Dr. UZI ETHROG Technion-Israel Institute of Technology Haifa, Israel

Photogrammetric Positioning of Supersonic Wind Tunnel Models

Hardware and analytical procedures for determining the threedimensional positions of free flight models is described.

INTRODUCTION

TESTING MODELS in wind tunnels is an important stage in planning new aircraft. The common type of supersonic wind tunnel consists of two long and thick steel tubes, one inside the other. The air speed inside the inner tunnel, where the model is tested, can exceed Mach 2. The external tunnel is built mainly for safety, and the air speed in it is much lower. Where the model is inserted for observation the cross-section of the tunnel is rectangular. The model inside the tunnel can be observed from both sides through two thick glass windows. Usually the model is mounted in the wind

is not required in static experiments, and also on the data acquisition and reduction method which is usually more complicated than in static testing, especially when three dimensional information is required.

Because of the short duration of the tests using the free flight technique (less than 0.1 second), the only way to study the model movements in the wind tunnel is by high speed photography and the only way to measure them is by photogrammetric methods. But the common photogrammetric technique of reconstructing the bundle of rays as it was at the moment of photography and by forming a photogrammetric model can not be used here for three reasons.

ABSTRACT: The confined space in a supersonic wind tunnel laboratory and the very short duration of the free flight tests cause serious difficulties in measuring the position of the models inside the wind tunnel during the test. To overcome these problems, an original and simple system was arranged for use with high speed photography, and an appropriate procedure for photogrammetric data reduction was developed. The position and orientation of the model are determined to an accuracy of a few millimetres, which exceeds the requirements for such tests.

tunnel near the windows by strings or struts or balance strings. During the last ten years a new technique, free flight testing in wind tunnels, has become a well-established practice. In principle, results obtained from free-flying models are more reliable than those obtained from models that are statically mounted in the wind tunnel. For certain types of experiments, such as dynamical stability tests or aircraft store trajectories, the investigation of the free flight of models is an absolute "must." However, though more reliable in principle, the accuracy of the free flight data depends very much on the dynamical scaling of the models, which

PHOTOGRAMMETRIC ENGINEERING AND REMOTE SENSING, Vol. 44, No. 10, October 1978, pp. 1261-1265. (1) The high speed camera takes between 600 to 3000 photographs per second. At such a speed only strip cameras can be used. The Hyiam 16 mm camera used in this work has a special prismatic compensator which rotates all the time synchronously with the film movement, thus producing photographs that look like frame photographs. It is not simple to calibrate such a camera. Furthermore, because of the compensator the inner orientation of the camera varies according to a periodic function, i.e., for each possible orientation of the rolling compensator there is a corresponding inner orientation of the camera. As the orientation of the compensator at the moment of photography is unknown, any information about the inner orientation is quite useless.

- (2) The usual procedure of finding the relative and absolute orientation of the photographs can not be used here. Because of the very narrow angle of the camera (the focal length of the camera is 100 to 150 mm while the photograph width is only 16 mm) there is a danger that the equations used for finding the orientation elements will be ill-conditioned.
- (3) The light rays which form the photographs pass on their way from the wind tunnel to the camera objective through two glass windows which are not planoparallel. In addition they are refracted by the air inside the wind tunnel as the density of the air and its refractive index are affected by the extreme changes in the air speed (from nearly zero at the window panes to more then Mach 2 at the center line of the wind tunnel only 25 cm from the windows). It is very difficult to correct for the effect of the refraction of the light rays.

A special photogrammetric method was, therefore, modified and adapted to cope with this problem. The positions of special marks on the model are determined by this method with the aid of known reference points on the front and rear windows of the test section.

THE APPARATUS

The free flying model is viewed in the test section of the supersonic wind tunnel through two pairs of plane, first-surface mirrors (Figure 1). The mirrors are set in special swivelling and adjustable frames in order to

obtain the best possible viewing angles and light conditions. The mirrors in each pair are mounted at an angle to each other, thus "splitting" the picture of the test section. This arrangement has two important advantages. First, the two pairs of mirrors together create a double periscopic system that allows photographing the split view of the test section at close quarters typical of a crowded wind tunnel laboratory. Second, this special arrangement of the mirrors provides a view of the model flying in the tunnel test section from two different angles and permits the use of a single camera to record the free flight trajectory practically in three dimensions. Such an arrangement eliminates the need for a second camera and for synchronization equipment.

Because of the high air speed and the steep gradient of the air pressure inside the supersonic wind tunnel, there is a danger that objects inside it can either affect the wind flow and therefore the test's results, or be destroyed during the test. Therefore, the reference points are marked on the exterior of the inner front and rear window panes (Figure 2). A special correction is applied to the measured data to allow for the refraction of the light rays by the thick window panes (25 mm).

The free flying model also bears marks on its surface in order to obtain as much flight data as possible about the trajectory of the model's center of gravity and about the model's spatial orientation. In the case of a model with pointed fore and aft ends, the ends themselves serve as markings; otherwise, fore and aft marks have to be added.



FIG. 1. Periscopic mirror system.



FIG. 2. Typical photograph with split view of the model and reference marks on window panes of the tunnel.

THE DATA REDUCTION METHOD

To acquire the coordinates of points on the flying model in the wind tunnel, the X, Y, and Z coordinates of the reference points marked on the outside faces of the front and rear windows of the test section must be established. The X-Y coordinates of these reference points are measured in the window planes. The Z coordinates of all the reference points in the front window equal zero and the Z coordinates of all the reference points on the rear window are all equal to the width of the inner tunnel. Referring to Figure 2 it can be seen that on each photograph the following data will appear:

- The images of at least six points on the front window of the tunnel's test section,
- The images of at least six points on the rear window, and
- The image of the free flying model and its marks.

From Figure 3 it is seen that the ray from point A on the model penetrates the glass plate of the front window at point a' and reaches point a on the photograph. Therefore, the point a is the image of both A and a'. Since at least four (usually six) reference points with known coordinates appear on each photograph, it is possible to find the X-Y coordinates of a' in the plane of the front window using the linear projective transformation formulae, i.e.,

$$X = \frac{a_1 U + a_2 V + a_3}{a_7 U + a_8 V + 1}$$
$$Y = \frac{a_4 U + a_5 V + a_6}{a_7 U + a_8 V + 1}$$
(1)

where

U,*V* are the coordinates of point *A* in the photograph plane,

X,*Y* are the coordinates of point a' in the window plane, and

 a_1 to a_8 are constants for a particular photograph.

The Z coordinate of point a' is of course equal to the Z coordinate of the front window plane. In an identical manner the coordinates of a'', which is on the same ray from A, can be found on the rear window plane. If another photograph, taken simultaneously with the first but from a different angle, is available one obtains another ray that passes through the point A. The intersection of these two rays should provide the coordinates of point A in the X, Y, Z space. The two



FIG. 3. Photograph in the wind tunnel.

reconstructed rays usually do not intersect but cross close to each other. The point A is, therefore, assumed to be the midpoint of a line perpendicular to both rays. The points of intersection of this line with the rays are designated by x_1 , Y_1 , z_1 and X_2 , Y_2 , z_2 . The six equations required to solve these unknowns are the two equations expressing the fact that point x_1, Y_1, z_1 is on the ray a'a''; the two similar equations for the other ray; and two additional equations expressing the fact that X_1 - X_2 , Y_1 - Y_2 , and Z_1 - Z_2 are proportional to the components of a vector given by the crossproduct of the vectors that describe the two rays. The coordinates of the point A are finally obtained by taking the average of X_1 and X_2 , Y_1 and Y_2 , Z_1 and Z_2 .

Accounting for the Refraction of Light Rays in the Wind Tunnel Windows

The reconstruction of the positions of the points on the model and on the test section windows from their images on the photographs is complicated by the fact that each ray passes through two thick glass windows. It is, therefore, refracted twice, and the analysis based on straight rays presented thus far is inexact unless this refraction is taken into account. The correlation for the refraction is described here.

In Figure 4 the points a', a'' are connected by the refracted ray that lies in a plane, provided the media through which it passes are homogeneous. The straight line passing through the point A will be defined by the two points G', G'' in the vicinity of a' a''. They lie on the perpendicular to the glass planes which pass through the points a', a'' and their distances from the glass surface depend on the penetration angle of the ray. The distance V from the inner surface of the windows is found as follows:

Referring to Fig. 4 it is seen that

$$\sin\beta = \sin\alpha/n \tag{3}$$



FIG. 4. Refraction of light ray between the points a' and a".

where n is the refraction coefficient for the ray penetrating the window glass from the air inside the wind tunnel.

Also,

$$U = t \cdot \mathrm{tg}\beta = t \frac{\mathrm{sin}\alpha}{\sqrt{n^2 - \mathrm{sin}^2\alpha}} \qquad (4)$$

and,

$$V = U \cot g\alpha. \tag{5}$$

Thus

$$V = t \, \frac{\cos\alpha}{\sqrt{n^2 - \sin^2\alpha}} \tag{6}$$

From the geometry of the wind tunnel for d >> t we get:

$$\cos\alpha \approx \frac{d}{\sqrt{d^2 + L^2}}$$
 (7)

and

$$\sin\alpha \approx \frac{L}{\sqrt{d^2 + L^2}} \tag{8}$$

Therefore,

$$V = \frac{t \cdot d}{\sqrt{n^2 (L^2 + d^2) - L^2}} \tag{9}$$

RESULTS AND CONCLUSIONS

It has been shown in this paper that the three dimensional trajectory of a free flying model in the test section of a supersonic wind tunnel and also its spatial orientation can be recorded by a single high speed camera and a double periscopic optical system. The simultaneous photography by a single camera of two different views of the model on a single frame saves the expense of a second camera and the costly synchronization equipment. The special data reduction method which was developed in this work enables one not only to find the coordinates of the model in the wind tunnel without calibrating the camera but also to compensate for the major part of the errors caused by lens distortion, refraction, and film shrinkage. Using Equation 1 compensates for the effect of the linear irregularities of the photographs, while the effect of the non-linear irregularities is largely compensated by using only the coordinates of the reference points nearest to the model image on the photograph in order to calculate the constants a_1 to a_8 in Equation 1.

Numerical experimentation with the method showed that a distortion of $\pm 40 \ \mu m$ in the location of a point on the photograph scale of 1:100 still permitted reconstruction

of the position in the wind tunnel to within 1 mm. A reduction in size as above (1:100) and the resulting accuracy (1 mm) are suitable for wind tunnel work. Actually, when the method was tested in static calibration tests much better results were obtained.

In these tests a ruler was photographed in several positions and orientations in the 40 cm \times 50 cm supersonic test section and its scale was reconstructed with an accuracy better than 1 percent. The maximum error in the X-Y plane was 1 mm while the maximum error in the Z coordinate was 5 mm. This accuracy and the simplicity of the hardware contribute towards a cost-effective method for wind tunnel free flight data acquisition. (Received October 4, 1977; revised and accepted May 22, 1978)

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3777 Long Beach Blvd., Long Beach, CA 90807

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Besides regular survey equipment, the geodetic survey group in the Netherlands earlier this year acquired an M&S Computing, Inc. interactive graphics design system.