

A Method of Determining Spectral Analytical Dye Densities

A large number of different color patches are exposed on the film.

INTRODUCTION

THE INFORMATION recorded on a multi-emulsion film (such as color or color-IR imagery) is in the form of various amounts of colored dyes. The spectral properties of these dyes are chosen in such a manner that, through the color subtractive process, almost any color visible to the human eye can be reproduced by various combinations of the three dyes. The remote sensing investigator is interested in being able to characterize the reflected energy from the ground by some measurement on a film image. This is accomplished through the use of a densitometer. The amount of dye present at each spatial location on the film is related to the amount of light incident on the film when it was exposed. To properly characterize the incident energy, a measurement of the dye content in each layer is necessary. The most exact representation of the dye content is through the measurement of the analytical densities of each of the layers for each spatial location on the imagery. The crucial data needed, to

ABSTRACT: A straightforward method for the user of color imagery to determine the spectral analytical density of dyes present in the processed imagery is presented. The method involves exposing a large number of different color patches on the film which span the gamut of the film's imaging capabilities.

From integral spectral density measurements at 16 to 19 different wavelengths, the unit spectral dye curves for each of the three dyes present were determined in two different types of color films. A discussion of the use of these spectral dye densities to determine the transformation between integral density measurements and analytical density is presented.

determine the dye content and, thus, the exposure, are the spectral densities of the dyes present in the developed imagery. This paper describes a procedure to determine these spectral dye densities.

DENSITOMETRY

Densitometry on multi-emulsion imagery has been described adequately elsewhere¹⁻³; thus, only a short review of the salient points will be presented here. Because of the colored dyes present in the imagery, any density measurement on a multi-emulsion imagery is wavelength dependent. Spectral density is a measure of density at a particular wavelength. Any density measurement on a multi-emulsion film is an integral density measurement. It is an integral density measurement because the measured density depends on the individual dye densities of each of the film layers. The integral spectral density ($D(\lambda)$) for a multi-

emulsion image can be expressed mathematically in terms of the individual yellow (Y), magenta (M), cyan (C), and base (B) densities as

$$D(\lambda) = DY(\lambda) + DM(\lambda) + DC(\lambda) + DB(\lambda) \quad (1)$$

where λ is the wavelength of the density measurement.

The densities on the right hand side of Equation 1 are known as spectral analytical densities and can be written

$$\begin{aligned} DY(\lambda) &= K_y \cdot Y(\lambda) \\ DM(\lambda) &= K_m \cdot M(\lambda) \\ DC(\lambda) &= K_c \cdot C(\lambda) \end{aligned} \quad (2)$$

where $Y(\lambda)$, $M(\lambda)$, and $C(\lambda)$ are the unit spectral dye densities for the yellow, magenta, and cyan dyes respectively. K_y , K_m , and K_c are the parameters which characterize the amount of dye present in the image.

If integral density measurements are taken at λ_1 , λ_2 , and λ_3 (three different wavelengths), three equations for the dye parameters (K) can be generated:

$$\begin{aligned} K_y &= C_{11}\Delta D(\lambda_1) + C_{12}\Delta D(\lambda_2) + C_{13}\Delta D(\lambda_3) \\ K_m &= C_{21}\Delta D(\lambda_1) + C_{22}\Delta D(\lambda_2) + C_{23}\Delta D(\lambda_3) \\ K_c &= C_{31}\Delta D(\lambda_1) + C_{32}\Delta D(\lambda_2) + C_{33}\Delta D(\lambda_3) \end{aligned} \quad (3)$$

where ΔD is the difference between the measured density and base density at that wavelength. Thus, the parameters K_y , K_m , and K_c can be determined from three integral spectral density measurements if the nine transformation constants (C_{ij} 's) are known. These nine constants are dependent only on the characteristics of the dyes in the film and not on the specific combination of dyes which represent a scene on the imagery. These nine numbers can be derived from a knowledge of the unit spectral dye densities. The only real problem is knowing the functions $Y(\lambda)$, $M(\lambda)$, and $C(\lambda)$, the unit spectral dye densities.

SPECTRAL DYE DENSITY

Methods of determining spectral dye densities in multi-emulsion films have been pioneered by scientists and engineers at Eastman Kodak^{4,5,6,7}. Curves are published by Eastman Kodak (and other film manufacturers) such as Figures 1 and 2 for each of the films produced by the company. If these curves really represented the spectral dye densities in the imagery used by the remote sensing investigator, no other work need be done. Unfortunately, this is not the case. These curves published by the film manufacturer are only representative spectral dye density curves and are not likely to be the actual dye density curves for the roll of film of interest. Small changes in the manufacturing process and environmental conditions produce small but significant changes in the spectral properties of the colored dyes in multi-emulsion films. Thus, if an investigator wishes to measure analytical densities, the spectral dye densities must first be determined for each processed roll of imagery.

The sequence of steps necessary to determine the spectral dye densities has been described by Brewer and Williams⁴. First, a large number of different color patches are exposed on the film. The spectral densities of the patches are then measured using a spectral

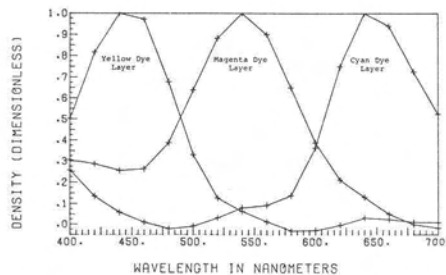


FIG. 1. Unit normalized published spectral dye densities for Kodachrome 64 color reversal film.

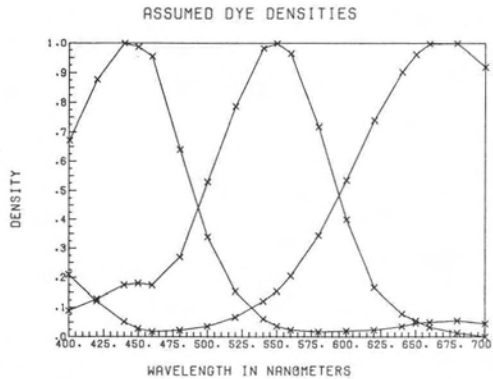


FIG. 2. Unit normalized published spectral dye densities for Kodak Aerochrome MS film 2448, Estar base.

densitometer. The densities are then analyzed using a characteristic vector approach to find the three linearly independent density vectors⁸.

A key to the success of this calibration technique lies in the exposing of color samples onto the film which adequately represents the color, value, and saturation possible on the film. We have chosen to use the Munsell color chips available commercially and, in a limited sample, a pocket in the *Manual of Color Aerial Photography*. Two hundred and forty-six chips were mounted on a card along with a number of gray samples. The photographed card represented the requisite number of color patches.

The types of color films investigated were Kodachrome 64 and Kodak Aerochrome MS film 2448, Estar base. The Kodachrome 64 was exposed using a 35 mm SLR camera. The Type 2448 was exposed using a 70 mm Hasselblad camera. This latter film would be of the most interest to the remote sensing community, though the techniques used to determine the spectral analytical dye densities are identical for all color films. The only difference in the analysis of the two films in this study was that three extra spectral density measurements were made on the Type 2448 patches.

DATA COLLECTION

The spectral density distribution of each color and gray patch was measured on a Gamma Scientific Microdensitometer. This densitometer is equipped with a monochromator so that densities at different wavelengths can be measured with a 0.5 nm band pass. The integral spectral density of a large number (330 for Kodachrome 64 and 425 for Aerochrome 2448) of the color patches on the developed film were measured every 20 nm between 400 and 700 nm. These data were coded and stored on a digital computer for further analysis. Additional density measurements at 450, 550, and 650 nm were made on the Aerochrome 2448.

CHARACTERISTIC VECTOR ANALYSIS

A mathematical analysis technique called characteristic vector analysis or eigenvector analysis has been developed and its application to optical response property determination has been described by J. L. Simonds⁸. This technique involves examining a number of sets of multivariate data and determining linear transformations of these data to a smaller number of parameters which contain essentially all the information contained in the original data set.

Characteristic vector analysis is easily adaptable to the determination of unit spectral dye densities. In this case, the multivariate data are the spectral dye density measurements at the various wavelengths. From these measurements, characteristic vectors are derived which describe the film density variation. The mathematical details on this transformation can be found in the Appendix. A relationship exists between the characteristic vectors and the spectral dye densities of the individual layers.

The characteristic vectors were then fit, in a least squares sense, to estimated unit dye densities. The published curves by Kodak for the two films were used as estimated dye densities.

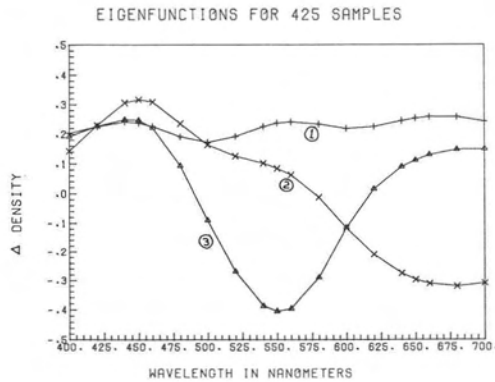


FIG. 3. The three most important characteristic vectors found by analyzing the entire data for Aerochrome 2448. These vectors account for 99.94 percent of the variance in the entire data set.

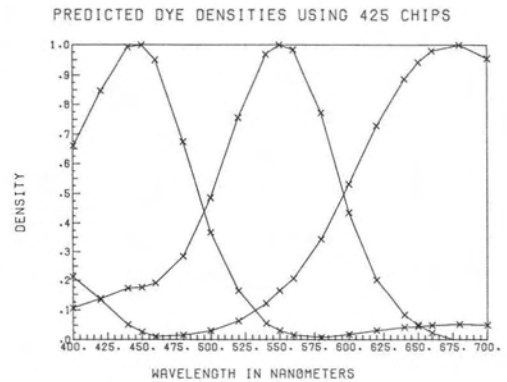


FIG. 4. The determined unit spectral dye densities using the entire data set for Aerochrome 2448.

RESULTS

The first run with each of the two data sets included all the sample color patches measured, 330 for the Kodachrome 64 and 425 for the Aerochrome 2448. From these data sets the variance-covariance matrix and characteristic values and vectors were calculated. Listed in Table 1 are the characteristic values for the first four vectors for each data set. Also listed are the percentages of the data set variances accounted for by each vector and the percentage of the data set variances which can be accounted for by linear combinations of the first n vectors. These values were calculated in accordance with Equations 9, 10, and 11 in the Appendix.

It can be seen that almost all (99.9 percent) of the variation in the data can be explained by the first three vectors. This implies that there are no other significant absorbers in the films other than the three dye layers. The three most important characteristic vectors (or eigenfunctions) are graphed in Figure 3 for film type 2448.

The characteristic vectors and the mean vector for each data set were fitted in a least squares sense to the estimated spectral dyes published by Eastman Kodak. The resultant spectral dye vectors were normalized to unity at the wavelength of peak absorption of each dye. These are the unit spectral dye densities. The determined unit spectral dye densities found by this method are plotted in Figure 4 for Aerochrome 2448 and in Figure 5 for Kodachrome 64. In addition to using the mean vector and the three most important characteristic vectors to determine the unit spectral dye density, the process was repeated for combinations of the mean vector and 4 through 16 (or 19) vectors, always taking the vectors in descending order of importance.

The integral spectral densities measured for each of the colored chips were then reconstructed from the derived unit spectral dye densities. In all cases the best fit of experimental data was found using the unit spectral dye densities derived using the mean vector and the first three characteristic vectors. This is illustrated in Table 2 for Kodachrome 64. The

TABLE 1. CHARACTERISTIC VALUES DETERMINED FROM USING ALL THE SAMPLES IN EACH OF THE DATA SETS

Vector Number	Kodachrome 64			Aerochrome 2448		
	Characteristic Value	Percent of Variance	Percent	Characteristic Value	Percent of Variance	Percent
1	12.241	91.72	91.72	8.056	89.64	89.64
2	0.877	6.56	98.29	0.723	8.03	97.67
3	0.221	1.65	99.94	0.203	2.26	99.93
4	0.005	0.04	99.98	0.004	0.05	99.98

TABLE 2. SUMMARY OF THE STANDARD DEVIATION FOR THE FITS OF THE MEAN AND 3 THROUGH 16 CHARACTERISTIC VECTORS FOR KODACHROME 64. THESE VECTORS WERE DETERMINED FROM ANALYSIS OF 330 COLOR SAMPLES.

No. of Vectors	Average Standard Deviation/Sample	Greatest One Sample Standard Deviation
3	0.09	0.29
4	0.09	0.30
5	0.10	0.31
6	0.10	0.31
7	0.10	0.31
8	0.10	0.31
9	0.10	0.31
10	0.10	0.31
11	0.10	0.31
12	0.10	0.32
13	0.11	0.34
14	0.11	0.34
15	0.11	0.33
16	0.12	0.36

average standard deviation for this fit considering all the colored chips was $\pm 0.089D$ for Kodachrome 64 and $\pm 0.022D$ for Aerochrome 2448.

The minimum necessary number of color patches required to give satisfactory estimates of the unit spectral dye densities was also investigated. The size of each data set was reduced by selecting the data from the color patches in a periodic manner. The unit spectral dye densities were then determined for these reduced data sets. Using these unit spectral dye densities, the integral spectral densities of the entire data sets were reconstructed. Tables 3 and 4 summarize the results of this analysis.

ANALYTICAL DENSITY TRANSFORMATION

The transformation between integral density and analytical density is described by Equation 3. The nine C constants are dependent on the actual unit spectral dye densities in the roll of film being analyzed. The C 's also depend on choice of three wavelengths used for the analysis. The nine C 's can be thought of as a matrix, and Equation 3 can be rewritten as

$$K = \tilde{C} \cdot \Delta D \tag{4}$$

The C -matrix can be calculated by taking the inverse of the matrix made up of the values of the derived unit spectral dye densities at the three desired wavelengths.

$$\tilde{C} = \begin{bmatrix} Y(\lambda_1) & M(\lambda_2) & C(\lambda_1) \\ Y(\lambda_2) & M(\lambda_2) & C(\lambda_2) \\ Y(\lambda_3) & M(\lambda_3) & C(\lambda_3) \end{bmatrix}^{-1} \tag{5}$$

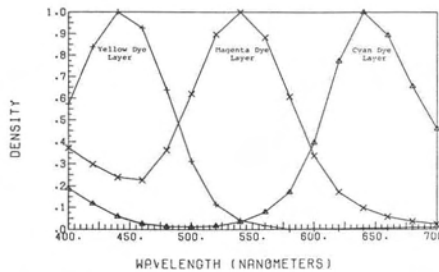


FIG. 5. The determined unit spectral dye densities using the entire data set for Kodachrome 64.

TABLE 3. COMPARISON OF THE STANDARD DEVIATIONS FOUND BY FITTING THE ANALYTICAL DYE DENSITIES DETERMINED FROM SMALLER DATA SETS TO THE MEASURED INTEGRAL DENSITIES OF 330 COLOR SAMPLES ON KODACHROME 64.

Number of Patches Used to Determine the Spectral Deviations	Average Standard Deviation/Sample
330	0.090
163	0.086
122	0.088
102	0.089
90	0.089

where $Y(\lambda)$, $M(\lambda)$, and $C(\lambda)$ are the derived unit spectral dye densities using the techniques described in this report.

An example of this type of calculation for the Aerochrome 2448 using measurement wavelengths of 450 nm, 550 nm, and 650 nm is

$$C = \begin{bmatrix} 1.000 & 0.176 & 0.027 \\ 0.031 & 1.000 & 0.164 \\ 0.046 & 0.051 & 0.943 \end{bmatrix}^{-1} = \begin{bmatrix} 1.005 & -0.177 & 0.002 \\ -0.024 & 1.013 & -0.175 \\ -0.047 & -0.047 & 1.070 \end{bmatrix}$$

Examples of using this C-matrix to transform integral density measurements into dye parameters K can be found in Table 5.

DISCUSSION AND CONCLUSIONS

It is the intent of most quantitative applications of densitometry to correlate a measurement on a film to some ground phenomena. Film exposure is the quantity which usually correlates with the amount of light reflected from the ground. It is beyond the scope of this paper to discuss the derivation of exposure from a number of density measurements, but it should be noted that the best derivation of film exposures is arrived at by using analytical densities². Thus, a crucial step in finding film exposure on a multi-emulsion film is a knowledge of the spectral densities of the dyes present in the film. The techniques described in this report are a reasonable method of determining these spectral dye densities in a color film.

Operationally, this type of calibration involves taking a picture of a card with a number of mounted color chips. The card used at the University of Wisconsin consists of 380 color chips (246 Munsell chips and 134 gray chips). This exposure could be taken at the beginning or tail of each roll of mission imagery. After processing, the spectral dye densities can be determined and thus the transformation parameters (Equation 3) can be found to calculate the analytical densities on a given roll of imagery. The actual number of color chips analyzed to derive the spectral dye densities does not seem to be critical. Eighty to ninety different color chips might be a reasonable number to use in this color calibration. This should be the first step in the analysis of color imagery. We believe this is a useful calibra-

TABLE 4. COMPARISON OF THE STANDARD DEVIATIONS FOUND BY FITTING THE ANALYTICAL DYE DENSITIES DETERMINED FROM SMALLER DATA SETS TO THE MEASURED INTEGRAL DENSITIES OF 425 COLOR SAMPLES ON AEROCHROME 2448.

Number of Patches Used to Determine the Spectral Densities	Average Standard Deviations/Sample
425	0.022
212	0.021
141	0.020
106	0.020
85	0.021
70	0.020
60	0.019
53	0.020

TABLE 5. MEASURED DENSITY AND ANALYTICAL DENSITY FOR AEROCROME 2448.

$\Delta D(450)$	$\Delta D(550)$	$\Delta D(650)$	K_u	K_m	K_c
0.5	0.5	0.5	0.42	0.41	0.49
0.5	1.0	2.0	0.33	0.56	2.07
1.0	0.50	0.5	0.15	1.93	0.42
1.0	0.50	0.5	0.92	0.40	0.46
1.0	1.5	2.0	0.74	1.15	2.02
1.5	0.5	2.0	1.42	0.12	2.05
1.5	2.0	0.5	1.16	1.90	0.37
2.0	0.5	1.0	1.92	0.28	0.95
2.0	1.0	1.0	1.84	0.79	0.93
2.0	2.0	2.0	1.66	1.63	1.95

tion technique to be used in conjunction with the normal radiometric calibration for multi-emulsion imagery.

ACKNOWLEDGMENTS

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APPENDIX

CHARACTERISTIC VECTOR ANALYSIS

The characteristic vector analysis assumes that there are a large number, m , of samples to be measured and each sample is measured in n levels (wavelengths). The measurements are the integral spectral densities for each of the color patches. An $m \times n$ matrix called a data matrix, \mathbf{D} , can be constructed from the densitometric data. Each row of the matrix corresponds to n spectral density measurements for each sample patch.

The mean vector can be calculated by taking columnwise averages of \mathbf{D} . The formula for the vector, \mathbf{Q} , is

$$q_j = \frac{1}{m} \sum_{i=1}^m d_{ij} \quad j = 1, 2, \dots, n \quad (1)$$

where q_j = the j th element of the $1 \times n$ row vector, \mathbf{Q} , and d_{ij} = the j th element of the i th row of the $m \times n$ data matrix, \mathbf{D} .

The mean vector is then subtracted from each row of the data matrix to obtain a mean cor-

rected data matrix, \mathbf{D} . This matrix contains information describing the departures from the mean of each data sample. Mathematically

$$\delta_{ij} = d_{ij} - q_j \quad i = 1, 2, \dots, m \quad j = 1, 2, \dots, n. \quad (2)$$

where δ_{ij} = the j th element of the i th row of the mean corrected data matrix, $\hat{\mathbf{D}}$.

A variance-covariance matrix, \mathbf{S} , can be calculated which describes the variation of the data about the mean and the correlation of the data levels with each other. If $m > n$, then pre-multiplying the mean corrected data matrix by its transpose will result in an $n \times n$ dimensional matrix which is the variance-covariance matrix, \mathbf{S} . This may be written as

$$S_{jk} = \frac{1}{m-1} \sum_{i=1}^m \delta_{ij} \cdot \delta_{ik} \quad j = 1, 2, \dots, n; \quad (3)$$

$$k = 1, 2, \dots, n.$$

where S_{jk} = the k th element of the j th row of the $n \times n$ dimensional variance-covariance matrix, \mathbf{S} .

The characteristic values (eigenvalues) are calculated from the determinant equation,

$$\det |\mathbf{S} - \lambda_i \mathbf{I}| = 0, \quad i = 1, 2, \dots, n. \quad (4)$$

where \mathbf{L} = $n \times n$ dimensional, diagonal matrix containing the characteristic values,
 \mathbf{I} = the $n \times n$ dimensional identity matrix, and
 λ_{ii} = the i th characteristic value, an element of the main diagonal of \mathbf{L} .

A characteristic vector corresponds to each characteristic value. The equation is

$$\mathbf{V}\mathbf{S} = \mathbf{L}\mathbf{V} \quad (5)$$

where \mathbf{V} = the $n \times n$ matrix of the characteristic vectors. Each row of this matrix is one distinct characteristic vector.

This equation can be written for each element of \mathbf{V} as

$$\sum_{i=1}^n v_{ki} S_{ij} = \lambda_{kk} v_{kj} \quad k = 1, 2, \dots, n; \quad (6)$$

$$j = 1, 2, \dots, n$$

where v_{ki} = an element of \mathbf{V} , and
 λ_{kk} = an element of the main diagonal of \mathbf{L} .

These characteristic vectors can then be normalized so that their length is equal to unity. The vectors so normalized are orthonormal basis vectors of the data set. They have the following properties:

- (1) The vectors are orthogonal, that is, they represent statistically independent forms of variation. The test for orthogonality requires that the following relationship be true:

$$\sum_{i=1}^n v_{ki} v_{ji} = 0 \text{ for } k \neq j \quad k = 1, 2, \dots, n; \quad (7)$$

$$j = 1, 2, \dots, n$$

- (2) A vector multiplied by itself is equal to unity. Mathematically stated,

$$\sum_{i=1}^n v_{ki} v_{ki} = 1 \quad k = 1, 2, \dots, n. \quad (8)$$

- (3) The percentage of the variation in the data set accounted for by each vector can be expressed as the ratio of its associated characteristic value to the trace of the covariance matrix. It is a mathematical theorem that the sum of all the characteristic values must equal a quantity called the trace of the covariance matrix. The trace can be written as

$$T = T_r(s) = \sum_{i=1}^n S_{ii} = \sum_{i=1}^n \lambda_{ii} \quad (9)$$

The proportion of the variability of the data accounted for by the i th characteristic vector is

$$P_i = \lambda_{ii}/T \quad (10)$$

From Equation 9 it is determined that

$$\sum_{i=1}^n P_i = \sum_{i=1}^n \lambda_{ii}/T = 1 \quad (11)$$

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Handbook of Remote Sensing Imagery of Connecticut Available

The *Handbook of Remote Sensing Imagery of Connecticut*, by Daniel L. Civco, William C. Kennard, and Michael Wm. Lefor, represents a comprehensive inventory of existing aerial photography and satellite imagery of the state.

The *Handbook* documents information on the types, coverage, and availability of remote sensing imagery acquired by various agencies at the federal, state, regional, and local levels. Coverage is illustrated on maps of the state at the approximate scale of 1:600,000. Other figures and tables are presented to supplement these coverage maps. When available, price and ordering information for the remote sensing imagery products is given.

The 98-page *Handbook*, published as Storrs Agricultural Experiment Station Bulletin Number 448, is available for a charge of \$2.00 (includes tax and postage) from:

Agricultural Publications Office, U-35
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The University of Connecticut
Storrs, CT 06268

Errata

Several errors have been reported for the July 1978 Yearbook issue of *Photogrammetric Engineering and Remote Sensing*.

On page 844, the second sentence in the last paragraph of the description of Geosurvey International should read "This inhouse computer has 512k bytes of C.P.U. core with 80 megabytes disc for data storage."

On page 896, the figure caption for the Autometric Award should read "Stanley C. Freden receives the Autometric Award on behalf of his co-authors Messrs Short, Lowman, and Finch from Clifford Greve representing the donor, Autometric, Inc."