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An Application of Photogrammetric Techniques to Building Construction*

Surveying procedures are employed to acquire the measurements and photogrammetric techniques are used to reduce the data.

Introduction

In the construction of precast slab buildings, a core tower is usually built first and the ground-fabricated floor slabs are then jacked up and pinned to the core at the inserts. The positions of these inserts have to be located very accurately so that floor loads are transmitted to the core at the correct design positions. The problem of locating the three-dimensional positions of the inserts within an accuracy of ± 5 mm is a typical problem in close-range photogrammetry, where two overlapping photographs are taken by using a metric camera. The conventional close-range photogrammetry technique is not used here for two reasons:

- The scale of the photograph, which shows the entire face of the core (about 90 metres in height) on the image format, is very small. Accordingly, one cannot reach the required accuracy by using such photographs.
- The instruments which are required for photography and for measuring the image coordinates or for plotting are very expensive.

Abstract: *Precast slab buildings have been used quite often in many areas. A core of the construction is first built and then the slabs of all floors are fabricated on the ground. These slabs are lifted and pinned on the core at certain positions called inserts. The positions of those inserts at the four sides are very important in the design. The distribution of the slab weight on the core depends mainly on the position of the inserts. During construction of the core, a very accurate instrument is used to insure these positions. However, after concrete settlement and creep the inserts positions on the core change. The three-dimensional position of the inserts can be determined by using close-range photogrammetry techniques. Knowing the three-dimensional coordinates of the inserts, the designer can judge whether it is safe to pin the slab on those inserts or if their positions have to be adjusted. This paper describes the method as applied to a 30-story precast slab building.*

The technique which is used in this article is an extension of the one developed by Abdel-Aziz and Karara (1974) in constructing the three-dimensional test area at the University of Illinois. In this approach the mathematical photogrammetric models (such as relative and absolute orientations) are applied to theodolite images which are formed analytically from theodolite measurements. The theodolite measurements are the horizontal and the vertical

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directions of each insert taken from two theodolite stations.

The main advantage of this approach is that only a theodolite is required for measurements and a calculator for data reduction. Moreover, the resulting accuracy of the inserts' coordinates is better than that obtained by using real photographs.

A complete description of the instruments used, the required measurements, and the data reduction is given below.

Instruments

Only three instruments are required to perform the measurements and the calculations of the inserts' positions. These instruments are

- A one-second theodolite,
- A line object of known length, and
- A calculator

These instruments are available in most construction firms.

Measurements

The measurements are taken, for each face of the building core, from two theodolite stations which are on a line parallel to the given axis at each face. At each theodolite station, one has to perform these sets of measurements:

- The vertical and the horizontal directions to each insert,
- The horizontal direction to the other theodolite station, and
- The vertical and the horizontal directions to the two ends of the known line object.

It is apparent that the vertical and horizontal directions are the only required measurements. In this way linear measurements, which have lower accuracy, have been avoided. Also, the vertical directions are measured according to the procedure given in Abdel-Aziz and Karara (1974) in which the theodolite spirit level screw of the vertical circle need not be adjusted. As a result, the vertical directions are measured without adjusting the spirit level of the vertical circle. Accordingly, the plane of the horizontal directions, which is the plane of the horizontal circle, is the zero reference of the vertical directions. This approach saves the time required for adjusting the spirit level of the vertical circle as well as increasing the accuracy of the inserts coordinates. The layout of the theodolite stations and the core is given in Figure 1.

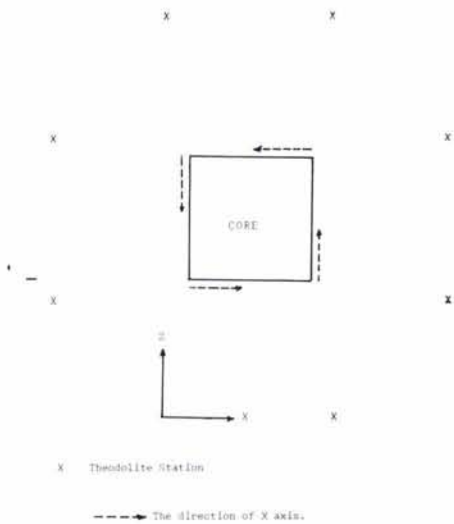

Fig. 1. Geometric layout of the core and the theodolite stations.

Data Reduction

Thus far, one has the vertical and the horizontal directions to all the inserts on each face of the core which are measured from two theodolite stations. These theodolite measurements are used to calculate the inserts coordinates X, Y, and Z. These coordinates are referred to a known system of axes in the field with an origin as the position of the first insert on that face. The X axis is a horizontal line defined on each face of the core. The Z direction is a vertical line (plumb bob direction). The procedure for computing the insert coordinates X, Y, and Z from the theodolite measurements is carried out as follows:

- Calculate the theodolite image coordinates for the theodolite measurements at each station,
- Perform the relative orientation of one of theodolite images of each face relative to the other one, and
- Calculate the inserts' coordinates.

Theodolite Image

A theodolite image is defined as a fictitious image which would be formed by a fictitious camera having the following orientation:

- The perspective center of the fictitious camera coincides with the point of intersection of the theodolite horizontal and vertical axes,
- The optical axis of that camera is parallel to the plane of the theodolite horizontal circle and perpendicular to the base line between the two theodolite stations, and

- The principal distance of the fictitious camera can be assumed equal to any value, C.

Knowing the horizontal and vertical directions of an insert measured from a theodolite station, one can calculate their corresponding theodolite image coordinates. The relations between the horizontal and the vertical direction α and β of an insert and the theodolite image coordinates, x and z, are given in Abdel-Aziz and Karara (1974). According to Figure 2 these relations can be written in these forms:

$$x = C \tan (90 - \alpha_0 + \alpha)$$
$$z = C \tan \beta / \cos (90 - \alpha_0 + \alpha)$$

where α_0 is the horizontal direction of the other theodolite station.

The main advantage of the theodolite images are

- No need for development and processing for theodolite image.
- No need for a comparator to measure the theodolite image coordinates, which are obtained from the measured vertical and horizontal directions.
- No limitations on the depth of focus, the format size, and the intensity of light for the theodolite image.
- Generally, the accuracy of the theodolite image coordinates is much better than photographic image coordinates. A one-second theodolite gives an accuracy in the x and z coordinates of $\pm$ 0.5 micrometres for a 100 mm principal distance.
- Four out of the five relative orientation parameters of the two theodolite images are known, having zero values.

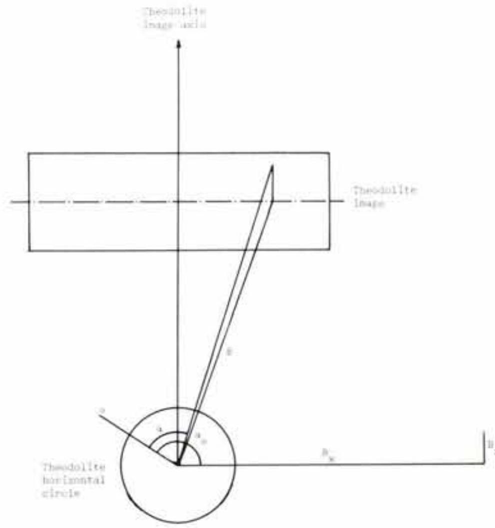

FIG. 2. The relationship between theodolite image coordinates, x and z, and the horizontal direction, α, and the vertical direction, β.

A sample of the horizontal and vertical directions taken from two theodolite stations (left and right) and their corresponding theodolite image coordinates, for a principal distance of 100 mm, is given in Tables 1 and 2.

RELATIVE ORIENTATION

The instruments used and the required measurements for calculating the coordinates of the two theodolite images have been given. Further, the orientation parameters of each theodolite image have been

TABLE 1. THEODOLITE DIRECTIONS AND THEODOLITE IMAGE COORDINATES FROM THE LEFT STATION

	Theodolite Directions						Theodolite Image	
	α			β			x	y
101	39	19	07	−00	42	07	7.2−17	−1.2284
103	43	22	03	−00	41	51	14.3434	−1.2299
105	45	57	14	−00	40	10	18.9836	−1.1893
107	48	34	00	−00	41	49	23.7527	−1.2503
109	52	19	20	−00	41	15	30.7969	−1.2558
1001	39	20	24	34	46	13	7.2295	69.6061
1003	43	20	02	34	33	16	14.2836	69.5671
1005	45	59	11	34	27	08	19.0424	69.5772
1007	48	31	11	34	07	19	23.6661	69.6320
1009	52	22	02	34	36	46	30.8829	69.5690
3001	39	22	04	64	37	06	7.2782	211.3310
3003	43	21	49	64	26	58	14.3365	211.3179
3005	45	59	32	64	15	40	19.0529	211.1556
3007	48	33	50	64	04	00	23.7475	211.3562
3009	52	21	05	64	37	45	30.8527	211.0883
α_0	125	12						

TABLE 2. THEODOLITE DIRECTIONS AND THEODOLITE IMAGE COORDINATES FROM THE RIGHT STATION

	Theodolite Directions						Theodolite Image	
	α			β			x	y
101	44	07	08	−00	24	55	29.3958	−0.7554
103	47	57	11	−00	25	25	22.2554	−0.7574
105	50	30	36	−00	25	10	17.6147	−0.7434
107	53	11	35	−00	26	32	12.8275	−0.7782
109	57	10	58	−00	25	06	5.7961	−0.7289
1001	44	08	02	33	54	58	29.3673	70.0775
1003	47	55	33	34	21	22	22.3053	70.0389
1005	50	32	16	34	36	12	17.5647	70.0550
1007	53	07	38	34	48	32	12.9394	70.1042
1009	57	13	59	34	57	49	5.7081	69.9982
3001	44	09	14	63	46	56	29.3292	211.6217
3003	47	56	58	64	11	11	22.2621	211.7955
3005	50	33	23	64	22	23	17.5312	211.6437
3007	53	10	26	64	32	35	12.8566	211.7894
3009	57	13	35	64	39	58	5.7198	211.5734
α_0	150	30	00					

discussed. Investigating the orientation parameters of the two theodolite images one finds that:

- The x axes of the two theodolite images, which are parallel to the base line, are parallel.
- The z axes of the two theodolite images, which are parallel to the theodolite vertical axes at the two theodolite stations, are parallel.
- The two theodolite images are on one vertical plane.

Accordingly, the relative orientation parameters which are required to orient the right theodolite image with respect to the left one have these values:

- The three rotation parameters around the three images axes having zero values; and
- The translation element along the Y direction, B_y, has a zero value.

The fifth relative orientation parameter, the difference of the height, B_z, between the two perspective centers of the theodolite images, can be determined either directly by using linear measurements or indirectly by using the coplanarity equation. The direct method is not used for the following reasons:

- The perspective center, which is the point of intersection of the theodolite vertical and horizontal axes, is not a well-defined point; and
- Even if it were defined, the value of B_z measured by linear measurements would not give the required accuracy.

The coplanarity equation for any two conjugate points of an insert having image coordinates, x and z, on the left image and $\bar{x}$ and $\bar{z}$ on the right image takes this form:

$$\begin{vmatrix} B_x & 0 & B_z \\ x & z & c \\ \bar{x} & \bar{z} & c \end{vmatrix}$$

The above equation takes the form

$$C\, B_x\,(z - \bar{z}) + B_z\,(x\bar{z} - z\bar{x}) = 0$$

where B_x is the horizontal projection of the distance between the two theodolite stations and is an unknown parameter.

The least squares solution of the above equation gives the value of the ratio

$$\frac{B_z}{B_x} \text{ as } \frac{B_z}{B_x} = -\,C\,\frac{\Sigma\,(z - \bar{z})}{\Sigma\,(x\bar{z} - z\bar{x})}$$

Finally, the relative orientation parameters are known and have these values:

- All the rotation parameters have zero values.
- The B_y prameters has a zero value and the B_z parameter can be calculated from the above equation for any assumed values of B_x.

The inserts' coordinates can be obtained by using these values of the relative orientation parameters and the exact value of B_x.

INSERT COORDINATES

The values of the relative orientation parameters which are required to orient the right theodolite image relative to the left one are given. Moreover, it was found that

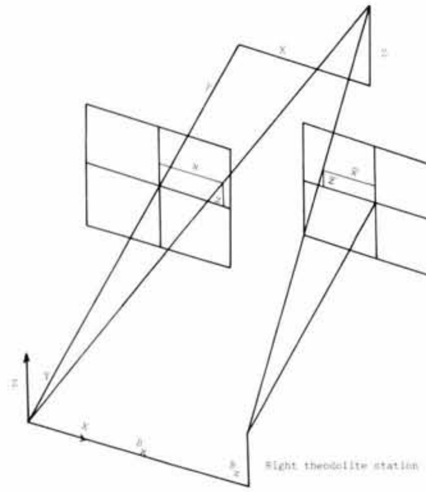

FIG. 3. The relationships between the insert coordinates, X, Y, and Z, and the theodolite image coordinates, x and z and $\bar{x}$ and $\bar{z}$.

all the relative orientation parameters except the B_z parameter have zero values. That means that the two theodolite images are on the same vertical plane and the image axes x and z of the left photo and $\bar{x}$ and $\bar{z}$ of the right photo are parallel. In such a case the relationship between the theodolite image coordinates x and z on the left photograph, $\bar{x}$ and $\bar{z}$ on the right photograph, and the inserts' coordinates X, Y, and Z according to Figure 3, are

$$X = x \frac{B_x}{x - \bar{x}}$$

$$Y = c \frac{B_x}{x - \bar{x}}$$

$$Z_1 = z \frac{B_x}{x - \bar{x}} \text{ and } Z_2 = \bar{z} \frac{B_x}{x - \bar{x}} + B_x$$

$$Z = (Z_1 + Z_2)/2$$

$$V_z = Z_1 - Z_2$$

where B_x is the horizontal distance between the two theodolite stations, which is an unknown value. The value of B_x can be obtained as follows:

- Assume any value for B_x such as $\bar{B}_x$ and obtain the ground coordinates of the two ends of the known line,
- Calculate from the ground coordinates the length of that known line, and
- The correct value B_x

$$= \bar{B}_x \frac{\text{True length of the known line}}{\text{Calculated length of the known line}}$$

The X, Y, and Z coordinates obtained by the above equations give the inserts' coordinates referred to a system of axes as given in Figure 3 with an origin at the perspective center of the left photograph, the X axis parallel to the given X axis, and the Z axis a vertical line parallel to the plump bob direction at the origin. A sample of the inserts coordinates, X, Y, and Z, and the accuracy of the measurements, V_z, which are calculated by using the given equations, are given in Table 3.

ADVANTAGE OF THE DEVELOPED TECHNIQUE

There are two main techniques which exist to perform this project. These are

- The photogrammetric technique wherein a camera is used for measurements and photogrammetric models are used for data reduction, and
- The surveying technique wherein a theodolite is used for measurements and surveying models are used for data reduction.

The developed technique is a combination of both techniques, where the measurements are taken according to the surveying technique and the photogrammetric technique is used for data reduction. A comparison between the surveying technique, the photogrammetric technique, and the developed technique is given next.

COMPARISON BETWEEN THE DEVELOPED TECHNIQUE AND THE SURVEYING TECHNIQUE

The inserts' coordinates obtained by using the developed technique and the surveying technique are functions of the theodolite directions and the estimated values

TABLE 3. COORDINATES OF INSERTS IN METRES

Inserts	Coordinates			
#	X	Y	Z	V_z
101	3.050	42.352	0.520	0.001
103	6.074	42.351	0.521	0.001
105	8.040	42.351	0.504	0.000
107	10.065	42.372	0.530	0.002
109	13.045	42.358	0.532	0.008
1001	3.062	42.353	29.481	0.001
1003	6.050	42.362	29.470	0.001
1005	8.063	42.341	29.460	0.002
1007	10.021	42.343	29.484	0.000
1009	13.082	42.360	29.655	0.005
3001	3.079	42.341	89.480	0.004
3003	6.072	42.351	89.495	0.001
3005	8.073	42.368	89.463	0.003
3007	10.056	42.345	89.499	0.005
3009	10.076	42.382	89.465	0.003

of B_z and B_x. The difference in the accuracy of the coordinates in the two techniques depends on the accuracy of the estimated values of B_z and B_x. The estimated values of B_z and B_x according to the developed technique are better than those obtained by surveying techniques for the following reasons:

- The value of B_z in the developed technique is estimated from the relative orientation of the two theodolite images. The coordinates of one insert are needed for estimating B_z and, if the core has n inserts, then n-1 extra observations exist for estimating B_z value by using a least squares solution.
- The value of B_z in the surveying technique is measured rather than computed and the measured value of B_z is of limited accuracy.
- The value of B_x is commonly measured in surveying technique by using a subtense bar. During measurements the subtense bar axis has to satisfy two geometrical conditions which are that (1) the subtense bar axis must be horizontal and (2) the middle point of the subtense bar and the theodolite station must be on a vertical plane perpendicular to the bar axis. In the developed technique there are no constraints on the substense bar axis.

As a result, one can expect that the accuracy of the inserts' coordinates by using the developed technique are better than those obtained by using the surveying technique.

COMPARISON BETWEEN THE DEVELOPED TECHNIQUE AND THE PHOTOGRAMMETRIC TECHNIQUE

The accuracy of the coordinates obtained by using the developed technique and the photogrammetric technique are functions of the accuracy of the image coordinates. A one-second theodolite gives an accuracy in the coordinates of the theodolite image of ± 0.5 micrometers for a 100 mm principal distance. This cannot be achieved by using real photographs.

As a result, the accuracy of the image coordinates in the developed technique is better than in the photogrammetric technique. Consequently, the inserts' coordinates obtained from the developed technique are more accurate than those obtained from the photogrammetric technique.

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