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Analytical Rectification Using Artificial Points

Data from a block aerotriangulation adjustment are used to project photograph points to the desired rectification plane, to which the photograph is then rectified.

Abstract: *A technique is presented that is operational in a production environment for rectification by computing coordinates of fiducials projected into object space. The major assumption is that data are available from a block aerotriangulation adjustment. The computational workload for a small programmable computer is trivial, and the flexibility for defining any plane of rectification is at the discretion of the photogrammetrist.*

Metrically accurate photo-base maps have been in demand for decades, and their use has received an increased impetus in recent years with the advent of the orthophotographic process. In areas of moderate to heavy relief, the only practical solution to accurate photographic imagery is the orthophoto. Deltas and floodplains, however, can be represented adequately with rectified photography. There are problems normally associated with the rectification process; the major difficulties are orientation and establishing a mean plane of rectification where ground control is of varying elevations. Establishing a mean plane by compensating for perspective distortion is presently accomplished by one or more of the following methods:

(1) analytical resection,
(2) "first and second tilt,"
(3) empirical fit, and
(4) ignoring the problem.

The most common method probably utilized is a combination of methods (3) and (4) for small projects. As projects increase in size, accumulation of error increases also and, generally, a tear in imagery is required. Often problems can be minimized by utilizing analytical resection, and a plane of rectification can be established analogically with a precision rectifier.

There are difficulties inherent with precision rectifiers:

(1) calibration,
(2) temperature,
(3) repeatability, and
(4) major capital investment.

The first difficulty, calibration, is of vital importance. Major equipment manufacturers normally calibrate a precision rectifier upon installation. Meticulous cleaning and gentle handling will necessitate recalibration every six to twelve months by a master technician. The second difficulty, temperature, varies according to equipment manufacturers; but there is a certain range of temperature within which a precision rectifier can maintain calibration. After that range is exceeded, operation must cease until the proper temperature can be re-achieved and the instrument has stabilized. Repeatability, the third difficulty is directly influenced by calibration and temperature. It follows, then, that these difficulties are in inverse proportion to that of the fourth.

Given a set of well-distributed ground points, first tilt and second tilt rectifier settings can be computed from various equations listed in the references. The accuracy of the rectified photograph is dependent on the terrain and imagery measurements, the geometric distribution of the control with respect to the photo, and the mechanical accuracy of the rectifier. The traditional process for "first and second tilt" rectifications has been to prick control on contact prints, measure distances with a scale, and reduce the data to angular units for tilt table settings and compensation for the Scheimpflug condition.

With the increasing availability of point transfer devices and aerotriangulation, the "first and second tilt" method has fallen in disfavor to empirical fits of triangulated ground points. This method uses the standard aerotriangulation process; triangulated ground points are plotted on a base sheet, and contact imagery marked with pass points is then rectified to a visual best mean fit with the base sheet. An alternate method is available to those organizations fortunate enough to have a fully-analytical capability and a precision rectifier. Resected camera station parameters may be used to derive angular units for tilt table settings, but there are disadvantages to this approach also.

First, aerotriangulation is usually performed on second generation imagery in order to avoid handling the original negatives. Final delivery products then must be third or fourth generation imagery with a drop in imagery quality easily noticed. Second, with the exception of precision rectifiers being able to receive absolute camera station parameters in machine units and correspondly re-scale for a specific plane of rectification; an empirical fit is the final rectification with an unknown and virtually always inclined plane of rectification. Finally, the only solution available for defining a specific plane of rectification is to achieve absolute orientation in a stereoplotter, set the elevation of the plane of rectification in the plotter, and compile salient imagery monoscopically from each photograph to be rectified. The monoscopic manuscripts then can be used for original negative rectification control at a probable cost unfeasible for competitive commercial mapping.

The ideal method for rectification should:

(1) Be able to use original negatives or original diapositives,
(2) Be able to define any plane of rectification,
(3) Provide an exact orientation without ambiguity,
(4) Utilize perimeter control, and
(5) Be efficient in a production environment.

Analytical rectification using artifical points will satisfy the above criteria. The approach presumes the use of full analytical techniques (using plate coordinates rather than model coordinates). There are many mapping organizations (to date) that do not have analytical facilities, but there are service companies capable of providing the necessary full analytical data. An attractive adjunct to this approach is that it requires only a trivial amount of computation on a digital computer. The major assumption in this paper is that classical full-analytical bundle adjustment aerotriangulation is a prerequisite to this approach.

The fundamental projective relation can be expressed as

$$\begin{bmatrix} X - X^c \\ Y - Y^c \\ Z - Z^c \end{bmatrix} = k \, [\boldsymbol{M}] \begin{bmatrix} x \\ y \\ c \end{bmatrix} \tag{1}$$

where X, Y, Z are the object space rectangular coordinates of a point,
X^c, Y^c, Z^c are the object space rectangular coordinates of the camera station,
k is a scalar,
$\boldsymbol{M}$ is the orientation matrix of direction cosines,
x, y are the image coordinates of a point, and
c is the focal length of the camera.

The above relates the X, Y, Z object space rectangular coordinates of a ground point to the x, y plate coordinates of its image. When the orientation of a camera is known, the direction of a photographed point may be determined from the measured plate coordinates of its image. For a detailed explanation of the orientation matrix and its elements derived from various orientation systems, the reader is referred to Chapter 2 of the *Manual of Photogrammetry,* Third Edition.

Simplifying terms from Equation 1,

$$(X - X^c) = k\,Q,$$
$$(Y - Y^c) = k\,R,$$
$$(Z - Z^c) = k\,S,$$

and solving for the horizontal coordinates of a ground point,

$$(X - X^c) = (Z - Z^c)\frac{Q}{S}, \text{ and} \tag{2}$$

$$(Y - Y^c) = (Z - Z^c)\frac{R}{S}\,. \tag{3}$$

The computation of Equations 2 and 3 will yield X, Y coordinates of a ground point for a given Z. The computation is simple and straightforward on a desk-top computer, and can be done on some advanced pocket computers. For manual calculations, the most tedious phase is the reconstruction of the orientation matrix, $\boldsymbol{M}$, if only the attitude angles such as ω, ϕ, κ, are listed in the aerotriangulation adjustment tabulation. If the reader has access to an aerotriangulation system that tabulates the orientation matrix, the option is most desirable for this application.

The data required for this approach are

(1) x, y, plate-centered image coordinates referenced to the principal point. These data are often used as input to an aerotriangulation system.
(2) X, Y, Z, ground point coordinates. These data are the primary output of an aerotriangulation system.
(3) X^c, Y^c, Z^c, ω, ϕ, κ, camera station parameters. These data are a secondary output of an aerotriangulation system after the photographs are resected.
(4) c, calibrated focal length of the camera.

An aerotriangulation adjustment will compute the coordinates of points imaged on two or more photos. The coordinates computed will have three components: horizontal (X, Y), and vertical (Z). If an elevation is known or assumed for a ground point, a horizontal coordinate may be computed by Equations 2 and 3, by using only one photo. In other words, rather than solve for the intersection of two or more rays in object space to derive the X, Y, Z of a ground point imaged on two or more photos, Equations 2 and 3 solve for the intersection of a single ray to a plane in object space at a given elevation Z.

The aerotriangulation has computed the X, Y, Z of a point from multiple images in a least squares adjustment; by re-computing the horizontal coordinates using Equations 2 and 3 with Z being known, the resulting coordinate will be slightly different from the least squares adjustment. The difference between these two coordinates shows the precision of the single ray approach (see Table 1).

The residuals in object space are a result of not correcting for systematic distortion and not using multiple intersections. For a nominal 5× enlargement of photo scale for rectified photo-base maps, the precision obtained with the single ray approach is more than adequate in rectifier space.

The primary concern in photo-base mapping is the final accuracy of the imagery at map scale. Photo-base map accuracy is specified by the allowable horizontal displacement of imagery from its true position. Rectified photographs are an attempt to transform a plane (the imagery) to a curved surface of varying elevations (the ground). The major distortions are perspective and Earth curvature. By choosing an elevation to represent a plane of rectification, Equations 2 and 3 will yield the horizontal position of a point at that elevation. By taking the difference in horizontal positions of a point computed in the aerotriangulation and the same point computed as a single ray at the chosen plane of rectification, that difference will represent the distortion or the accuracy of rectifying that point. Proceeding with all other points measured on the photo will give the individual distortions at those points for a rectification to the chosen plane (see Table 2).

An analysis of residuals computed as in Table 2 may give the photogrammetrist an indication of the rectification accuracy. (Such residuals can be quite misleading if all points measured happen to be near the same elevation in an area of moderate to heavy relief.) In addition to computing single ray coordinates of points used in the aerotriangulation, additional points may be computed for the same plane of rectification. Fiducial coordinates can be computed, and are quite easy to identify for rectification.

TABLE 1. SINGLE RAY PRECISION

Photo Scale	Object Space RMS (feet)	Image Space RMS (mm)	5× Enlargement Rectifier Space RMS (inches)	Number of Photos
1:12,000	0.62	0.006	0.003	60
1:12,000	0.50	0.005	0.003	33
1:12,000	0.24	0.006	0.001	29
1:12,000	2.61	0.006	0.013	127
1:24,000	6.91	0.006	0.017	82
1:24,000	4.47	0.006	0.011	160

TABLE 2. SINGLE RAY PLANIMETRIC ACCURACY

Area	Photo Scale	Object Space RMS (Feet)	Image Space RMS (mm)	5× Enlargement Rectifier Space RMS (Inches)
N.W. Kansas	1:50,000	44.08	0.008	0.053
		49.96	0.010	0.060
		43.59	0.011	0.052
S. Louisiana	1:30,000	10.37	0.042	0.035
		8.71	0.019	0.029
		3.77	0.014	0.013

It is interesting to note that the common practice in commercial aerotriangulation projects designed for a 5× enlargement for rectification or stereocompilation is performed from plate measurements referenced to only four fiducials. The majority of firms that offer aerotriangulation services for ordinary applications offer unit prices based on reading only four fiducials (corners when available). For rectification projects designed for a 5× nominal enlargement factor, reading four fiducials implies the requirement for a tilting easel measuring 45 inches by 45 inches. In order to facilitate reducing one of the dimensions of the tilt table, the use of a transformation algorithm for a camera's flash plate or calibration report is useful. Simply stated, a three-parameter transformation is used to merge the flash plate coordinate system into the image plate coordinate system used in single ray computations. After the transformation is completed on a plate-by-plate basis, not only the four corner fiducials used in the original aerotriangulation mensuration are available for single ray computation, but also all marks observed *only once on the flash plate* (or calibration) may be computed. The photogrammetrist may then compute the object space coordinates of corner fiducials, side fiducials, *reseau* marks, saw teeth, side lap notches, and film shrinkage marks. The photogrammetrist then has a great latitude in the point distribution he can make available to the rectifier operator, and design the optimum geometry for any given equipment configuration. Furthermore, the photogrammetrist may select any plane of rectification desired by stating it in terms of Z as an argument in Equations 2 and 3. Finally, because object space coordinates are computed for fiducials and other marks on the camera cone face, there is no need to perform rectification from diapositives used in aerotriangulation; the original negatives will perform admirably well and with better resolution.

In conclusion:

(1) Single ray allows indiscriminate use of original negatives or diapositives,
(2) Any plane of rectification can easily be computed,
(3) Rectification orientation is unique,
(4) Geometry used is based on an aerotriangulation output,
(5) Fiducials and saw teeth are quite well defined and allow rapid identification during orientation.

This technique is a refinement for production rectification. In areas of moderate to heavy relief the single ray approach will not replace the orthophoto process; single ray is attractive only for projects normally performed by rectification.

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REFERENCES

Abdel-Messih, M. A. 1963. Projective Nets Corrected for Radial Distortion for Graphical Rectification in Aerial Photogrammetry, *Photogrammetric Engineering*, Vol. XXIX No. 5, pp. 893–895.

Altenhofen, Robert E. 1944. The AMS Method of Tilt Analysis, *Photogrammetric Engineering*, Vol. X, No. 4, pp. 265–276.

———— 1945. Reply to Comments of Mr. Singleton and Others, *Photogrammetric Engineering*, Vol. XI, No. 3, pp. 231–235.

Anon, (AMS) 1946. The Rectification of a Tilted Aerial Photograph, *Photogrammetric Engineering*, Vol. XII, No. 3, pp. 297–307.

Boge, W. E. 1965. Resection Using Least Squares, *Photogrammetric Engineering*, Vol. XXXI, No. 4, pp. 701–714.

Brewer, R. K., and A. K. Heywood. 1972. Coastal Boundary Mapping, *Proceedings of the 38th Annual Meeting*, American Society of Photogrammetry, pp. 182–191.

Brown, Duane C. 1957. *A Treatment of Analytical Photogrammetry with Emphasis on Ballistic*

Camera Applications, RCA Data Reduction Technical Report No. 39, ASTIA No. 124144.

Church, Earl. 1944. *Notes on the Rectification of Tilted Aerial Photographs,* Bulletin No. 14, Syracuse University.

__________ 1947. Tilt Analysis Based on Vertical Control Data, *Photogrammetric Engineering,* Vol. XIV, No. 3, pp. 463–473.

Clark, H. W. 1949. The Geometry of Photorectification, *Photogrammetric Engineering,* Vol. XV, No. 2, pp. 288–310.

Coblentz, C. S. 1939. Restitution of Aerial Enlargements for Area Determination, *Photogrammetric Engineering,* Vol. 5, No. 3, pp. 138–146.

Dickson, R. A., Low-Cost Analytical Aerotriangulation for Rectification of Aerial Photographs, *Proceedings of the 40th Annual Meeting,* American Society of Photogrammetry, pp. 252–261.

Estes, J. M. 1967. The Anharmonic Method of Rectification, *Photogrammetric Engineering,* Vol. XXXIII, No. 10, pp. 1171–1174.

Eves, Howard. 1944. A Geometrical Note on the Isocenter, *Photogrammetric Engineering,* Vol. X, No. 4, pp. 233–235.

Gracie, G. 1967. *Mensurational Procedures in Terrestrial Photography,* Vol. 1, Raytheon/Autometric Operation, Naval Air Systems Command, Contract No. N00019-67-C-0024.

McNeil, G. T. 1947. Tilt by the Graphical Pyramid Method, *Photogrammetric Engineering,* Vol. XIII, No. 3, pp. 403–414.

Merritt, E. L. 1947. Space Resection, *Photogrammetric Engineering,* Vol. XIII, No. 3, pp. 415–441.

Moric, M. 1966. Projective Methods, *Photogrammetric Engineering,* Vol. XXXII, No. 5, pp. 849–855.

Reddle, V. L. 1954. A Graphical Method of Determining Rectification Range, *Photogrammetric Engineering,* Vol. XX, No. 5, pp. 829–831.

Roth, Edwin A. 1953. Simplified Graphs and Tables for Use With Optical Rectifiers, *Photogrammetric Engineering,* Vol. XIX, No. 4, pp. 630–637.

Singleton, Robert R. 1945. Comments on "The AMS Method of Tilt Analysis", *Photogrammetric Engineering,* Vol. XI, No. 3, pp. 231.

Sternagle, Beryl D. 1968. *Aerial Triangulation Adjustments,* Unpublished Manuscript, U.S. Army Topographic Command.

Traenkle, C. A. 1954. Resection in Space by Projective Transformation, *Photogrammetric Engineering,* Vol. XX, No. 1, pp. 139–159.

__________ 1956. Reduction Process of Resection Problems by Photogrammetric Rectifiers, *Photogrammetric Engineering,* Vol. XXII, No. 4, pp. 741–749.

Underwood, P. H. 1947. The Determination of Tilt from Scale Check Lines, *Photogrammetric Engineering,* Vol. XIII, No. 1, pp. 143–155.

Visser, J. 1959. Systematic Procedure for Affine Rectification, *Photogrammetric Engineering,* Vol. XXV, No. 1, pp. 151–155.

Wilson, K. R., and J. Vlcek. 1970. Analytical Rectification, *Photogrammetric Engineering,* Vol. XXXVI, No. 6, pp. 570–575.

BOOK REVIEWS

Neblette's Handbook of Photography and Reprography, Seventh Edition. Edited by John M. Sturge. Van Nostrand Reinhold Co., New York, 21 × 28 cm, 641 pages, 486 illustrations, hard cover, 1977, $47.50.

Work on the Seventh Edition of Professor C. B. Neblette's *Handbook* was interrupted by his untimely death in 1974. However, this completely new volume, now published 50 years after the First Edition, continues to reflect his capabilities as an educator in organizing and inspiring the writing of up-to-date informative technical material. This continuity is largely due to the editing of John M. Sturge. The 24 chapters, written by 26 experts, cover a range of photographic subjects too long to list here. Silver halide black-and-white and color processes are well represented; so are silverless, electrographic, and other reprographic systems, in considerable depth of theory and practice. Dr. Edwin Land has co-authored a very complete chapter on one-step B/W and color processes. Micrographics and microfiche are included. If the chapter on Aerial Photography and