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Methods and Possibilities for Digital Differential Rectification*

Examples of digitally produced orthophotos are displayed, the economics of digital rectification are discussed, and a proposed design for a future system is presented.

Photogrammetry has very successfully used digital techniques for calculating geometrical relationships between measured points in order to determine orientation, to perform aerial triangulation adjustment, and to control analytical plotters or off-line optical orthophoto devices.

Digital image processing techniques permit the extension of the digital treatment of photogrammetry to differential image elements. Image processing offers special advantages outside of those already known for digital processing of point information. These advantages concern the enormous potential to stretch image contrasts contained in the emulsion, but concealed to the eye, in an optimal manner.

Within the framework of digital image processing, which is of concern to many disciplines, the topic of digital rectification is of prime concern to geometrical image processing and to photogrammetry.

At the University of Hannover our first experiences with digital rectification were made with Landsat satellite images in a simplified manner (see Figure 1).

The Landsat images are already in digital form on computer compatible tapes provided by NASA or the European receiving station at Fucino, Italy.

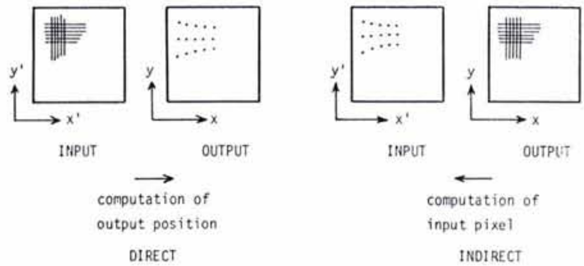
The stored image, picture element by picture element (or pixel), can be reproduced by a digital write device. Image coordinates of identifiable features whose ground coordinates are known can be measured, e.g., in a comparator. A set of polynomial coefficients a_0 to b_5 can be determined by calculation, and then an output position can be calculated for each input pixel. The disadvantage of this direct method of calculation is that from the output positions and their photographic densities a rectangular output grid must be chosen for which the densities must be interpolated in an off-line process.

KEY WORDS: **Computers; Differential equations; Digital data; Digital Mapping; Elevation; Images; Landsat; Models; Rectification; Scanners**
ABSTRACT: Experiences with digital differential image rectification at the University of Hannover are described. Digital rectification of Landsat images can be performed at marginal cost on large computers. Aerial photographs can likewise be rectified on the basis of a digital elevation model at comparable cost if a coarse resolution is chosen. Rectification to photographic resolution capability, however, requires that special scanners and pipeline processors be built. Suggestions for such devices are given.
REFERENCE: Konecny, G., "Methods and Possibilities for Digital Differential Rectification," *Photogrammetric Engineering and Remote Sensing,* Journal of the American Society of Photogrammetry, ASP, Vol. 45, No. AP6, June, 1979

A regular output grid is immediately obtained in the indirect method of digital rectification. There, after the determination of coefficients c_0 to d_5, an input pixel location may be computed for the transfer of densities.

The experiences with Landsat were also adapted to digital tape images of an aircraft multispectral scanner, the Bendix M²S. The

* Presented at the Commission III Symposium, International Society for Photogrammetry, Moscow, USSR, August 1978.



$$x = a_0 + a_1x' + a_2y' + a_3x'y' + a_4x'^2 + a_5y'^2 \qquad x' = c_0 + c_1x + c_2y + c_3xy + c_4x^2 + c_5y^2$$

$$y = b_0 + b_1x' + b_2y' + b_3x'y' + b_4x'^2 + b_5y'^2 \qquad y' = d_0 + d_1x + d_2y + d_3xy + d_4x^2 + d_5y^2$$

FIG. 1. Simplified digital rectification.

original image contains large displacements due to panoramic distortion. These distortions are removed successfully for flat terrain with a simple polynomial rectification model.

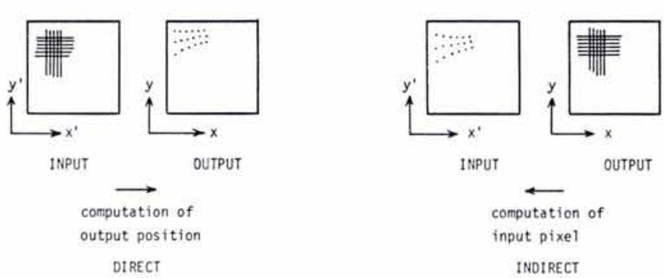
These programs were operational in Hannover in 1975. Examples are shown in Konecny and Schuhr (1975) using a line printer employing overprints characters allowing 16 grey shades. Later we improved the digital output facility using the Optronics P-1700 digital read-write device. Examples are shown in Schuhr (1976).

While Landsat images generally do not contain significant relief displacements, such displacements must be taken into account for images taken from aircraft. We have therefore modified our programs for the use of collinearity equations.

In the direct method, the height information to be utilized must be stored in a digital terrain model (DTM) based on image coordinates. Since this is usually difficult to attain, the indirect method is preferred. There the DTM can be stored as a function of a rectangular output grid (see Figure 2).

The present functioning of our programs utilizing a DTM is expressed by Figure 3.

An image in digital form, either from satellites, from aircraft scanners, or digitized



$$x = (z-z_0)\frac{a_{11}(x'-x'_c) + a_{21}(y'-y'_c) - a_{31}c}{a_{13}(x'-x'_c) + a_{23}(y'-y'_c) - a_{33}c} + x_0\ ; \quad x' = -c\,\frac{a_{11}(x-x_0) + a_{12}(y-y_0) + a_{13}(z-z_0)}{a_{31}(x-x_0) + a_{32}(y-y_0) + a_{33}(z-z_0)} + x'_c$$

$$y = (z-z_0)\frac{a_{12}(x'-x'_c) + a_{22}(y'-y'_c) - a_{32}c}{a_{13}(x'-x'_c) + a_{23}(y'-y'_c) - a_{33}c} + y_0\ ; \quad y' = -c\,\frac{a_{21}(x-x_0) + a_{22}(y-y_0) + a_{23}(z-z_0)}{a_{31}(x-x_0) + a_{32}(y-y_0) + a_{33}(z-z_0)} + y'_c$$

OFF-LINE

1) INPUT $z = f(x', y')$ from storage
for grid $d'(x', y') = d(x, y)$
2) calculation of regular grid

ON-LINE

1) & 2) $z = f(x, y)$ from storage
$d(x, y) = d(x', y')$

FIG. 2. Digital rectification using collinearity equations.

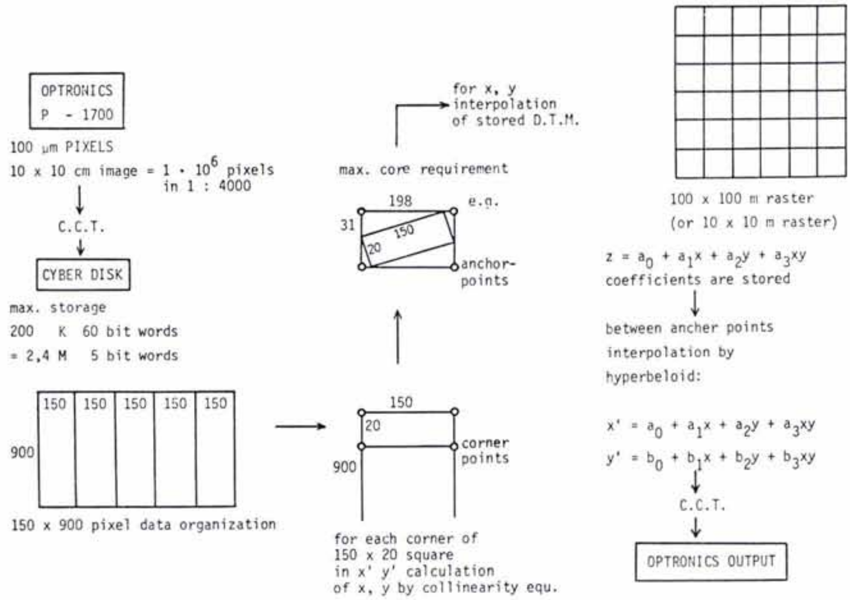


FIG. 3. Off-line digital rectification.

by means of the Optronics is transferred to the disk of the C.D.C. Cyber 73/76 computer available in Hannover. There it is stored in data blocks of 150 by 900 pixels.

In order to speed up calculation, the strict geometrical solution is applied to anchor points of blocks 150 by 20 pixels only. This serves two purposes: First, the computing speed; second, to determine the maximum storage requirement in core to be used for the indirect rectification of each image block.

The DTM is originally stored for the whole image in the form of a rectangular grid. The grid must be so dense that elevation interpolation is possible by a first order polynomial.

This interpolated height is then used for the anchor point solution; the image of the small 150 by 20 block is transferred by a polynomial indirect solution.

The resulting computer compatible tape is written out on the Optronics.

The time and cost figures for digital rectification with the different programs were as follows:

> A Landsat image containing 1.5 M pixels was rectified in 1976 with a polynomial fit, pixel by pixel in 12 minutes on the CDC Cyber 73/76 for a cost of DM 3.000,—. Today this image can be rectified in 1.8 minutes at ⅛ the cost, that is DM 456,— with a collinearity model applied to anchor points.

The technique is applicable to the rectification of any type of imagery for which an appropriate program module for the respective geometric model has been inserted.

Digital rectification is required for tape images of digital sensors such as Landsat or M^2S, but it can also be applied to photographic images if they have been digitized on the Optronics.

To test the applicability of digital rectification to aerial photography, an aerial photograph at the scale 1 : 4000 shown in Figure 4 was digitized on the Optronics at 100 μm pixel size. The scanned Optronics output, slightly deformed in affinity, is shown in Figure 5.

Known techniques of image processing, such as contrast enhancement by a local high pass filter, may be applied to the digital image or edges may be extracted. Our general image processing software contains about 70 modules, which may be applied at will for a particular purpose. These will not be described further, because they are immaterial to the rectification process.

An existing 1 : 5 000 map of the area has been used to interpolate DTM information (see Figure 6). This information is stored in a rectangular grid (see Figure 7). Based on this an orthophoto at the scale 1 : 5 000 was produced (see Figure 8). This orthophoto may also be generated with a digitally determined coordinate grid (see Figure 9) or local contrast enhancement may be utilized, before printing out the orthophoto (see

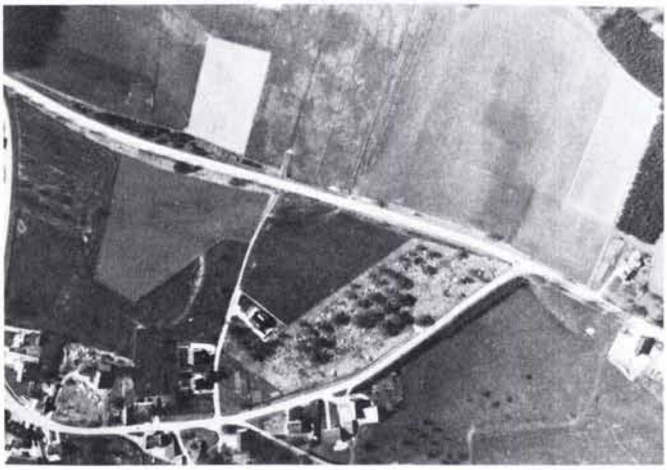


FIG. 4. Original aerial photograph.

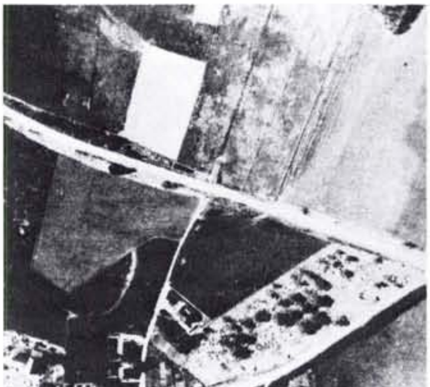


FIG. 5. Scanned image.

Figure 10). Another example is the printout of edge enhancement in orthophoto form (see Figure 11).

A geometrical test was executed for the orthophoto. The theoretical accuracy limit imposed by digitization is ± 0.3 pixels. At a number of check points the resultant RMS error was ± 1.3 pixels. By using well defined points only, it could be reduced to ± 0.8 pixels. The explanation for this gap between theoretical and practical accuracy is that, for this example, the DTM grid was taken too coarse (it is 100 m).

The foregoing examples demonstrate that it is technically feasible to generate digital orthophotos from aerial photography.

FIG. 6. Map of area for extraction of DTM.

202.0	197.0	185.0	197.0	204.0	212.0	207.5
203.2	204.0	194.0	200.0	213.0	222.0	213.0
199.0	205.0	205.6	208.0	218.0	227.0	227.0
196.0	203.0	213.0	218.5	225.2	228.0	233.5
202.0	209.0	215.5	220.8	223.5	220.0	231.0
194.0	200.0	206.0	210.2	209.0	216.0	223.5

FIG. 7. DTM grid.

The next remarks shall be made with respect to the economic feasibility of the digital rectification process. Table 1 compares the amount of image data contained in images of different resolution for the 23 by 23 cm format. The demonstrated capability for Landsat and for the 100 μm-pixel size-orthophoto contains $5 \cdot 10^7$ bits per image. It is satisfactory for printing image processed orthophotos by photolithography in an enlarged format of 40 by 40 cm.

The generation of digital orthophotos may be of particular advantage for the production of color orthophotomaps. Photolithography only requires a low resolution input of three color separations. These should, however, be image processed to allow for the proper balance of color. Such control could best be executed by digital means. Off-line image processing for photolithography is therefore a potential tool today.

If the digital rectification is to be applied to images of the highest resolution, as is required for visual analysis at high magnification, a pixel size of 12.5 μm is necessary, and the amount of data rises to $3 \cdot 10^9$ bits.

Currently there exist various economic and technical limitations for the digital processing of such images. The first limitation is the read-write speed for the input

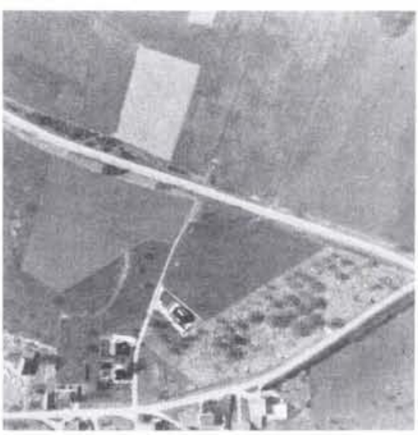


FIG. 8. Digital orthophoto.

FIG. 9. Digital orthophoto with coordinate grid.

FIG. 10. Contrast enhanced orthophoto.

FIG. 11. Edge enhanced orthophoto.

TABLE 1. IMAGE DATA.

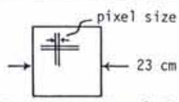


Image Type	Pixel Size (μm)	Resolution (lp/mm) = 1000 / Pixel Size $2\sqrt{2}$	Pixels/Line for 23 cm	Pixels/Image 23 × 23 cm	Bits Per Image
Photogrammetric Aerial Photo	12,5	26	18 400	$338 \cdot 10^6$	$3{,}04 \cdot 10^9$
	25	13	9 200	$84 \cdot 10^6$	$7{,}56 \cdot 10^8$
	50	7	4 600	$21 \cdot 10^6$	$1{,}89 \cdot 10^8$
LANDSAT Image	100	3,5	2 300	$5 \cdot 10^6$	$4{,}7 \cdot 10^7$
Photolithography Requirement	167	2,1	1 377	$1{,}9 \cdot 10^6$	$1{,}5 \cdot 10^7$

TABLE 2. DIGITAL READ AND WRITE LIMITATIONS

Device	Speed	Time for Scanning 23 × 23 cm Image at Resolution (Pixel Size) 12,5 μm	25 μm	50 μm	10 μm	167 μm
Slow Drum	7 KHz	$13{,}4^h$	$3{,}3^h$	50^m	12^m	$4{,}5^m$
Fast Drum*	1 MHz	$5{,}6^m$	$1{,}4^m$	21^s	5^s	$1{,}9^s$
Electron Drum	500 KHz	$11{,}2^m$	$2{,}8^m$	42^s	10^s	$3{,}8^s$

* With CCD-arrays up to 5 Mhz possible

and output of images. Only fast drums offer a satisfactory input-output rate for high resolution (see Table 2).

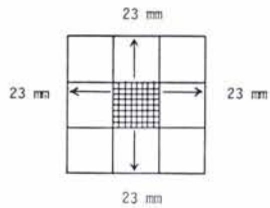
The next limitations are transfer rates within the computing system and its peripherals, as well as the storage requirements (see Table 3).

With respect to storage devices currently available, only high digital density tapes offer sufficient storage capability and sufficient transfer speed.

Devices, such as a fast scanning and printing drum and the high digital density tape intermediate storage, are extremely expensive. They would be worthwhile only if processing of images, geometric and otherwise, could be carried out in a data stream, that is, on-line. For on-line processing another limitation is imposed by the maximum core storage requirement for the input-image in the indirect rectification process. It is determined by the size of

TABLE 3. DIGITAL DATA TRANSFER RATE AND STORAGE LIMITATIONS

Device	Transfer	Storage	Remark
Drum	8 msec/track	2 M byte = $16 \cdot 10^6$ bit	too small
Disk	400 tracks × 20 surfaces 20 msec/rotation $8 \cdot 10^6$ bit/sec 1 MHz	88 M byte = $700 \cdot 10^8$ bit expandable to 400 M byte	1 photo at 25 μm 1 photo at 50 μm
C.C.T. (magn. tape)	8 tracks $3 \cdot 10^3$ bit/cm/track $8 \cdot 10^6$ bit/sec 1 Phot at 25 μm in 84 sec	720 m $1{,}7 \cdot 10^9$ bit	2,5 photos at 25 μm experimental
H.D.D.T. (magn. tape)	28 tracks $3 \cdot 10^5$ bit/cm/track $8 \cdot 10^7$ bit/sec 1 photo at 12,5 μm in 34 sec	2800 m $2{,}3 \cdot 10^{12}$ bit	900 photos at 12,5 μm suitable



RESOLUTION IN PIXEL SIZE	LINES	NO. Of PIXELS
12.5 μm	5520	30,5 M
25 μm	2760	7,6 M
50 μm	1380	1,9 M
100 μm	690	475 K
167 μm	414	171 K

Input block 69 x 69 mm
in random
acess
memory (core)

$$dx' = \frac{x'}{z}\, dz_0 + \frac{x'}{z}\, dz + y' d\kappa + \frac{x'y'}{c}\, d\omega - (1 + \frac{x'^2}{c^2}\, c \cdot d\phi$$

$$dy' = \frac{y'}{z}\, dz_0 + \frac{y'}{z}\, dz + x' d\kappa - (1 + \frac{y'^2}{c^2})\, c \cdot d\omega + \frac{x'y'}{c} \cdot d\phi$$

$dz_0 < 10\,\%z$; $dz < 10\,\%z$; $d\kappa < 10^0$; $d\omega < 5^0$; $d\phi < 5^0$

dx', dy' < 8 mm < 8 mm < 20 mm < 23 mm < 23 mm

FIG. 12. Required storage for digital rectification on-line.

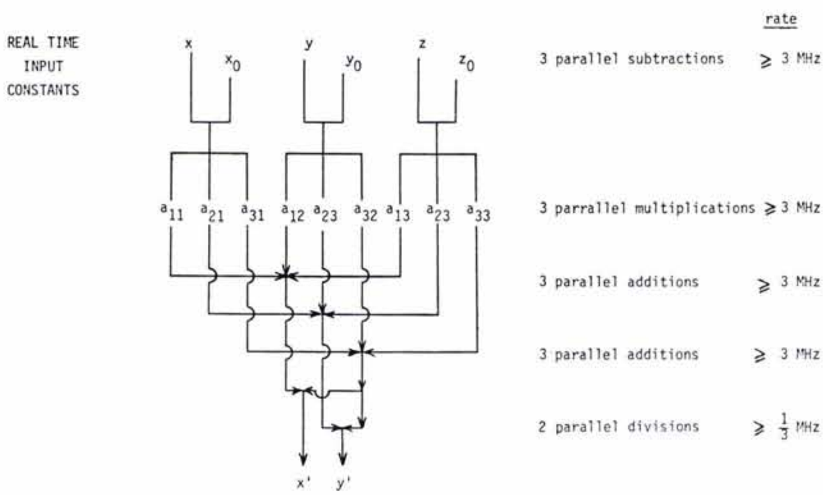


FIG. 13. Perspective processor for digital rectification.

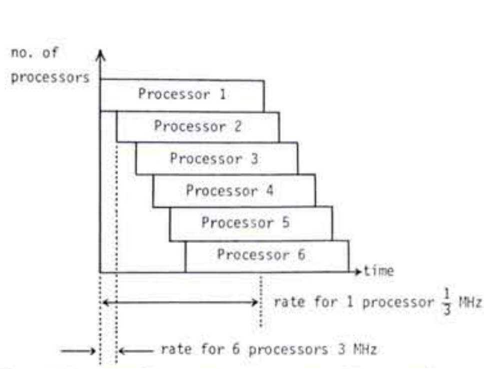


FIG. 14. Pipeline arrangement of several processors.

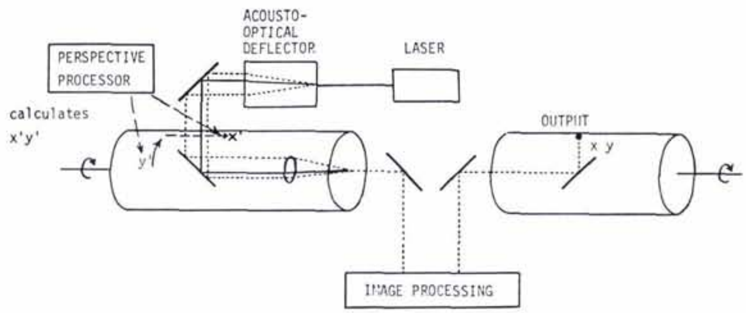


FIG. 15. Suggested on-line system for digital rectification.

expected image deformations, which for aerial photography result in a requirement to store a 69 by 69 mm image in core, unless anchor points are priviously determined in an off-line manner (see Figure 12).

Finally a limitation exists in computing the collinearity equation in real time. Digital hardware is currently capable of solving the total of 14 arithmetic operations at 1/3 MHz rates if they are performed in parallel (see Figure 13). U. Helava last year in Hannover designed such a processor, which he calls the digital projector. Such a perspective processor design may be applied several times. A number of these processors may be arranged in a pipeline mode. Six processors operating at 1/3 MHz rate could increase the total flow rate to 3 MHz (see Figure 14). Such a perspective processor system could be integrated with an output and an input drum. The image on the input drum can be addressed in real time by an acousto-optical laser deflector. A suggestion to that effect was presented at the last conference on the use of digital components in photogrammetry in Hannover in February, 1978 (Konecny, 1978; Lütjen, 1978) (see Figure 15). The result would be a digital image processing system capable of producing high quality digital orthophotos at very high speeds and at low unit cost.

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(Received January 23, 1979; accepted February 11, 1979)

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