

# Densitometry on Multi-Emulsion Imagery

Basic concepts of color densitometry and film calibration procedures are reviewed.

## INTRODUCTION

INVESTIGATORS in the fields of remote sensing and photogrammetry are beginning to realize the potential benefit of digital analysis of film imagery<sup>1-5</sup>. Film imagery has long been known to be a good qualitative interpretation tool. With the advent of the digital computer and proper calibration procedures, it is now possible to analyze film imagery in a quantitative manner<sup>6,7</sup>. Before meaningful interpretations can be assigned to digital processed imagery, densitometry as applied to multi-emulsion film must be understood. The purpose of this paper is to review basic concepts in densitometry and to discuss film calibration procedures in order to allow the remote sensing investigator to better utilize densitometric analysis of photographic imagery.

---

*ABSTRACT: Basic concepts of color densitometry and film calibration procedures are reviewed with special emphasis on the specific application to the remote sensing investigator. The differences between and the instrumentation to measure the spectral, broad band, specular, diffuse, integral, and analytical densities are discussed. An explanation of equivalent neutral density and methods of determining this type of density are presented.*

*Methodologies of using analytical densities for the remote sensing community are detailed. The use of analytical densities in the construction of characteristic curves is discussed. Comments are made on reasons for the use of analytical densities in the analysis of film imagery and on proper application of the exposure values derived from the characteristic curves.*

---

## STRUCTURE OF MULTI-EMULSION FILMS

There are a great many multi-emulsion films available to the researcher. These films fall into two general types: color films and color-infrared (color IR) films. The color films are in general sensitive to light in the visible part of the electromagnetic spectrum and the color IR films are sensitive to the near-infrared as well as the visible part of the spectrum. There are also some special purpose two layer films (such as Kodak SO-224). Procedures for analyzing each of these types are essentially the same. The only real difference is in the interpretation of the data derived from the densitometric analysis of the imagery. Because the analysis procedures are similar for all types of multi-emulsion imagery, only a detailed description of the analysis procedures for color films will be presented. Extensions to the analysis of color IR imagery will be indicated in the appropriate sections.

Information is recorded on a color image in the form of various amounts of colored dyes. The colored dyes used are yellow, magenta, and cyan. At each spatial location on the processed imagery there exists some concentration of yellow, magenta, and cyan dyes. In general the concentrations of each of these dyes differ from each other as well as differing from the dye concentrations at adjacent parts of the imagery. It is the intent of densitometry on multi-emulsion imagery to infer the amounts of each dye at each spatial location of interest on the imagery. From these data the amount and (to some extent) the spectral distribution of the incident energy on the unexposed film can be inferred.

The structure of a typical color film is shown in Figure 1. From the point of view of the user, the three important layers are the ones that are sensitive to blue, green, and red light. Curves representing the sensitivities of these layers can be found in Figure 2. After the film has been exposed to visible light and processed, yellow, magenta, and cyan dyes are formed in their respective layers. In a color reversal film (such as Type 2448) the amount of dye formed in each layer depends inversely on the amount of light sensitizing each layer. An object that appears blue in nature will, after processing, cause an image to contain a relatively small amount of yellow dye and proportionately larger amounts of magenta and cyan dyes. Because magenta and cyan dyes absorb the green and red light, the image will appear blue in normal lighting conditions. To infer the amounts of the dyes present in the imagery, some type of measurement must be made on the developed transparency.

DENSITY

Light absorption in a photographic image is expressed in terms of optical density. For black and white imagery it is usually defined as

$$D = \log_{10} \frac{1}{T} = \log_{10} \frac{P_o}{P} \tag{1}$$

where  $D$  is the optical density,  $T$  is the transmittance, and  $P_o$  and  $P$  are the incident and transmitted light fluxes respectively.

For black-and-white emulsions, the density of the transparency is related to the amount of metallic silver in the image. This in turn is related to the amount (and spectral distribution) of the incident radiation on the unexposed film. Color film images are more complex; thus, density definitions must be carefully examined.

The transmitted light flux through a piece of film can be written as

$$P = P_o 10^{-D} \tag{2}$$

Usually the incident flux and density are not independent of wavelength; thus, Equation 2 is more correctly written as

$$P(\lambda) = P_o(\lambda) 10^{-D(\lambda)} \tag{3}$$

where  $(\lambda)$  indicates that the preceding term depends on wavelength; i.e., Equation (2) is independently valid for each wavelength. If  $P_o(\lambda)$  was incident on the film diagrammed in Figure 1, the transmitted flux,  $P(\lambda)$ , through the film can be calculated. Assuming the

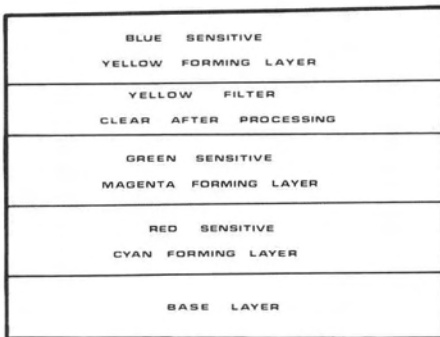


FIG. 1. Schematic of the film structure for Kodak Aerochrome MS film 2448, Estar base.

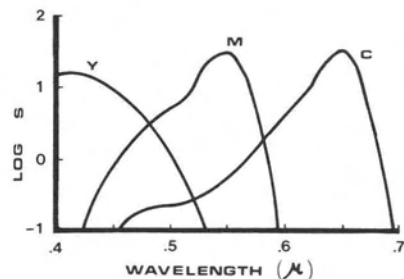


FIG. 2. Spectral sensitivity curves for Kodak Aerochrome MS film 2448, Estar base.

incident light is impinging on the bottom of the transparency, then the transmitted light through the base layer can be written as

$$P_b(\lambda) = P_o(\lambda)10^{-DB(\lambda)} \tag{4}$$

where  $DB(\lambda)$  is the base density and  $P_b(\lambda)$  is the transmitted flux through the base material. The incident flux on the cyan layer is the light transmitted through the base layer. Thus the transmitted intensity on the upper side of the cyan layer is

$$P_c(\lambda) = P_b(\lambda) \cdot 10^{-DC(\lambda)} \tag{5}$$

or

$$P_c(\lambda) = P_o(\lambda)10^{-DB(\lambda)} \cdot 10^{-DC(\lambda)} \tag{6}$$

where  $DC(\lambda)$  is the density of the cyan layer and  $P_c(\lambda)$  is the transmitted flux through the cyan layer. The magenta and yellow layers can be treated similarly, and the total transmitted flux can be written as

$$P(\lambda) = P_o(\lambda)10^{-DB(\lambda)} \cdot 10^{-DC(\lambda)} \cdot 10^{-DM(\lambda)} \cdot 10^{-DY(\lambda)} \tag{7}$$

where  $DM(\lambda)$  and  $DY(\lambda)$  are the densities of the yellow and magenta layers, respectively.

Equation 7 can be rewritten as

$$P(\lambda) = P_o(\lambda)10^{-[DB(\lambda) + DC(\lambda) + DM(\lambda) + DY(\lambda)]} \tag{8}$$

If the terms in the bracket are set equal to  $D(\lambda)$ , i.e.,

$$D(\lambda) = DB(\lambda) + DC(\lambda) + DM(\lambda) + DY(\lambda), \tag{9}$$

the form of Equation 8 is exactly that of Equation 3. The  $D(\lambda)$  is the overall density or *integral* density of the transparency. An integral density is one which is the sum of individual component densities. Densitometers measure an integral density. The component densities are called *analytical* densities. In a multi-layered film the densities of the individual layers as well as the base ( $DB$ ,  $DC$ ,  $DM$ , and  $DY$ ) would be analytical densities. There is no way of directly measuring the analytical densities in a multi-layered image. In a multi-layered film one of the tasks of densitometry is to determine the analytical densities from a multitude of integral density measurements.

Since density is a function of wavelength, Equation 1 can be written as

$$D(\lambda_a) = \log_{10} \frac{P_o(\lambda_a)}{P(\lambda_a)} = \log_{10} \left( \frac{1}{T(\lambda_a)} \right) \tag{10}$$

where  $\lambda_a$  is a particular wavelength. Density expressed in this fashion is called *spectral* density. Spectral density is a measure of density at a particular wavelength. Either the incident flux is monochromatic or the densitometer records a monochromatic part of the transmitted flux. Many of the densitometers in use perform neither of the above functions. These densitometers employ white light as the incident light flux and either detect the transmitted light directly or through a particular color filter. The densities these types of instruments measure are *broad band* densities. A broad band density is a density measurement which integrates the effects of the spectral density variation over a wide range of wavelengths. Kodak wratten filters 92, 93, and 94 (Figure 3) are commonly used for color

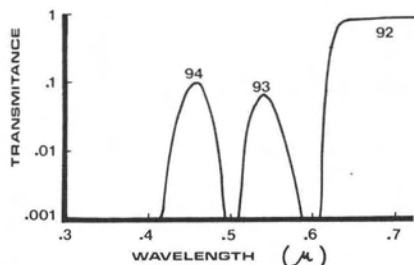


FIG. 3. Transmittance vs wavelength curves for Kodak Wratten filter numbers 92, 93, and 94.

separation on color films. A density measurement with one of these filters in the light path would be a broad band density measurement. Because of the broad band nature of the density measurement, the mathematical expression of broad band density is different from Equation 10. Broad band density is not only a function of the incident and transmitted light flux, but is also a function of the transmittance of the filter and the sensitivity of the densitometer. For broad band density, designated as  $D_f$ , Equation 10 can be written as

$$D_f = \log_{10} \frac{\int P_o(\lambda) \cdot F(\lambda) \cdot S(\lambda) d\lambda}{\int P(\lambda) \cdot F(\lambda) \cdot S(\lambda) d\lambda} \quad (11)$$

where  $D_f$  is the broad band density measured through filter  $F$ ,  $F(\lambda)$  is the transmittance function of the filter (if any) in the system, and  $S(\lambda)$  is the relative sensitivity of the densitometer to light flux. The integral extends over all wavelengths where the densitometer is sensitive. If the filter is sufficiently narrow, then the integrals in Equation 11 are not necessary and Equation 11 reduces to Equation 10. Thus, it is usually possible to make spectral density measurements on an instrument that normally has been making broad band density measurements if a sufficiently narrow band filter is substituted for the broad band filter (with an associated degradation of the signal-to-noise ratio for the system).

Depending on the optical geometry within a densitometer, the instrument will measure either *specular* or *diffuse* density. If the incident light flux is collimated and the transmitted flux is measured through a small aperture, the density measured is specular. For example, a spot micro-densitometer is an instrument that measures specular density. For diffuse density measurement, incident light flux is extended (doubly diffuse) or collimated, and the transmitted flux is measured with an extended angular acceptance. Diffuse densitometers are used extensively by the photographic industry for control of film processing. Figure 4 is a schematic of the optical geometry for these two types of densitometers. For most remote sensing applications, specular density measurements are used for analysis.

#### ANALYTICAL DENSITY

There are many different ways of representing analytical density in a multi-layered film. The representation used by most photo scientists involves a calculation of an Equivalent Neutral Density (END). Procedures for END calibration of color densitometers have been discussed extensively in the literature<sup>8-11</sup>. This method of analytical density representation is not as well suited for remote sensing purposes as it is for control of the photographic process. The END of a given dye patch in a layer of color film is the visual density the patch would have if the required amounts of the remaining two dyes were added or subtracted so that the modified patch was a neutral gray<sup>14</sup>. An equivalent neutral density is a *visual analytical density*. The type of density best suited to the remote sensing investigator is a *spectral analytical density*.

If one could measure the density as a function of wavelength for a single layer, an analytical spectral density measurement would result. It is usually not possible to do this unless one has the capability to deposit a single layer of dye on some substrata. Eastman Kodak uses this technique as the first step in END calibration<sup>11</sup>. For the remote sensing investigator, only an integral spectral density measurement is possible. The relationship between integral spectral density and analytical spectral density has been given in Equation 9.

$D(\lambda)$  is the integral spectral density;  $DY(\lambda)$ ,  $DM(\lambda)$ , and  $DC(\lambda)$  are the analytical spectral densities of the yellow, magenta, and cyan layers, respectively; and  $DB(\lambda)$  is the spectral density of the base material which also includes other residual stain<sup>11</sup>. Some investigators

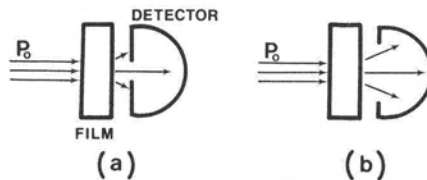


FIG. 4. Schematic of the optical geometry for (a) specular density measurement and (b) diffuse density measurement.

have reported that Equation 9 does not exactly describe the density relationship in a multi-layered film<sup>14,15</sup>. These investigators point out that there are inter-layer interactions in a multi-layered film which may cause small amounts of different colored dye to be formed in the image. These are small effects and for most purposes Equation 9 does adequately describe the relationship between integral and analytical densities in a three-layered film. The problem to resolve is how to determine  $DY$ ,  $DM$ , and  $DC$  from a measurement of the integral density  $D(\lambda)$ .

If Equation 9 represented the only knowledge of the film characteristics, any number of integral density measurements would not be sufficient to determine the analytical densities of the layers. Fortunately, this is not the case. The spectral properties of the dyes present in the transparency are known (or can be determined). With this added information, the analytical densities can be determined.

Let  $Y(\lambda)$ ,  $M(\lambda)$ , and  $C(\lambda)$  represent the unit spectral densities for the yellow, magenta, and cyan dyes in a film. Unit spectral densities are analytical spectral densities which have been normalized so that their peak value is 1.0. Figure 5 is a typical set of such curves for a color reversal film. The relative spectral properties of the dyes in these types of films are independent of concentration. The analytical density for the yellow dye layer can then be represented as some constant,  $K_y$ , times the unit spectral density function for the yellow dye, i.e.,

$$DY(\lambda) = K_y Y(\lambda) \quad (12)$$

Similarly, the analytical spectral densities of the magenta and cyan layers can be represented as

$$DM(\lambda) = K_m M(\lambda) \quad (13)$$

and

$$DC(\lambda) = K_c C(\lambda) \quad (14)$$

and Equation 9 can be rewritten as

$$D(\lambda) = K_y Y(\lambda) + K_m M(\lambda) + K_c C(\lambda) + DB(\lambda) \quad (15)$$

If the functions  $Y(\lambda)$ ,  $M(\lambda)$ ,  $C(\lambda)$ , and  $DB(\lambda)$  are known, it is possible to determine the analytical densities from measurements of the integral density at three different wavelengths. If the integral density measurements are taken at  $\lambda_1$ ,  $\lambda_2$ , and  $\lambda_3$  (the three different wavelengths), a set of three simultaneous equations can be solved for the dye parameters,  $K$ . If we let  $\Delta D(\lambda) = D(\lambda) - DB(\lambda)$ , the solutions to these equations can be written

$$\begin{aligned} K_y &= C_{11}\Delta D(\lambda_1) + C_{12}\Delta D(\lambda_2) + C_{13}\Delta D(\lambda_3) \\ K_m &= C_{21}\Delta D(\lambda_1) + C_{22}\Delta D(\lambda_2) + C_{23}\Delta D(\lambda_3) \\ K_c &= C_{31}\Delta D(\lambda_1) + C_{32}\Delta D(\lambda_2) + C_{33}\Delta D(\lambda_3) \end{aligned} \quad (16)$$

where the nine constants ( $C_{ij}$ 's) are dependent only on the characteristics of dyes in the film and not on the specific combination of dyes which represent a scene on the imagery. Thus, from three integral spectral density measurements [ $D(\lambda_1)$ ,  $D(\lambda_2)$  and  $D(\lambda_3)$ ], three

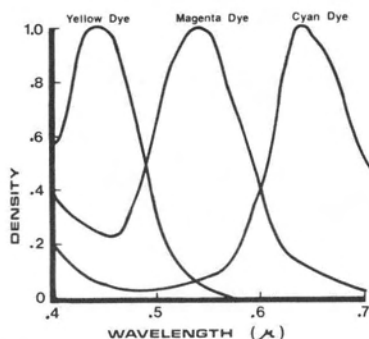


FIG. 5. Unit spectral density curves for the three dyes in a typical color reversal film.

parameters ( $K_y$ ,  $K_m$ , and  $K_c$ ), dye constants can be calculated. The analytical spectral density for each of the layers can be found using Equations 12, 13, and 14.

The three  $K$ 's ( $K_y$ ,  $K_m$ , and  $K_c$ ) can be thought of as analytical densities themselves since they do not depend on wavelength, but just depend on the amount of dye formed in each layer. A dye constant  $K$  represents the spectral analytical density at the peak absorption for that dye. Because the  $K$ 's can be related to the amount of dye in each of the layers, these are the parameters to be derived for each spatial location on the film to characterize a scene.

A problem arises in the derivation of the three  $K$ 's if a spectral densitometer is not available for analysis. Many of the instruments available to the remote sensing investigator are broad band densitometers. Substituting Equation 15 into Equation 8 and the resulting expression into Equation 11, the resulting relation between measured broad band integral density and analytical density is

$$D_f = \log_{10} \frac{\int P_o(\lambda) \cdot F(\lambda) \cdot S(\lambda) d\lambda}{\int P_o(\lambda) \cdot F(\lambda) \cdot S(\lambda) \cdot 10^{-K_y Y(\lambda) - K_m M(\lambda) - K_c C(\lambda) - DB(\lambda)} d\lambda} \quad (17)$$

Because of the presence of the integrals in the numerator and denominator of this expression, it cannot be reduced to the form of Equation 15. The only way to reduce this equation to one that is similar to Equation 15 is to make the filter function very narrow (allowance of only a small range of wavelengths to be transmitted). In this very special case, where only a narrow range of wavelength is transmitted at some wavelength  $\lambda_a$  through filter  $F$ , Equation 17 reduces to

$$D_f = \log_{10} \frac{P_o(\lambda_a) \cdot F(\lambda_a) \cdot S(\lambda_a)}{P_o(\lambda_a) \cdot F(\lambda_a) \cdot S(\lambda_a) \cdot 10^{-K_y Y(\lambda_a) - K_m M(\lambda_a) - K_c C(\lambda_a) - DB(\lambda_a)}} \\ = \log_{10} \left\{ \frac{1}{10^{-K_y Y(\lambda_a) - K_m M(\lambda_a) - K_c C(\lambda_a) - DB(\lambda_a)}} \right\} \quad (18)$$

$$D_f = K_y Y(\lambda_a) + K_m M(\lambda_a) + K_c C(\lambda_a) + DB(\lambda_a) \quad (19)$$

Now Equation 19 is in the same form as Equation 15 and if three different narrow band filters are used, the  $K_y$ ,  $K_m$ , and  $K_c$  can be derived by substitution into Equation 16. It should be noted that the nine constants in Equation 16 will depend not only on the spectral properties of the dyes in the film, but also on the choice of wavelengths for measurements. This is especially true for broad band densitometers fitted with a variety of narrow band transmission filters. There will be a different set of nine constants for Equation 16 for each densitometer and set of three filters.

#### SPECTRAL PROPERTIES OF THE DYES

To solve Equation 16 for the values of  $K_y$ ,  $K_m$ , and  $K_c$ , a knowledge of spectral densities of the three dyes at three different wavelengths is necessary. Representative curves such as Figure 5 are published by the film manufacturers. Unfortunately, these are only representative curves. The actual spectral properties of the colored dyes in a particular roll of film vary from these curves at the time of manufacture and can vary again depending on time and other environmental conditions. It is impossible to predict the spectral densities of the dyes at the time density measurements are to be taken on a particular roll of film. Thus, some type of calibration must be made. Two different methods of determining the spectral densities of the dyes have been used by the photographic industry.

The oldest method involves exposing particular combinations of monochromatic light so as to force the film to form only one dye after processing<sup>16</sup>. For example, exposure of film Type 2448 to monochromatic light at 550 nm and monochromatic light at 660 nm with sufficient strength will result in only yellow dye being formed after processing. This calibration would be put on each roll of film before processing. After processing, spectral density measurements would then be made at the three wavelengths of interest for each of the dyes. These nine numbers then would be substituted into equations of the form like Equation 15. The film base/stain density would also have to be measured at the same three wavelengths. This can be accomplished by measuring the density of an overexposed portion of the film. This method of dye calibration has many drawbacks. Often it is difficult to get pure dyes on the developed imagery. It is also not clear that the dyes formed in this fashion are representative of the combination of dyes present in a scene exposed through a camera lens. The dyes formed from the calibration are due to excessive exposures of monochromatic light;



thus, the spectral properties of the dyes and/or the residual stain may be different than the dyes formed due to lower light levels. Due to these and other problems, another technique has been found to determine the spectral properties of the three dyes.

The method generally accepted by the photographic industry to determine the spectral properties of dyes in multi-layered films involves exposing a series of different colored patches on the film which adequately represent the expected range of colors for the dye images on the film. A detailed description of the method for determining the analytical spectral dye densities is beyond the scope of this paper, but can be found elsewhere<sup>8-13</sup>. A brief description will be related here.

The procedure involves taking pictures of a large number of colored patches on the film which contain many different combinations of dye concentrations. It is desirable that these patches adequately represent in color, saturation, and density the practical range of all dye combinations that the film is capable of producing. A minimum of 90 samples is recommended. The set of Munsell color chips in the rear of the *Manual of Color Aerial Photography*<sup>21</sup> is adequate.

After processing, the integral spectral densities of these patches are measured at a number of different wavelengths (17 to 32 measurements in the visible). Using a mathematical technique known as characteristic vector analysis<sup>12,13,17</sup>, values found for  $Y(\lambda)$ ,  $M(\lambda)$ , and  $C(\lambda)$  at the measurement wavelengths can be found. Characteristic vector analysis involves four steps:

- (1) The variance-covariance matrix describing the variation of the measured densities from their own mean is determined.
- (2) Characteristic vectors are determined which describe this variance. These vectors are spectral density vectors which are linearly independent.
- (3) The characteristic vectors are fitted to published dye curves in a least squares sense. These new dye vectors are called assumed dyes.
- (4) If linear combinations of the assumed dyes can adequately describe the original density measurements, the assumed dyes are deemed to be the unit spectral analytical dye densities of the three layers.

Once the vectors  $Y(\lambda)$ ,  $M(\lambda)$ , and  $C(\lambda)$  have been determined, Equation 16 can be derived for that roll of imagery and that particular densitometer.

The characteristic vector approach is the best method to determine the spectral densities of the dyes if the investigator has access to a digital computer. The accuracy of the results using the characteristic vector analysis approach is predicated on the assumption that combinations of dyes in the calibration patches are characteristic of the combinations of the dyes in the rest of the imagery. For this approach to be valid, it is crucial that the developed calibration patches be a large number of different combinations of dye densities. This can usually be accomplished by taking a picture of a card at the beginning (or end) of a roll of imagery which has a large number of different colored chips (Munsell color chips for example).

The characteristic vector approach has another real advantage. If broad band integral density measurements are taken on the calibration patches at three different wavelengths, it is possible to use Equation 16 directly. The products of the characteristic vector approach are the functions  $Y(\lambda)$ ,  $M(\lambda)$ , and  $C(\lambda)$  at the measurement wavelengths. If the density measurements are taken through broad band filters (such as described in Figure 3), the resultant functions will not actually be the unit spectral densities of the three dyes, but some numbers proportional to the dye densities. When the densities in the scene of interest are measured through the same broad band filters on the same densitometer, these proportionality constants are compensated for by the nine constants in Equation 16. Thus, from three broad band measurements the correct three dye constants or analytical densities can be derived. It should be noted again that the nine constants will depend on the instrument and choice of three broad band filters and cannot be directly transferred to another instrument and/or set of filters.

#### EQUIVALENT NEUTRAL DENSITY (END)

A great deal of research time has been invested by the photographic industry to standardize terminology concerning, and methods of determining, human perception of color. Since color is a visual characteristic, a colorimetric standard observer was agreed on by the CIE (Commission Internationale de l'éclairage). The CIE colorimetric system is based on the use of three primary colors which, when mixed together in the appropriate amounts, can match any

color perceived by the standard observer. The amounts of the three CIE primary colors which are required by the standard observer to match (so he would perceive the same color) unit energy of each wavelength are shown in Figure 6.

An END for a dye layer is defined as the visual density that would be perceived by the standard observer if the required amounts of the remaining two dyes were added or subtracted so that the color was a neutral gray. The values of  $K_y$ ,  $K_m$ , and  $K_c$  in a color film are related to the END's at that spatial location through the curves in Figure 6. A digital computer is needed to derive the relationship<sup>13</sup>.

The condition that a combination of dye concentrations in the three layers will appear neutral gray is that the tristimulus values  $X$ ,  $Y$ , and  $Z$  be some linear multiple of the tristimulus values for the source illumination.

$$\begin{aligned}\int P\bar{x}T d\lambda &= \alpha \int P\bar{x} d\lambda \\ \int P\bar{y}T d\lambda &= \alpha \int P\bar{y} d\lambda \\ \int P\bar{z}T d\lambda &= \alpha \int P\bar{z} d\lambda\end{aligned}$$

where  $P$  is the energy flux from the source illumination,  $T$  is the transmittance of the film, and  $\bar{x}$ ,  $\bar{y}$ , and  $\bar{z}$  are the tristimulus values (Figure 6). After some mathematical manipulation, the conditions for observing neutral gray are reduced to

$$\begin{aligned}\int \left[ P\bar{x} - \frac{\int P\bar{x} d\lambda}{\int P\bar{y} d\lambda} \cdot P\bar{y} \right] T d\lambda &= 0 \\ \int \left[ P\bar{z} - \frac{\int P\bar{x} d\lambda}{\int P\bar{y} d\lambda} \cdot P\bar{y} \right] T d\lambda &= 0\end{aligned}\quad (20)$$

remembering  $T$  is defined as

$$T(\lambda) = 10^{-D(\lambda)}$$

Using Equation 15,  $T(\lambda)$  is

$$T(\lambda) = 10^{-[K_y Y(\lambda) + K_m M(\lambda) + K_c C(\lambda) + B(\lambda)]}\quad (21)$$

The procedure for finding the END of the yellow layer can be sketched as follows:

- (1) holding  $K_y$  fixed, vary  $K_m$  and  $K_c$  to satisfy Equations 20 and 21, and
- (2) calculate the visual density using transmittance which satisfies the above condition.

$$D = \log_{10} \frac{\int P \cdot S d\lambda}{\int P \cdot T \cdot S d\lambda}$$

where  $S$  is the relative sensitivity of the eye<sup>18</sup> and  $T$  is the transmittance which will satisfy Equations 20 and 21.

It is usually not necessary to go through this complex procedure for every combination of  $K_y$ ,  $K_m$ , and  $K_c$  to find the corresponding ENDS. There seems to be a linear relationship between the ENDS and the analytical densities, though Saint<sup>19</sup> points out this is not necessarily so. If a linear relationship holds true, i.e.,

$$D_{\text{END}} = a_1 + a_2 D_{\text{ANALYTICAL}}$$

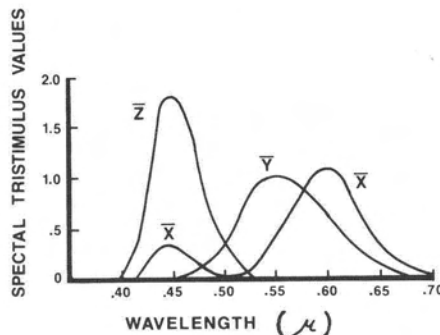


FIG. 6. Spectral tristimulus values.



Then only a small number of END calculations need be made to derive the relationship for the whole roll of imagery. The END calculations do not add any additional information to the quantitative analysis of imagery; thus, they are not recommended for use by the remote sensing community.

#### CORRELATION TO EXPOSURE

The intent of most remote sensing applications of photography is to infer some ground phenomena from a measurement (or interpretation) on the film imagery. It is beyond the scope of this paper to discuss the various techniques of correlating film imagery with ground phenomena, but it should be noted that film density is not the parameter to be correlated with the reflected energy from the ground. The dye densities formed in films depend in a non-linear way on not only the amount of energy and its spectral distribution striking the unexposed film, but also on the processing of the imagery.

Radiometric calibration procedures are discussed extensively in reference texts<sup>16,22</sup>. The intent of a radiometric calibration is to derive the relationship between film density (or dye constants) and the light impinging on the unexposed film. This relationship is called a characteristic or  $D$ -log  $E$  curve. It is called a  $D$ -log  $E$  curve because for a limited range of exposures (incident energy) there is a near linear relationship between density and the logarithm of the incident energy. An example of such a curve can be found in Figure 7. Note there is a different characteristic curve for each layer of the film.

Once a characteristic curve has been generated for a specific roll of imagery, the use of it in the analysis procedure is relatively straightforward. After the analytical densities have been found for a particular spot on the imagery, the characteristic curve is then used to find the three exposures (one for each layer) which represent the energy incident at that spot on the unexposed film. These exposures are the parameters to be correlated with the ground phenomena of interest.

The accepted procedure for radiometric calibration of black-and-white or multi-emulsion films involves exposing the unexposed film before processing to a range of known exposures. This is usually done with the aid of an instrument known as a sensitometer. A sensitometer projects spatially uniform radiance that has some known spectral distribution onto the film. The usual spectral distribution of the energy is similar to that of skylight (black body temperature of approximately 5400 °K).

Between the light source and the film some type of varying density material is inserted. This varying density material is called a *step wedge*. The picture of the step wedge on the roll of film to be calibrated is called a *film wedge*. Because differing amounts of light are transmitted through the step wedge, differing amounts of light expose the film wedge. If the density at each point on the step wedge is known, the exact exposure at each point on the film wedge can be calculated.

There are two different methods of calculating film wedge exposure values. The equivalent exposure method involves a knowledge of the film sensitivity relationship, and the traditional method does not. Each method will be described below.

Traditionally, the exposure at a point on the film wedge is represented as

$$E = t \cdot \int P(\lambda) \cdot d\lambda = \int \epsilon(\lambda) d\lambda \quad (22)$$

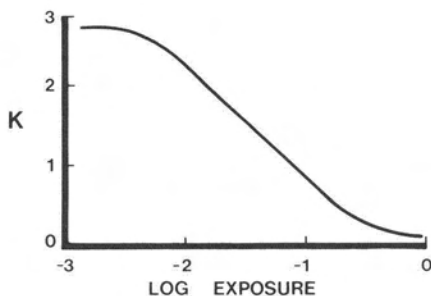


FIG. 7. Typical characteristic curve for one layer of a multi-emulsion film.

where  $E$  is the film wedge exposure,  $t$  is the exposure time, and  $\epsilon(\lambda)$  is the energy density incident on the film wedge.  $\epsilon(\lambda)$  will vary from location to location on the film wedge depending on the step wedge density values. One can represent the step wedge densities as  $n$  different functions,  $DS_n(\lambda)$ . The spectral density of a good quality step wedge does not vary with wavelength. For these types of step wedges, the densities on the wedge can be represented as  $n$  density values  $DS_n$ . The exposure on the film wedge at the corresponding locations,  $E_n$ , can then be written as

$$E_n = 10^{-DS_n} \int \epsilon_0(\lambda) d\lambda \quad (23)$$

The exposure for the light incident on the step wedge ( $E_0$ ) can be written as

$$E_0 = \int \epsilon_0(\lambda) \cdot d\lambda \quad (24)$$

Combining Equations 23 and 24,

$$E_n = 10^{-DS_n} \cdot E_0 \quad (25)$$

Thus, the exposure on the film wedge is related to the exposure without the step wedge and the exposure through the step wedge densities. These exposures would be valid for each of the layers of a multi-layered film.

The purpose of a radiometric calibration is to relate a density measurement on a film image to the amount of energy incident on the film. The analysis scheme described above derives an exposure value for a location on a photographic image which is the number of ergs/cm<sup>2</sup> with a spectral distribution similar to the energy distribution used for the sensitometric calibration. If the incident energy reflected for the scene on the film does not have a spectral distribution similar to skylight, it is not clear what the exposure values mean in an absolute sense. The exposure values have a meaning only in a relative sense (i.e., when comparisons are made between exposure values derived from incident energy spectral distributions which are similar). It is fortunate that most of the quantitative applications of photography have dealt with problems where only a relative calibration has been necessary. If comparisons among film types, between films and electro-optical sensors, or between incident energy spectral distributions are to be made, a different expression for exposure might be better. The equivalent exposure scheme derives a different expression for exposure.

Instead of trying to relate the energy density (total number of ergs/cm<sup>2</sup>) to film density, another value related to energy density is used in the equivalent exposure scheme. The intent of this scheme is to derive an expression for exposure which is as unambiguous as possible. There are many energy distributions with differing amounts of ergs/cm<sup>2</sup> which will give the same exposure value as defined in Equation 22. The equivalent exposure scheme defines exposure as the equal energy spectrum which will sensitize the film the same as the incident energy spectrum. The exposure thus defined is a weighted average energy spectra.

An equal energy spectra is an energy distribution whose intensity does not vary with wavelength, i.e., a distribution which has the same number of ergs/cm<sup>2</sup>  $\mu$  at every wavelength. An example illustrating the relationship between the film sensitivity ( $S$ ), incident radiation ( $\epsilon$ ), and equivalent exposure ( $E$ ) can be found in Figure 8. The sensitivity curve is indicative of such a curve for the yellow layer of a color film. The scale to the right of the diagram is used for both energy distribution curves.

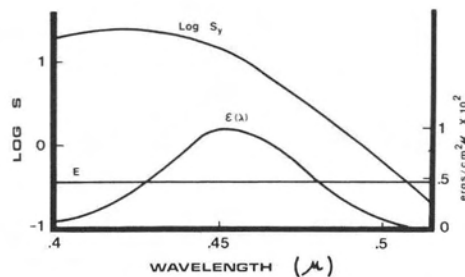


FIG. 8. A schematic showing the relationship between film sensitivity ( $s$ ), incident exposure ( $\epsilon(\lambda)$ ), and equivalent exposure ( $E$ ).

If an equivalent exposure is to have the same effect on a film layer as the real incident energy flux, then it must satisfy the following relationship:

$$\int E \cdot S(\lambda) d\lambda = \int \epsilon(\lambda) S(\lambda) d\lambda \quad (26)$$

where  $E$  is the equal energy spectra,  $\epsilon(\lambda)$  is the incident energy spectra,  $S(\lambda)$  is sensitivity of the film layer, and the integral is over all wavelengths.

Since  $E$  is a single valued number and does not depend on wavelength, Equation 28 can be rewritten as

$$E = \frac{\int \epsilon(\lambda) \cdot S(\lambda) d\lambda}{\int S(\lambda) d\lambda} \quad (27)$$

This is the equivalent exposure due to the incident energy distribution  $\epsilon(\lambda)$ . The units of  $E$  are the same as  $\epsilon(\lambda)$ . If energy flux (ergs/cm<sup>2</sup> sec  $\lambda$ ) are used for  $\epsilon(\lambda)$ , then  $E$  will have the same units. Usually, exposure time is included in the  $\epsilon(\lambda)$  and, thus, the units of  $E$  would be ergs/cm<sup>2</sup> $\mu$  (energy density).

Equation 25 is still valid if the expression for  $E_o$  is modified. In this scheme,  $E_o$  is

$$E_o = \frac{\int \epsilon_o(\lambda) \cdot S(\lambda) \cdot d\lambda}{\int S(\lambda) d\lambda} \quad (28)$$

In this analysis scheme,  $E_o$  is a function not only of the spectral distribution of incident flux but also of the film sensitivity.

In a multi-layered film such as color of color IR imagery, there will be a different exposure for each layer. The exposures on the film wedge which correspond to each step on the step wedge are given by

$$\begin{aligned} E_n^y &= 10^{-DS_n} \cdot E_o^y \\ E_n^m &= 10^{-DS_n} \cdot E_o^m \\ E_n^c &= 10^{-DS_n} \cdot E_o^c \end{aligned} \quad (29)$$

where  $E_n^y$ ,  $E_n^m$ , and  $E_n^c$  are the film wedge exposures which correspond to the  $n$ th step on the step wedge for the yellow, magenta, and cyan dye layers, respectively. The expressions for  $E_o^y$ ,  $E_o^m$ , and  $E_o^c$  can be derived from Equation 28.

$$\begin{aligned} E_o^y &= \frac{t \cdot \int P_o(\lambda) S^y(\lambda) d\lambda}{\int S^y(\lambda) d\lambda} \\ E_o^m &= \frac{t \cdot \int P_o(\lambda) S^m(\lambda) d\lambda}{\int S^m(\lambda) d\lambda} \\ E_o^c &= \frac{t \cdot \int P_o(\lambda) S^c(\lambda) d\lambda}{\int S^c(\lambda) d\lambda} \end{aligned} \quad (30)$$

where  $S^y(\lambda)$ ,  $S^m(\lambda)$ , and  $S^c(\lambda)$  are the sensitivities of the yellow, magenta, and cyan layers, respectively.

#### CONSTRUCTION OF THE CHARACTERISTIC CURVES

After the film wedge has been processed, it is essentially an image of the step wedge. It is a very special image of the step wedge. This is an image in which the investigator or photo-scientist knows the exposure for each of the layers for each spatial location on the film wedge. If the density at each spatial location on the film wedge is measured, a set of characteristic curves for the roll of film can be constructed.

A characteristic curve is the relationship between film density and film exposure, or more usually the relationship between density and  $\log_{10}$  of exposure. The densities to be used for the construction of the characteristic curve are the analytical densities (as defined in Equation 16) on the film wedge. The exposures to be used are the known exposures on the film wedge at the location where the density measurements are made (Equation 25 or 29). The result will be three  $K$  vs  $\log_{10} E$  curves, one for each layer.

A radiometric calibration is exposed on the film by the person or firm processing the film. The numbers usually supplied the researcher are (1) the step wedge densities and (2) the black body temperature of the lamp used to expose the film wedge. This is half the information needed to construct the  $K$ - $\log E$  curves. The values used to construct the characteristic

TABLE 1

Step Wedge								
Step $n$	$DS_n$	$D(450)$	$D(550)$	$D(650)$	$K_y$	$K_m$	$K_c$	$\log E_n$
1	0.6	0.145	0.244	0.278	0.1	0.2	0.25	-0.6
2	0.8	0.255	0.311	0.347	0.2	0.25	0.31	-0.8
3	1.2	0.883	0.575	0.848	0.78	0.4	0.78	-1.2
4	1.6	1.500	1.212	1.251	1.3	0.95	1.1	-1.6
5	2.0	2.246	2.082	1.861	1.9	1.7	1.6	-2.0
6	2.4	2.924	2.544	2.369	2.5	2.05	2.05	-2.4
7	2.8	3.633	3.207	3.903	3.1	2.6	2.5	-2.8
8	3.0	4.032	3.513	3.143	3.45	2.85	2.7	-3.0

curves are not the exposure values, but the logarithm of the exposure. Taking the logarithm of Equation 29:

$$\begin{aligned}\log_{10} E_n^y &= -DS_n + \log E_0^y \\ \log_{10} E_n^m &= -DS_n + \log E_0^m \\ \log_{10} E_n^c &= -DS_n + \log E_0^c\end{aligned}\quad (31)$$

Thus, the log of the exposure on the film wedge corresponding to a location on the step wedge with density  $DS_n$  is the negative of the step wedge density plus a constant. If the step wedge density were equal to 1, then the log of the exposure at the corresponding location on the film wedge would be  $-1$  plus  $\log_{10} E_0$  for the layer of interest. For most purposes, only the relative exposure is desired and in those cases the second term ( $\log_{10} E_0$ ) is not needed, i.e., for the above example the film wedge log exposure would be  $-1$ . Figure 7 is an example of a characteristic curve with relative log exposure plotted along the abscissa.

This procedure is best illustrated by an example. Assume the step wedge has eight steps. At corresponding locations on the film wedge, spectral densities are measured at 450 nm, 550 nm, and 650 nm. The step wedge densities are 0.6, 0.8, 1.2, 1.6, 2.0, 2.4, 2.8, and 3.0. Thus,  $DS_1 = 0.6$ ,  $DS_2 = 0.8$ ,  $DS_3 = 1.2$ , etc. The next step in the procedure would be to calculate  $K_y$ ,  $K_m$ , and  $K_c$  for each step on the film wedge from the density measurements at 450, 550, and 650 nm. This is done with the aid of Equation 16. Typical values for the  $C_y$ 's are

$$\begin{array}{lll} C_{11} = 1.011 & C_{12} = -0.151 & C_{13} = -0.0305 \\ C_{21} = -0.0782 & C_{22} = 1.032 & C_{23} = -0.1465 \\ C_{31} = -0.0121 & C_{32} = -0.130 & C_{33} = 1.020 \end{array}$$

Table 1 summarizes the density readings on the film wedge and the calculations of the  $K$  values.

The characteristic curves plotted from these data can be found in Figure 9. To find the *absolute* log exposure that corresponds to a particular analytical density, one needs only to add the proper  $\log E_0$  value to the *relative* log exposure derived from the graph. An analytical density of 1.8 for each of the layers would correspond to

$$\begin{aligned}\log E^y &= -1.95 \\ \log E^m &= -2.10 \\ \log E^c &= -2.10\end{aligned}$$

In turn this corresponds to relative exposures of

$$\begin{aligned}E^y &= 0.01122 \\ E^m &= 0.00794 \\ E^c &= 0.00794\end{aligned}$$

To find the absolute exposure, these terms must be multiplied by the corresponding  $E_0$  which would be supplied by the firm exposing the film wedge or calculated using Equation 30.

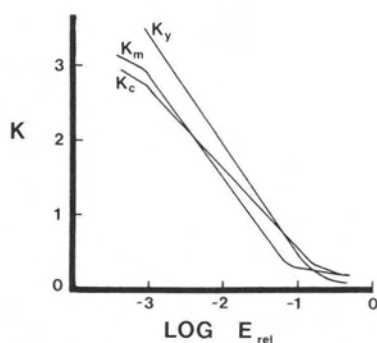


FIG. 9. Characteristic curves for the data illustrated in Table 1.

#### SUMMARY

The intent of densitometry on multi-emulsion imagery is to derive some measurement of the incident radiation exposing the film. A multitude of steps are involved in the correct transformation between density measurements to film exposure. This paper has described the important considerations a user of multi-emulsion imagery should be aware of in order to correctly apply densitometry to these types of films.

The three most important considerations are the correct use of a spectral densitometer, the proper calibration to determine the spectral dye densities, and the correct use of a radiometric calibration. The first and third considerations are discussed in some detail within the paper. The calibration for the spectral dye densities is discussed elsewhere<sup>13</sup>. Quantitative analysis of multi-emulsion imagery is not always an easy task but, if the proper procedures are followed, reliable data will result.

#### REFERENCES

1. Smedes, H. W., *et al.*, 1971. "Digital Computer Mapping of Terrain by Clustering Techniques Using Color Film as a Three-Band Sensor." *7th Symposium on Remote Sensing of Environment*. Vol. III, pp. 2057-2072.
2. LeSchack, L., 1971. "ADP of Forest Imagery." *Photogrammetric Engineering*, 37(8):885-898.
3. Hoffer, R., *et al.*, 1971. "ADP of Multiband and Multi-Emulsion Digitized Photos." *Photogrammetric Engineering*, 38(10):989-1000.
4. Coggeshall, M. E., R. Hoffer, and J. S. Berkebile, 1974. *A Comparison Between Digitized Color and Infrared Photography and Multispectral Scanner Data Using ADP Techniques*. LARS Information Note 033174, Purdue University.
5. Buckner, J., and R. Maciel, 1969. *Three-Band Source Spectra from Color Film* EGG Technical Report No. B-3968, EGG-1183-484.
6. Scarpace, F. L., and A. W. Voss, 1974. "Analysis of Multi-Layered Film." *Proceedings*, ASP-ACSM Convention, March.
7. Lillesand, T., F. L. Scarpace, and J. L. Clapp, 1975. "Water Quality in Mixing Zones." *Photogrammetric Engineering and Remote Sensing*, XLI(3), March.
8. Brewer, W. L., and F. C. Williams, 1954. "An Objective Method for Determination of Equivalent Neutral Densities of Color Film Images. I. Definitions and Basic Concepts." *J. Optical Soc. Am.*, 44(6), June.
9. Morris, R. H., and J. H. Morrissey, 1954. "An Objective Method for Determination of Equivalent Neutral Densities of Color Film Images. II. Determination of Primary Equivalent Neutral Densities." *J. Optical Soc. Am.*, 44(6), July.
10. Holland, F. H., *et al.*, 1954. "An Objective Method for Determination of Equivalent Neutral Densities of Color Film Images. III. Densitometer Calibrations." *J. Optical Soc. Am.*, 44(6), July.
11. Simonds, J. L., 1963. "Application of Characteristic Vector Analysis to Photographic and Optical Response Data." *J. Optical Soc. Am.*, 53(8), August.
12. Friederichs, G. A., 1976. *Determination of the Spectral Dye Density Distributions of Color Films*, M.S. Thesis (unpublished), University of Wisconsin-Madison.
13. Friederichs, G. A., and F. L. Scarpace, 1977. "A Method of Determining the Spectral Dye Densities in Color Films." *Proceedings*, Spring ASP-ACSM Convention.
14. Sant, A. J., 1970. "Procedures for Equivalent Neutral Density (END) Calibration of Color Densitometers Using a Digital Computer." *Photographic Science and Engineering*, 44(5), September.

15. Jackson, J. E., and R. H. Morris, 1975. "An Application of Multivariate Quality Control to Photographic Processing." *American Statistical Association Journal*, June.
16. Evans, R., W. Hanson, and W. Brewer, 1953. *Principals of Color Photography*. John Wiley and Sons, Inc., New York.
17. Overhage, C. (Ed), 1950. "Principals of Color Sensitometry." *J. SMPTE*, 54, June.
18. Stimson, A., 1974. *Photometry and Radiometry for Engineers*. John Wiley and Sons, Inc., New York.
19. *Manual of Remote Sensing*, 1975. American Society of Photogrammetry.
20. Evans, R. M., 1948. *An Introduction to Color*. J. Wiley and Sons, Inc., New York.
21. *Manual of Color Aerial Photography*, 1968. American Society of Photogrammetry.
22. *SPSE Handbook of Photographic Science*, 1973. J. Wiley and Sons, Inc., New York.

(Received July 7, 1977; revised and accepted June 19, 1978)

## BOOK REVIEW

*Remote Sensing: A Handbook for Archeologists and Cultural Resource Managers*, by T. R. Lyons and T. E. Avery. Cultural Resources Management Division, National Park Service, U.S. Department of Interior, Washington, D.C. 109 pages, 54 figures, 8 color plates, 9 tables, soft cover, 1977.

**T**HIS HANDBOOK is a primer in remote sensing for the cultural resource manager. Within the next five years the National Park Service intends to publish several supplements, which will cover, in detail, many of the basic principles outlined in the handbook and will assist the scientist and manager to maintain a state-of-the-art acquaintance with remote sensing technology.

The ten sections of the handbook describe basic characteristics and geometry of aerial photographs (Sections 1-4); some selected mapping techniques (Section 5); utilization of some non-photographic remotely sensed data (Section 6); interpretation techniques, with emphasis on archeologic subjects in the southwestern U.S. (Sections 7 and 8); photogrammetry (Section 9); and a summary, including specifications for archeological photography (Section 10). Each topic is treated in an elementary way and could be considered unsatisfactory to the serious scientist. However, as presumably intended by the authors and the Park Service, the book provides a general review of the scope, usefulness, and economies of remote sensing techniques in cultural analysis. The supplements should assist the serious scientist and the manager in more details and provide a clearer understanding of the possible ad-

vantages or disadvantages of remote sensing.

The format of the book is well conceived. The inclusion of many demonstrative figures, pictorial examples, and color plates of archeological features is most helpful and informative.

Errors and inconsistencies exist in the copy provided the reviewer. For example, on Figures 1-5 the electromagnetic spectrum is expressed in microns and on Figures 1-6 in micrometers and spelled as micrometres on page 5. The change in terminology could cause unnecessary confusion. Consistency in an elementary handbook should be maintained. Other examples of fuzzy definition include meanings of "multispectral *photography* vs multispectral *scanning*," "density slicing," "physiography," "x- and/or y-axis" on photos, "triangulation procedures," and many other terms used but not adequately defined. The supplements will correct these definitions.

The book has limited value to the remote sensing expert, is an introduction to the beginning or uninitiated scientist, and provides an overview for the manager.

—Ernest I. Rich  
Stanford University