed initial values for ω_o, φ_o, K_o (usually zero);
X, Y, Z are the projected model coordinates;
dX, dY are the misclosures in the X and Y directions computed from the known grid coordinates and the measured one; and
V_x, V_y are the residuals for the measured corrdinates in x and y directions.

The experiment by Stewardson (1972) was based on monocular grid measurement (nine points each in two planes), and the range of ω_o and φ_o lay between -3 grads and $+3$ grads. The geometrical model used is the space intersection model based on the equation:

$$\frac{X_1 - X_2}{Z_1 - Z_2} = \frac{X_1 - X_o}{Z_1 - Z_o} \qquad \frac{Y_1 - Y_2}{Z_1 - Z_2} = \frac{Y_1 - Y_o}{Z_1 - Z_o} \tag{6}$$

where X_1, X_2, Y_1, Y_2 are the X,Y coordinates of the same projected image points in two planes $Z = Z_1$ and $Z = Z_2$ and $Z = Z_1 - Z_2$.

Practically speaking, the intersection method is more economical and accurate for determining the PC coordinates than the resection method because no grid plate is required and it is independent of the inner orientation parameters. The intersection method has therefore been used. The purpose of this section is to study the behavior of the PC under actual production conditions on an A10 stereoplotter, especially in the Z direction, for the following reasons:

- The allowable reliable samples are from large-scale mapping (Table 4) where the limiting design factor is the height accuracy and the control configuration is nearly a full control model in height.
- The above studies (Maarek, 1971; Maarek and Konecny, 1973; Stewardson, 1972) show a close agreement in the behavior of the PC in the X and Y directions.
- The behavior of the PC coordinates in the Z direction is far more critical, in strip and block adjustment, than its behavior in planimetry.

TABLE 11. COMPARISON STUDY ON THE P.C. BEHAVIOUR

Publications		Maarek, 1971 Maarek and Konecny, 1973	Stewardson, 1972			
Method Equation		Resection (5)	Intersection (6)			
Instrument		A-8	A-8	A-7	A-7	A-10
f (mm)		150	150	150	150	150
B (mm)		180	150	0	240	0
Order of errors (um)	X_o	The range of V* is -11 to 11	37	10		25
	Y_o		28	23		20
	Z_o		60	26		25
σ (μm)	σ_{Xo}	5 to 11**	8	5	7	5
	σ_{Yo}	5 to 11	8	5	7	5
	σ_{Zo}	3 to 5	17	5	9	5
Nature of the error		Rand Sample with σ_o*	Accidental error			

* Residuals for the observations (reduced to grid scale) with standard deviation of unit weight $\sigma_o = 4.2\ \mu m$

** X_o, Y_o and Z_o are computed from the variance-covariance matrix solving least squares adjustment (Maarek and Konecny, 1973).

SAMPLE USED

The sample used was composed of about 270 models of large-scale mapping adjusted according to the procedure described earlier. The numerical absolute orientation is used to compute σ_{OH}. σ_{OH} is computed for each stereo-model for two large samples and is listed in Table 7. From this table, it is easily seen that σ_{OH} is on the order of $\pm$ 7 μm. The following points should also be noted:

- An A10 stereoplotter is used for AT.
- The PC coordinates are determined by a spatial intersection method based on Equation 6 employing glass plates.
- A complete stereo grid with grid plates (in the Z direction plane adopted during AT) is utilized if a major discrepancy is noted in the strip-formation residuals and can be attributed to the PC behavior. This procedure is also carried out if an overall maintenance of the A10 stereoplotter is requested. The observed machine coordinates (*X*, *Y*, *Z*) together with the known grid coordinates are subjected to the NAO. One of these stereo grid calibrations (20 points) gave results of 3.2, 4.0, and 4.5 μm for standard deviations of the observed *X*, *Y*, and Z coordinates respectively.

COMPUTATION OF σ_{ZO}

All the PC coordinates are used as check points while solving the NAO for each model, because the absolute orientation parameters are computed by using only the tie point coordinates. From the solutions, the values of the PC coordinates (ground system) are made available twice for each intermediate PC in any strip. From these double values the mean is computed and its residual (ΔZ_{PC}) could also be obtained. The residuals can serve to compute σ_{ZO}, which will reflect two factors: (1) the height accuracy of the tie point coordinates (σ_{AH}), and (2) the accuracy and stability of the PC coordinate in the Z direction.

σ_{ZO} is computed from three samples, and it is listed in Table 12. From this table the scatter of ΔZ_{PC} is on the order of $\pm$ 40 μm with an average σ_{ZO} in the order of 20 μm. As noted earlier, σ_{AH} is about equal 22 μm.

ANALYSIS OF THE RESULTS

In a sample of about 270 models, the PC accuracy in the Z direction behaves as any pass point in the model, and the A10 in the Z direction is a stable stereoplotter.

PRACTICAL RESULTS OF PHOTOGRAMMETRIC BLOCKS FOR SMALL-SCALE MAPPING

At Terra Surveys Limited, small-scale topographic mapping consists of relatively large photogrammetric blocks (150 to 500 models). It is generally carried out as part of Canadian International Development Agency contracts. One of the projects for which the author performed most of the numerical adjustment was in Ghana, West Africa. The mapping scale was 1:50,000 and the contour interval (C.I.) was 50 ft. The contract area lay between 1°W and 3°W longitude; and 5°N and 7°30′N latitude and covered approximately 80 sheets of 1:50,000 scale with 1° central meridian.

PHOTOGRAPHY AND GROUND CONTROL

The aerial photography system consisted of a Wild RC-9 camera equipped with a superwide-angle lens and statoscope registration for recording the height difference between the exposure stations and the isobaric surfaces. The photography consisted of 48 lines and three cross lines at scale ranges between 1:45,000 and 1:55,000; and relatively large-scale photography (1:15,000 to 1:20,000) for the identification of the horizontal control points.

TABLE 12. σ_{ZO} (μm)

	ΔZ_{PC} in (μm)										
Sample	± 5	± 10	± 15	± 20	± 25	± 30	± 35	± 40	>40 <-40	Average σ_{ZO}	Total ΔPC
1, N_{PC}	23	21	13	7	16	7	2	0	2	20	91
2, N_{PC}	6	11	9	6	14	6	10	4	4	26	70
3, N_{PC}	28	36	26	11	5	2	0	1	2	14	111

N_{PC} = the number of PC used; ΔZ_{PC} = Difference between the computed value of the PC and its mean in Z-direction.

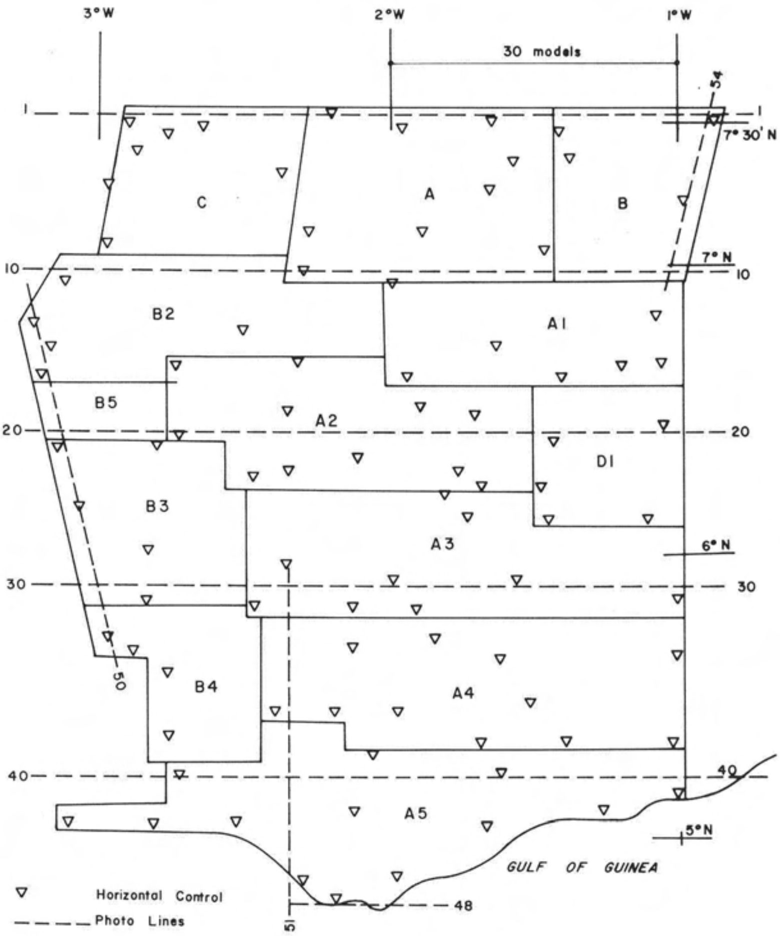

FIG. 7. Ghana block adjustment.

The horizontal control net consisted of 102 geodetic points. The western border control points were recently established and they are of approximately third-order accuracy. The location and the distribution of the horizontal control are shown in Figure 7.

The vertical control consisted of few trigonometric and mostly barometric levelling points. The meteorological conditions are the limiting factor for the accuracy of the barometric levelling. A fair estimate of its accuracy (expressed as an estimated mean square error) is on the order of ± 5 ft. in the coastal and populated areas and ± 10 ft. in the vegetation areas.

The vertical control points are distributed every five to ten models, (Block D1 as an example, Figure 8). However, the distribution in block "A", the first block to be adjusted, was every two or three models to allow the following tests:

- Evaluating the accuracy of the barometric levelling values by using some of them as check points;
- Determining the bridging distance in the height direction;
- Finding a simple method for using the statoscope data with polynomial block adjustments;
- Testing the validity of the fitting accuracy; and
- Obtaining a good approximation for the statoscope unit.

Following the tests, it was decided that:

- The bridging distance for the height adjustment could be as great as ten models;
- Only the reliable statoscope data should be used as a check for bridging distances of about six models and be incorporated in the height adjustment for bridging distances greater than six models.

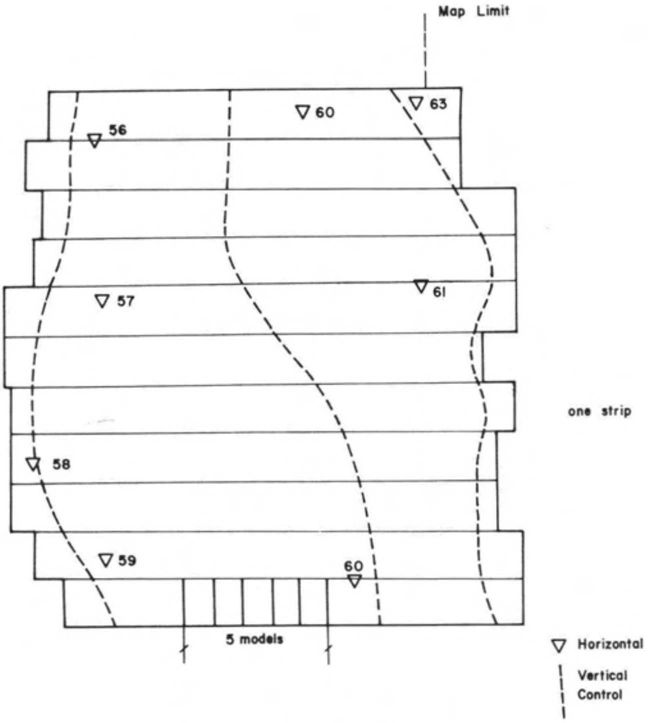

FIG. 8. Block D1 - Ghana.

STATOSCOPE

The method developed for using the statoscope is based on the following considerations:

- It is important that a reasonably accurate horizontal adjustment of the block be available prior to the introduction of the statoscope data.
- In the first height-adjustment run, the vertical control for each strip should be divided into bands and any vertical control points between these bands should be used as check points. Only a limited number of tie points should be incorporated between the strips. From this adjustment, the height of the exposure stations is determined.
- Because each strip, or part of a strip, begins and ends at a known elevation, the height of the exposure station in the first and the last model, after horizontal adjustment, represents the correct height of the exposure station above sea level.
- By adding the height of the exposure station of the first model to the difference in elevation of each subsequent exposure (obtained from the statoscope data), a first approximation of a correct elevation for each exposure station is obtained.
- The elevation computed in the last step is in fact not absolutely correct because it is influenced by the slope of barometric surfaces and other errors which, it can fairly be assumed, propagate in a linear fashion. Therefore, at the end of each strip or part of a strip a "closing" error could exist which will be distributed linearly.
- After compensating for this linear component, a final block adjustment is performed, using the elevations of the exposure stations as vertical points, as well as the barometric levelling points.

TRIANGULATION AND ADJUSTMENT PHASE

The triangulation phase is similar to the procedure described earlier, except for the following changes:

- Great care is taken in the identification of the horizontal control points;
- Three pass points are used in the common overlap areas between models;
- A strict numbering system is adopted for coding the various types of measured points; and
- The average preparation time is about 10-15 minutes per model while the aerial triangulation time required is 20-30 minutes per model.

The measurements obtained on the A10 stereoplotter are processed in the conventional manner by using strip formation (Schut, 1967) and polynomial block adjustments (Schut, 1966). The following checks are carried out during the adjustment phase:

TABLE 13. THE LIMIT FOR THE RESIDUALS OF STRIP FORMATION

Model Scale	Maximum Error (μm) Y-Coord.	Z-Coord.	Area
1:20,000	± 80	± 120	Non-vegetation
	± 120	± 150	Vegetation

Coord. = Coordinate

(1) The maximum residuals in the Y and Z coordinates at the strip formation stage are checked to be within the limits given in Table 13. If the residuals exceed the listed values in this table, the following steps are taken:
   (a) Reset the models using five pass points.
   (b) Employ a semi-numerical procedure for the relative orientation (Bervoets, 1962).
   (c) The operator should supply information about the residual Y-parallax, and the quality of the pass, tie, and control points.

(2) The block adjustment is performed by using a modified version of Schut (1966), which allows relative weights between tie and control points and can handle blocks containing up to 500 models. To check the blunders and mistakes either in the photogrammetric or control (field) data, the following polynomial adjustments are performed:
   (a) 122 POLY to check the blunders in the horizontal control points, using only the well-identified points (three or more points in the first strip). The nature of the error in the transformed horizontal control (check points) will serve as a guide to the mistakes and blunders in the horizontal control points.
   (b) 211R or 222 POLY, to check any discrepancy in the vertical control data, by using only the vertical control points in bands.
   (c) Utilization of the statoscope data as described earlier.
   (d) Final block adjustment by 222 POLY, or in some cases for large strips (number of models greater than 25 and of the vertical control bands over four), a third-degree polynomial height adjustment is used.

RESULTS OF THE GHANIAN PROJECT

The triangulation procedure and numerical adjustment described earlier has been used in the area of southwestern Ghana shown in Figure 7. The area has been subdivided into 13 blocks containing 3,876 models according to:

- The configuration of the horizontal control points;
- Positions of the cross lines;
- Non-utilization of the cantilever part of strips (uncontrolled according to height);
- Core and size of the block adjustment program;
- Vertical control data received from the field; and
- Adjacent strips and models between photogrammetric blocks (to avoid horizontal cracks between blocks).

The following results were obtained from the Ghanian project:

- In the vegetation area, model setting was mostly on the order of 5 percent.
- The cross lines were flown in order to provide supplementary vertical control for use in height adjustment on the block edges (Blocks B, B3, B5) or to improve the bridging distance in height (Blocks A4, A5, B4).
- Block B and Cross Line 54, Block B5 and Cross Line 50, and Block B3 and Line 50 were adjusted separately, while a combined block adjustment of Block A4 and Line 51 and Block A5 and Line 51 was carried out.
- The relative accuracy computed from the residuals on the tie points and expressed as the standard deviation in planimetry was on the order of 50 to 70 μm at image scale. The maximum errors on the tie point coordinates were kept on the order of 8, 8, and 5m in easting, northing, and height, respectively.
- All the blocks marked A (e.g., A, A1, A2, A3, A4, A5) were adjusted first, due to their favorable horizontal control distribution.
- One or more lines of photography (usually two) are used as common lines between adjacent blocks and at least two or three models are used as common models between strips in the flight direction. Common points from previously adjusted blocks are weighted equally with

the tie points in the next block. These common points are used to detect any serious error in the horizontal adjustment of the blocks, to ensure that plotting is continuous, and to improve the statistical reliability of σ_{FP}.

- The data rejection rate, for either tie or control points, is an essential factor in the quality of the triangulation and the control points used. The rejection rate is about 5 percent and listed in Table 14 for the first half of the project area.

EVALUATION OF RESULTS AND COMPARISONS

The results obtained were listed in Table 15, and are based on the assumptions given earlier. The following is an evaluation of those results.

- σ_{FP} for A blocks (e.g., A, A1, A2, A3, A4, A5) ranges between 54 to 119 μm at the image scale with an average of 80 μm, while σ_{FP} for the other blocks varies between 85 to 152 μm with an average of 120 μm. σ_{FP} at ground scale varies from 9 to 25 ft.
- From Table 3, σ_{CP} is about 48.0 ft. and about two to five times σ_{FP}. Hence, the planimetric block adjustment accuracy in the Ghanian project could satisfy the requirement of 1:50,000 in planimetry. Also, σ_{FP} is in agreement with Forster (1975).
- A formula from a recent study (Forster, 1975) on the planimetric accuracy of photogrammetric blocks in international reports shows that a reasonable σ_{XY} can be obtained as a function of σ_{OXY} and the number of horizontal control points (n) and stereo-models used. The definitions of σ_{XY} and σ_{OXY} follow:

  σ_{XY} is a reasonable estimate of the standard planimetric error at the photograph scale of the point determined.

  σ_{OXY} is the standard error of unit weight of the precision of the block system used. Using σ_{OXY} equal to 15 μm, σ_{XY} and σ_{FP}/σ_{XY} are obtained and shown in Table 15. One may conclude that σ_{FP}/σ_{XY} is on the order of 1.3, and the planimetric accuracy of the 13 adjusted blocks are in agreement with the international reports shown in Forster (1975).
- σ_{FH} is highly consistent through most of the blocks and it ranges from 38 to 58 μm with an average of 47 μm. σ_{FH} at ground scale is 6.2 to 9.5 ft.
- From Table 3, σ_{CH} is 14.4 ft., which is obtained from σ_{MH} and σ_{CL}. σ_{CL} is computed on the assumption that the topographic plotter (e.g., the B8) used has a C-factor equal to 900.
- σ_{CH} (14.4 ft.) is between 1.5 and 2.3 σ_{FH}. Hence, the height accuracy of the Ghanian block adjustment is highly suitable for 50 ft. C.I. mapping. The ratio of σ_{AH} to σ_{FH} in the project was about 1.2 to 1.4; this was obtained from trial adjustments of the density vertical control, block A. In the trial adjustments, ten model bridging distances are used with more than 14 vertical control points in three or four bands per individual strip, while the rest of the vertical control points serve as check points.

DIGITIZATION

In recent years, photogrammetric technology has moved into a new phase in which digital components are increasingly integrated with semi-automatic and automatic systems. These systems deal with the field of digitization. Digitization describes the geometrical representation of a part of the earth's surface based on measurements of discrete points of the surface and the corresponding interpolation rules. The products of digitization are profiles, spot heights, contour lines, and, to a certain extent, planimetric maps. The object of this section is to report on our experience in this field using a simple system composed of:

- A conventional stereoplotter (A8 or A10);
- An electronic recording device (Gradicon);
- A Hewlett-Packard Calculator (H.P.C.);

TABLE 14. REJECTION RATE FOR GHANA PROJECT

Points		A	B	C	A1	D1	A2	A3	B2	Total	%
		Blocks									
Horizontal	Total	12	65	78	13	28	24	41	99	360	6
	Rejected	0	1	0	0	1	4	7	7	20*	
Vertical	Total	321	210	342	415	221	414	516	449	2888	3
	Rejected	11	5	12	10	6	11	16	13	84	
Ties	Total	614	296	501	462	511	637	992	539	4552	5
	Rejected	9	4	28	30	22	40	48	40	221	

* 3 ground control rejected, and 17 tie pts. from previous adjusted blocks.

TABLE 15. SUMMARY OF GHANA'S BLOCK ADJUSTMENT

		Planimetry					Height	
Blocks	N_M	n_1	n	σ_{FP} (μm)	σ_{XY} (μm) (Forster, 1975)	$\frac{\sigma_{FP}}{\sigma_{XY}}$	n	FH μm
A	273	12	12	65	78	0.8	310	39
B	144	64	06	123	99	1.2	205	38
C	195	78	11	85	81	1.1	330	45
A1	280	13	9	74	95	0.8	405	43
D1	200	27	7	123	98	1.3	219	39
A2	454	20	18	111	78	1.4	403	47
A3	426	34	18	119	77	1.5	500	51
A4	434	39	27	92	66	1.4	478	52
A5	435	31	30	54	71	0.8	432	48
B2	316	92	12	132	86	1.5	436	46
B5	215	62	11	152	82	1.9	284	50
B3	226	56	10	117	86	1.4	359	58
B4	278	84	13	107	84	1.3	345	58
13	3876	612	186				4702	

n = number of control; n_1 = n+ tie points used as control from previous adjusted blocks.

- An interface between the above three components combined with a key punch or a magnetic tape drive; and
- A Calcomp pen plotter.

The early development of the system produced mainly digitized profiles by using a compiled manuscript as a base for data collection. Recently, however, no difficulty has been encountered in the production of digital terrain models composed of height information in the form of either profiles or spot heights followed by contour lines.

The sample used for testing the performance of the system components is shown in Table 16.

OUTLINE OF THE SYSTEM

This section will list and discuss the experience gained by using the above-mentioned system and give some basic information on its performance.

- The system is off-line. The digitized sample is produced either on a tape or cards from the stereoplotter, and the digital plotting is carried out on the Calcomp pen plotter.
- The mode of digitization is a point mode to produce profiles and spot heights which can be used for the production of contour lines by a linear interpolation method.
- The choice of the linear interpolation method was preferred over others (Leberl, 1973), because we use mainly square or rectangular grids as patches for the digitized sample points. This coincides with previous work (Leberl, 1973; El-Ghazalli, 1974; Makaroric, 1973) on this subject, from which it is concluded that, when a regular point grid is used for digitization, the choice of the interpolation method has little effect on the height accuracy and the difference in accuracy between interpolation methods is fairly small. It is a simple, convenient, and efficient procedure.

TABLE 16. DIGITIZED SAMPLE USED

		Scale				
Sample	N_M	Photo	Map	C.I. (ft.)	Purpose	Mode
1	20	1:3000	1:480	2	Profiles, Spot-heights	Production
2	7	1:24000	1:12000	10	Height and Planimetry	Test

TABLE 17. GRID AND RANDOM TOLERANCE FOR DIGITIZED HEIGHT DATA

Terrain	Scale of Photo	Grid Distance: Direction of the Flight	Grid Distance: Across the Flight	Random Tolerance
Flat	Large	1/20 B	1/40 B	0.2‰ H
Broken and not Flat	Large or Medium	1/40 B	1/80 B	0.3‰ H

B = The base of stero-model
H = The flying height, ‰ = 1/1000

- The criteria for choosing new digitized points in addition to those specified in the regular pattern are governed mainly by the topographical rise and fall of the terrain.
- Our experience in planimetric digitization is limited and at an early stage; however, the profiles, spot heights, and contour lines are now satisfactory for production purposes. Manual editing is performed rarely for profiles but occasionally for spot heights.
- Our experience gained from the shape and size of the grid patch and the distances between cross or longitudinal profiles has shown that, for a satisfactory representation of the terrain heights, an additional digitized point is required on the grid patch or the profile with the specified distance shown in Table 17.
- A random tolerance, on the order shown in Table 17, is recommended to fulfill the height accuracy requirement in comparing a digitized height and line map product.

MATHEMATICAL TRANSFORMATION AND COMPUTER PROGRAMMING

The transformation formulas used to transform the machine coordinates from either relative or absolute models to other local cartesian coordinates systems depend on the four methods utilized in digitizing the samples. The mathematical equations used in those methods are

$$X_G = \lambda\,(X \text{ Cos } K - Y \text{ Sin } K) + X_o \quad (7a)$$
$$Y_G = \lambda\,(X \text{ Sin } K + Y \text{ Cos } K) + Y_o \quad (7b)$$
$$Z_G = \lambda\, Z \quad (7c)$$
$$Z_G = \lambda\, C_o + C_1 X + C_2 Y \quad (8)$$

where λ ; Scale Factor
K; Swing Angle
X, Y, Z; The machine coordinates
X_o, Y_o, Z_o; Three shift components
X_G, Y_G, Z_G; The ground coordinates
C_o, C_1, C_2; The coefficients of first order polynomial

The four methods used for digitization are shown in Table 18, and the following brief remarks regarding these methods and their mathematical transformations (M.TR.) have been added:

TABLE 18. METHOD USED FOR DIGITIZATION

Method	Stereo Plotter Mode	H.P.C. Used	Purpose	Equations Used	Transformations for
A	RO + AO	Yes	Profiles	7A,7B,7C	NCL
B	RO + AO	No	Profiles Spot Heights	7A,7B,8	NCL
C	RO	No	General	2,3A,3B 7A,7B	AO + NCL
D	RO + AO	No	General	(Jaksic, 1974)	NCL

NCL = New center-line
RO = Relative Orientation
AO = Absolute Orientation

- M.TR. for method A, is a first-order conformal transformation and direct scale in height (Equations 7a, 7b, and 7c). It is an on-line method because the computation for Equation 7 is done with the H.P. calculator which is connected to the stereoplotter. The digitization is performed after the relative orientation (RO) and absolute orientation (AO) of the models is done. The function of the M.TR. is to orient the digitized data (profiles and cross sections) to a new local coordinate system (e.g., center lines of road or river).
- M.TR. for method B is a first-order polynomial in planimetry and height (Equations 7a, 7b, and 8). Method B is similar to method A, with exception that the mathematical computation is carried out on a mini-computer. Method B corrects for the slope of the terrain.
- M.TR. for method C, is based on the numerical procedure given in the section on absolute orientation. In addition to performing the AO, it has the option of scaling and rotating the digitized sample to a new coordinate system by using a first-order conformal transformation. Also, it has another option: to interpolate linearly the produced spot height in order to form contour lines.
- Method D is a complete software program developed at NRC (Jaksic, 1974), applying a generalized first-order polynomial in planimetry to allow for affine and conformal properties for the transformation.

The software programs used can be divided into two main parts: (1) Computer programs for the M.TR. listed in Table 18 which have been developed in the Company with the exception of computer program used for method D, and (2) standard software programs developed by Calcomp (1968).

Finally, we should point out that, although little has been said in this section about utilization, correction, figure field, and contouring programs, these programs are in the developmental stage and will form the subject of a later article. Also, an economical complete software package for an automatic cartographic data system, especially for small- and medium-scale mapping, represents a considerable challenge to the photogrammetrists.

Conclusions

From the analyses of the results and comparisons given in the previous sections, the following conclusions are drawn:

(1) Graphs for the over-correction factor for the lateral tilt omega during relative orientation are of practical importance especially in case of incomplete stereo-models.

(2) From Table 2, only 15, 9, and 20 percent of B, Ω, and Φ, respectively, need minor corrections if the procedure of the numerical absolute orientation (NAO) is applied.

(3) The time required for absolute orientation has been halved by using the NAO. Also, this procedure computes two powerful statistical estimators, namely σ_{OH} and σ_{AH}.

(4) The second-degree polynomial adjustment appears very satisfactory for both height and planimetric requirements in large-scale mapping. Also, the height control has no influence on the planimetric accuracy and vice versa. This conclusion holds true in the case of small-scale mapping.

(5) The evaluation and analyses of different statistical estimators are of great importance in the field of practical aerial triangulation.

(6) From Table 10, it is easy to conclude that the results obtained from the large-scale aerial triangulation projects are slightly better or similar to the published results in this field.

(7) The accuracy of the perspective center in the Z direction behaves as any model point in the stereo-model, and the A10 stereoplotter (in Z direction) is a stable stereoplotter.

(8) A large project of small-scale mapping, which covered approximately 80 mapping sheets at a scale of 1:50,000 in Ghana, has been discussed. This project contains 3,876 stereo-models adjusted in 13 blocks and the results are given in Table 15. From this table, a combination of barometric levelling and statoscope (with bridging distance up to ten models) proved sufficient as vertical control for the height block adjustment for 50-foot contour interval mapping.

(9) The second-degree polynomial adjustment appears satisfactory for both planimetry and height requirements in the case of small-scale mapping. However, in some cases for larger strips (25 models) a third-degree polynomial in the height adjustment is used.

(10) The practical results which emerged from the Ghana project and their evaluation could serve as a good example for photogrammetric block adjustment in small-scale mapping.

(11) Digitized profiles and cross sections, and also to a certain extent spot height and

contour lines digitization, are straightforward products, since they require little time for manual editing.

(12) It may, however, be generally concluded that the basic problems in the field of numerical photogrammetry are the same in university, research, government, and industrial organization.

Acknowledgment

The writer expresses his thanks to all his colleagues of Terra Surveys Limited, Ottawa, Canada, for their contributions and for allowing him the necessary time to make this paper possible.

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New Sustaining Members

Northern Aerial Survey

NORTHERN AERIAL SURVEY was incorporated in 1973 and started business with one first order plotter and one operator. The company is located at 3025 Airport Road, Hastings, Michigan, 49058, (Phone: 616/945-3216).

Charles F. Murphy is the President of Northern Aerial Survey, which is located in the airport at Hastings in 3200 square feet of floor space with two plotters, two operators, and adequate peripheral equipment (drafting tables, light tables, and scribing equipment). They report having a modest photo lab with a 48″ × 48″ camera, film processor, and miscellaneous supporting equipment.

Northern Aerial Survey has been growing since incorporation and is building an enviable list of satisfied clients on jobs from a 20 acre trailer park to a complete map for an Air Force Master Plan Study.

Texas Instruments, Inc.

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Texas Instruments, Inc. address is P.O. Box 5621, Dallas, Texas, 75222, ATT: Francis J. Buckmeier, Ecological Services.