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Numerical Cadastral Survey

A numerical photogrammetric method for determining the coordinates of section and quarter corners, relocating and restoring section corners and monuments, checking the establishment of a subdivision, and determining the coordinates of urban lots is described.

(Abstract appears on following page)

THE OBJECTIVE of a Cadastral Survey is to determine the position of property corners and other information so as to establish ownership. This information, among other things, is most useful for tax purposes. The position of property corners should be so determined that they may be restored on ground in case of disputes. In the U.S.A. this is mostly done by dividing the country into a number of townships*, each 36 square miles in area, based on an initial point and a standard baseline. Each township is then divided into 36 sections. Each section is divided into 4 squares. The section corners are established by the federal agency or representative. The lot (property) corners in the township are tied to the closest section corner or quarter corner (Figure 1). The establishment of quarter and section corners, though theoretically simple, presents practical problems such as rough wooded terrain, etc.

As development proceeds and subdivision occurs, the plots or maps showing the subdivision and its distance and bearing from the closest section or quarter corner are filed with the Record of Surveys. In case of dispute, the lot corner can be located on the ground with tolerable accuracy.

As these lots are further developed and become urban lots, they are re-surveyed for taxation, etc.

In practice, the subdivision plots are usually prepared at 1 in. = 100 ft scale and the distances are given to an accuracy of 0.01 feet and the bearing to one second. Thus, the mapping accuracy is 0.02 feet or 0.6 cm. Since ±0.02 feet is somewhat unrealistic, we should select ±0.05 feet or ±1.5 cm for station accuracy. A relative accuracy of ±1.5 cm and an absolute accuracy of ±3 to 5 cm is a realistic value for the position of lot corners. Given developments in analytical photogrammetry and computer software, it is possible to get an accuracy of ±5 micrometres at

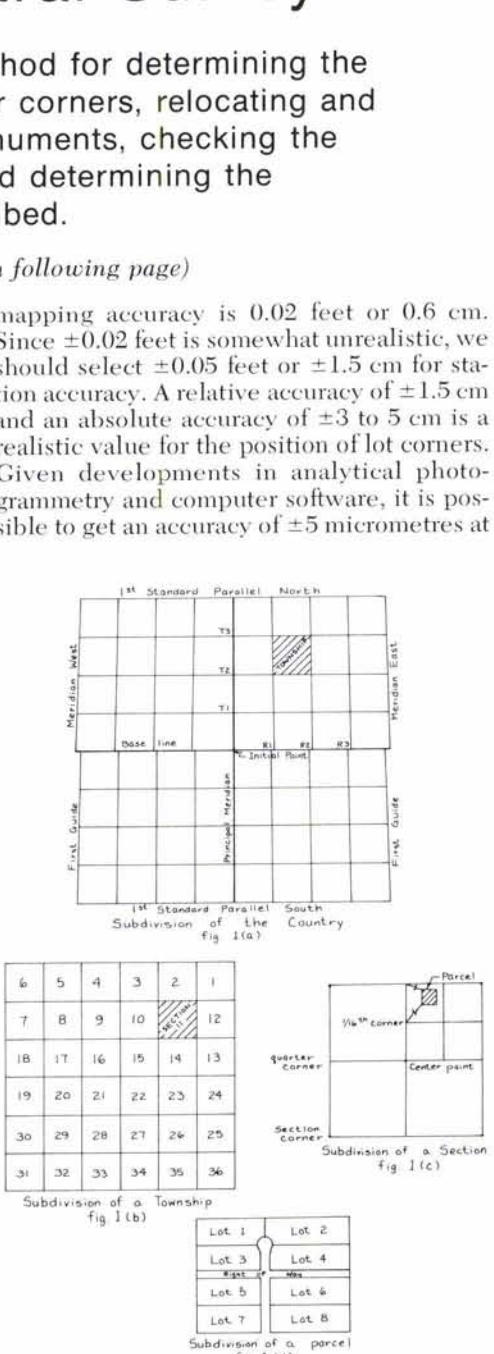

FIG. 1. Cadastral survey system.

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* Surveys also are performed by metes and bounds.

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ABSTRACT: A neglected field in the area of surveying in the U.S.A. is the cadastral survey. The plentiful supply of land in the past caused this neglect. In recent years, however, urban development has proceeded very rapidly and, consequently, land is becoming a scarce factor.

The objective of a cadastral survey is to locate boundaries of land parcels owned by individuals, monument their boundaries, survey these boundaries, and produce plans or plots which establish ownership and also give information and relocate the boundaries if and when disputes arise. The numerical cadastral survey assigns coordinates to the monuments defining the property boundary on a national, state, or city system. The advantage of establishing such coordinates is that they can be stored in computers and retrieved to plot plans on any desired scale. The coordinates can be also used to relocate the monuments or corners, as well as to restore lost monuments or corners.

The normal method of establishing coordinates is by detail traversing from high order control. This is both tedious and time-consuming. Photogrammetric and inertial survey methods can be used to establish these coordinates.

The objectives of this paper are to give (1)Such methods of determining the coordinates of section and quarter corners. The section and quarter corners are the intersecting points of a rectangular system used in the Cadastral Survey of the U.S.A.; (2)methods of relocating and restoring section corners and monuments; (3)methods of checking the establishment of a subdivision and for determining the coordinates of the lots in the subdivision; and (4)methods for determining the coordinates of urban lots. I have developed a new system, the stereoscipic digitizing system, to reduce the burden of work involved in determining the coordinates of lots in urban areas.

REFERENCE: Jeyapalan, K., "Numerical Cadastral Survey," *Photogrammetric Engineering and Remote Sensing*, Journal of the American Society of Photogrammetry, ASP, Vol. 45, No. AP9, September, 1979

the photo scale. This means that a ground accuracy of ±1.5 cm will require a photographic scale of 1:3000 This same photography can also be used to produce 1 in. = 100 ft scale maps by a two-and-one-half times enlargement or 1 in. = 50 ft scale maps by a five times enlargement. Thus, plotting instruments such as Wild B8 and Kern PG2, which have a maximum enlargement factor around two-and-one-half times, or the Kelsh, Wild A8, and other similar instruments, which have an enlargement factor of 5, can be used. Also, two to three photographs at this scale will stereo-cover an eighth of a section.

In order to get a position accuracy in the 3 to 5 cm range, a photograph scale ranging from 1:6000 to 1:9000 will be required. At a scale of 1:8400, three photographs are sufficient to provide stereo-cover for a section. Using PG2, Kelsh, and other plotting instruments and a photographic scale of 1:6000, topographic maps at a scale of 1 in. = 200 ft or 1 in. = 100 ft can be made. Such topographic maps can be used for earthwork calculations in connection with urban engineering work, such as laying pipe lines, construction work, etc.

Usually the subdivision map or plot filed with the Record of Surveys has not been checked independently on the ground. If the 1:3000 scale photos are used to plot a map at the original subdivision map scale, the necessary check would be provided.

Recently, there has been increased emphasis on tying section corners to the state plane coordinate system. When this is done, lot corners can also be tied to the state plane coordinate system. If both the section or quarter corners and the lot corners are in the same coordinate system, then any disputed or lost corner can be reestablished by surveying from the nearest section or quarter corner. By using the quarter corners as control points, analytical photogrammetry can determine the coordinates of a lot corner from 1:3000 scale photography.

The coordinates of the lot corners, those of the section corners and the quarter section corners, and such other relevant information can be stored in the computer, retrieved at any time, and then used to plot the Cadastral Survey map of the section at any desired scale using computer graphics. The resulting map, together with the photographs and topographic maps, can be used in the valuation of urban lots. In the valuation of urban lots, an orthophoto of the section will be most helpful.

Photogrammetric methods also can be used to determine the state plane coordinates of the section corners (Brown, 1977). The 1:6000 to 1:9000 scale photography can be used to determine the State plane coordinates of both section and quarter corners.

If at any time the section and quarter corner are lost, they also can be re-located by photogrammetric methods using 1:6000 to 1:9000 scale photography.

Two major problems in analytical photogrammetry are how to obtain accurate coordinates of control points and how to identify the control points and pass points on the photographs. In order to obtain the best pointing accuracy and avoid misidentification, targeting should be done. Cadastral surveys require the targeting of all points and photographic exposures at predetermined positions. The type and size of targets depends on the ground surface and the required pointing accuracy. All this is both expensive and time-consuming (Lafferty, 1971; Lafferty, 1973). However, if the 1:3000 scale photography is used, then many fence corners can be identified, especially with a zoom stereoscope.

Processing large-scale photography requires expensive computers and qualified operators, both of which can be avoided by using an inexpensive digitizer attached to a desk calculator.

This research study is organized in the following sections:

- Determination of section and quarter section corner coordinates,
- Relocation of section and quarter corners and mining claim monuments,
- Methods of checking the accuracy of a 20 acre subdivision, and
- Procedure for determining the lot corner coordinates of an urban system.

Determination of Section and Quarter Corners

As mentioned earlier, each township of 6 by 6 miles is divided into 36 sections, one square mile each. Ideally, the four corners defining the township should be established by second order triangulation from existing 1st order triangulation points. The third order triangulation procedure can then be used to establish the section and quarter section corners (Figure 2).

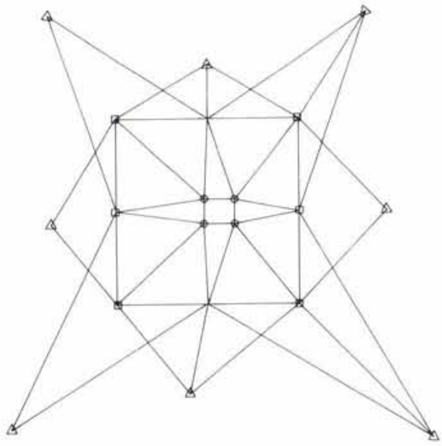

Fig. 2. Triangulation, trilateration method of establishing coordinates.

When triangulation procedures are used to establish coordinates of township and section corners, urban development, vegetation, etc., causes obstruction and delay. Therefore, an alternative method for establishing these coordinates is very desirable. The auto surveyor and analytical photogrammetry are two such methods.

The Auto Surveyor

The auto surveyor is a new development in the area of intertial navigation. After correcting for drift, etc., it is expected to give the coordinates of a position with an accuracy of $\pm 10^1$ cms (Brown, 1977). The observation procedures are similar to a traverse. The system is initialized at known station A and transported to another known station B in a motor vehicle via stations whose coordinates are unknown. The vehicle is stopped at these stations, and relevant information on x, y, and z coordinates are recorded from the on-board computer. The vehicle is also frequently stopped for updating zero-velocity. The closing errors obtained at station B are then used to compensate the drift errors in the x, y, and z coordinates of the unknown stations.

Experiments with the system have indicated that, over long distances, the accuracy obtainable is of a second order traverse standard. Hence, it is suitable for establishing township corners but in its present state is unsuitable for the short traverses used to establish section and quarter section corners.

ANALYTICAL PHOTOGRAMMETRY

Analytical photogrammetry uses aerial photographs taken along a predetermined flight line at predetermined intervals using a high-precision wide-angle camera. The film is then carefully processed and glass diapositives are made from it.

The location of both known and unknown points are identified on the diapositives. The coordinates of these points together with the fiducial marks on the diapositives are measured to 0.001 mm accuracy by a comparator. If the photo coordinates of the fiducial marks are known, then these coordinates can be transformed into photo coordinates. Errors due to film shrinkage, lens distortion, atmospheric refraction, etc., are then corrected and the plate coordinates are transformed into ground coordinates by analytical triangulation programs. Computer capability enables 1000 photographs to be processed at one time.

Experiments have shown that it is possible to obtain ground coordinates to an accuracy of 0.005 mm at photo scale. Thus, if the ground control coordinates are accurate, then it is possible to determine the section and quarter corner coordinates of sufficient accuracy merely by choosing the appropriate photo scale. The larger the photo scale, the more photographs that need to be processed and the costlier the project. Since the coordinates of section corners require an accuracy of 3 to 5 cm, the 1:6000 to 1:9000 scale photographs can be used to determine these coordinates.

Relocation of Section and Quarter Corners

In cadastral survey projects it is often necessary to relocate the corners and, if they are lost, restore them. The normal procedure of restoring lost corners is to traverse from the nearest relocated corner and establish a position either by single or double proportion.

An alternative method is to target all the relocated section corners and the estimated positions of lost corners and azimuth markers placed nearby. The coordinates of these targets can then be determined by analytical photogrammetry using 1:6000 to 1:9000 scale photography. By using the coordinates of relocated corners, the coordinates of the lost corners can be determined and established by ground survey methods from the estimated location by using the azimuth marker for direction.

In order to test this alternate method, a subdivision of section 17, 19, and 20 of Township 2 north Range 15 east, Mt. Diablo base and meridian was undertaken. The section and some quarter corners were relocated and targeted (Figure 3). The estimated positions of center points and 1/16th corners and lost quarter corners were targeted. The targets were black plastic squares 2 feet in size. They were each marked at the center with a white circle 6 inches in diameter. The 6 inch diameter gave an image size of about 50 mm on the 1:3000 photographs.

The photography was taken at 1:6000 (about 25 photos) and 1:3000 scales (about 100 photos). The targets were photo identified on the large-scale photos and an analytical aerial triangulation was carried out using the small-scale photograph (22 photos).

Table 1 shows the coordinates of the estimated corner points obtained by analytical triangulation in section 19, and also the calculated coordinates and the moves required to set them on the ground. These measurements were used to relocate or set the points on the ground. The estimated accuracy of the new points is of the order of 2 to 3 feet. This large error is partly due to the fact that the pass points were not premarked or targeted and partly because of the use of small-scale photography. If a higher accuracy is desired, then large-scale photographs should be taken after restoring the lost corners and setting subdivision points. Then an aerial triangulation should be carried out preferably with pass points also targeted.

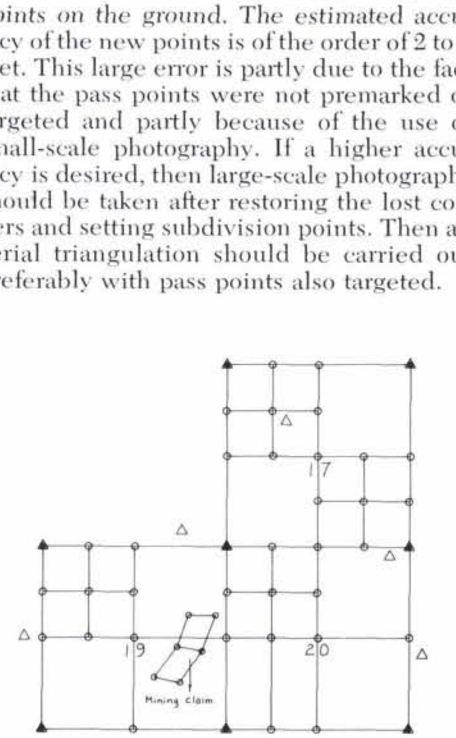

FIG. 3. Relocation of section and quarter corners.

TABLE 1. MOVES FOR CORRECT POSITIONING IN SECTION 19

Points	Coordinates		Move			
	Triangulated	Proportioned	Bearing			Distance
NW 1/16 19 25	552145.1 2037075.8	552299.9 2036884.6	S51°	01′	08″E	246.0′
N 1/16 24-19 26	552189.7 2035588.8	552296.29 2035573.04	S 8°	24′	38″E	107.8′
W 1/16 18-19 27	553608.1 2036698.1	553590.5 2036868.65	N84°	06′	18″W	171.5′
1/4 18-19 49	553473.7 2038093.0	553602.0 2038180.7	S34°	20′	25″W	155.4′
C 1/4 19 39	550970.5 2038568.4	551004.8 2038311.6	S84°	30′	54″E	358.4′
1/4 19-30 31	548386.1 2038417.1	548406.3 2038242.6	S83°	23′	05″E	175.7′
1/4 19-20 59	550604.0 2040770.4	550995.9 2040833.7	S 9°	10′	16″W	397.0′
M.C.M. 43	550097.10 2040236.96	(1/4 19-20) 550604.0 2040770.4	N46°	27′	41″	735.87
W 1/16 19 18	550787.67 2036749.23	551009.20 2036900.49	N34°	19′	30″	268.24′
1/4 24-19 19	550914.75 2035814.18	551013.64 2035589.35	N 2°	16′	25″W	245.62′
N 1/16 - 19 40	552122.86 2038165.44	552303.40 2038196.15	N 9°	39′	13″E	183.13′
N 1/16 19-20 64	552105.45 2040823.05	552310.51 2040818.88	N 1°	09′	54″W	205.1′
M.C.M. 63	552008.08 2040270.30	(section corner 17, 18, 19, 20) 553550.679 2040615.679	N12°	37′	12″	1580.8

M.C.M.—Mineral Claim Monument

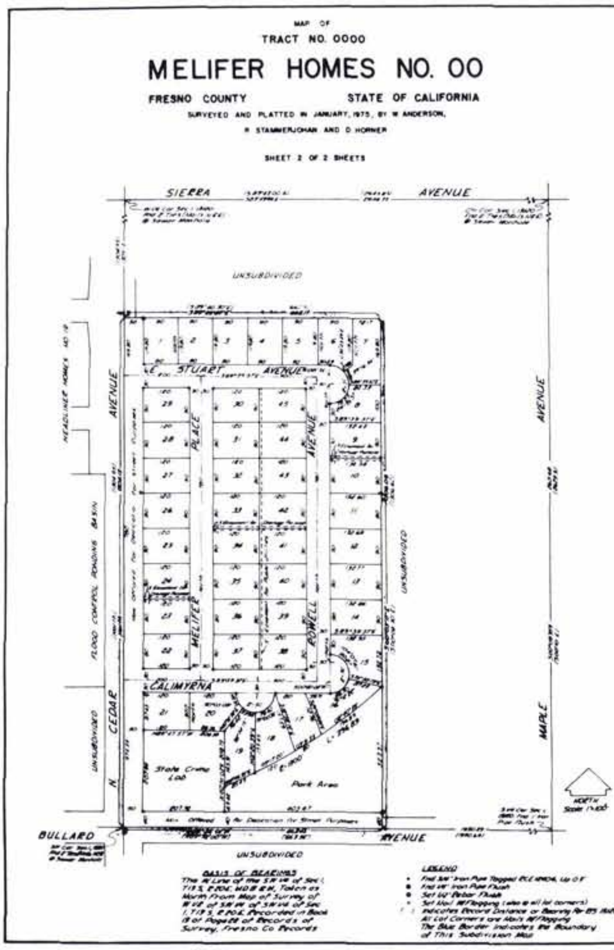

FIG. 4. Subdivision map prepared by ground survey.

SUBDIVISION OF A 20 ACRE LOT

In order to check the accuracy of photogrammetric survey results and solve the problem involved, a 20 acre lot was subdivided from start to finish by field survey methods. Initially, a contour map showing one foot contours was prepared using plane table methods. The subdivision was designed on a map at a scale of 1 in. = 100 ft and then the lot corners were set by ground survey methods to the accuracy specified by the design map (Figure 4). Using the distances and bearings shown on the map, the coordinates of the lot corners as well as those of section corners were determined.

Using the 1:6,000 scale photographs and Kelsh plotting instruments, a one foot contour map of the area was prepared which compared satisfactorily with that by plane table method. Since black and white gives the maximum contrast and since the area is sandy, the 2 foot square black plastic target with 6 inch diameter of white dots at the center was again used. These targets were then placed over the lot corners and fixed to the ground with eight nails. The 2-foot target gave an image size of 0.1 mm at the 1:6000 photo scale and the 6 inch white dot gave an image size of 0.05 mm at 1:3000 photo scale photography. At places where the targets could not be positioned, two targets were set in the neighborhood and offsets were measured. Then photography at a scale of 1:3000 was taken to provide stereo coverage of the area. Using a Wild A9 stereoplotter* the positions of the targets were plotted by photogrammetric methods and a subdivision map

* The wide angle photograph was reduced to be adopted in the Wild A9 stereoplotter.

was prepared (Figure 5). With the digitizer attached to the Wild A9 the model coordinates of the targets were obtained and then transformed to give the ground coordinates.

Table 2 gives the results. The large errors are due to computation blunders in ground survey methods, indicating the value of an independent check. The small errors may be due to setting errors in ground survey methods. By eliminating the computation blunders and then using the residuals between the two methods, standard errors of ±0.27 ft in x and ±0.38 ft in y were obtained. This shows that photogrammetric methods could be effectively used to check the ground survey and also to determine the position coordinates of the lot corners.

This project also was done using analytical photogrammetric techniques. The Kern monocomparator, which gives the plate coordinates to one micrometre, was used. The coordinates generated by the monocomparator were processed through two computer programs developed for this project. The first program gives the coordinates by a sequential method and the other program by a simultaneous least-squares adjustment method. The standard errors obtained using the residuals between the ground control coordinates and coordinates obtained by the sequential triangulation method are ±0.36 ft in x and ±0.46 ft in y. The standard errors obtained using the residuals between the ground and simultaneous triangulation method are ±0.22 ft in x and ±0.26 ft in y.

Survey of an Urban Lot

In order to solve the problems involved in the photogrammetric surveys of a residential

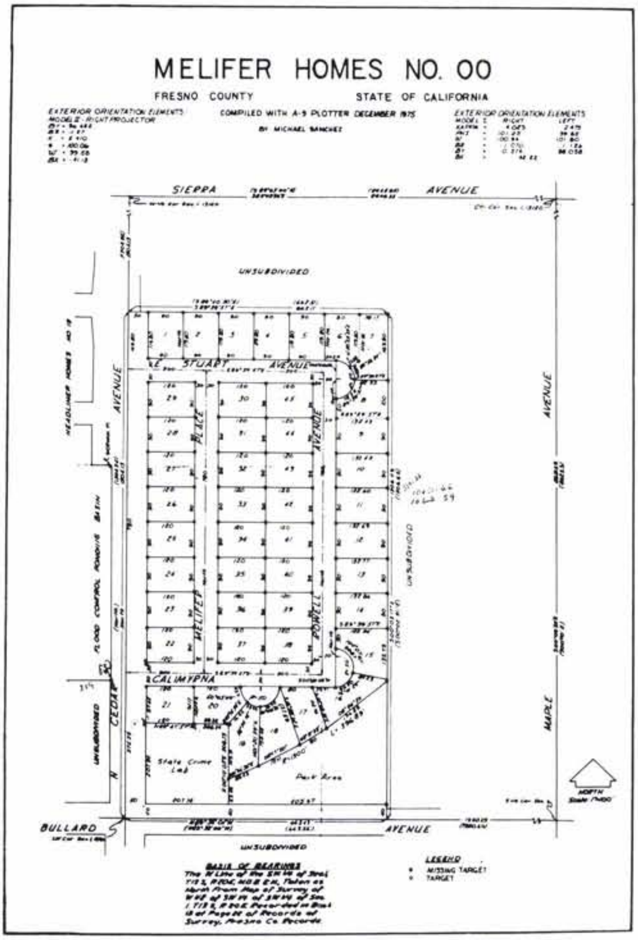

FIG. 5. Subdivision map prepared by photogrammetry.

TABLE 2(A). DIFFERENCE BETWEEN PHOTOGRAMMETRIC AND GROUND COORDINATES (LATITUDE)

Listing of Transformed Model and Ground Coordinates	Field Latitude	Model Latitude	Residual
SE COR PROP			
(SW COR PARK)			
SE COR LOTS 15/16	10358.74	10363.612	−4.872
E COR LOTS 14/15	10491.46	10491.706	−0.246
E COR LOTS 13/14	10581.46	10581.793	−0.333
E COR LOTS 12/13	10671.46	10671.889	−0.429
E COR LOTS 11/12	10761.46	10761.781	−0.321
E COR LOTS 10/11	10851.46	10851.300	0.160
E COR LOTS 9/10	10941.46	10941.610	−0.150
E COR LOTS 8/9	11031.46	11031.502	−0.042
W COR LOTS 9/10	10942.24	10942.265	−0.025
W COR LOTS 10/11	10852.24	10852.364	−0.124
W COR LOTS 11/12	10762.24	10762.230	0.010
W COR LOTS 12/13	10672.24	10672.534	−0.294
W COR LOTS 13/14	10582.24	10582.438	−0.198
W COR LOTS 14/15	10492.24	10492.528	−0.288
SW COR LOT 15	10442.24	10441.435	0.805
W COR LOTS 15/16	10380.04	10380.258	−0.218
(@ Cul de Sac)			
CR LOT 16	10342.24	10341.539	0.701
S COR LOTS 16/17	10236.74	10236.906	−0.166
S COR LOTS 17/18	10194.02	10197.088	−3.068
S COR LOTS 18/19	10137.95	10137.468	0.482
W COR 19/PARK	10101.54	10101.434	0.106
(@ Crime Lab)			
S COR LOTS 20/21	10247.16	10247.021	0.139
NW COR LOT 21	10345.03	10344.878	0.152
N COR LOTS 19/20	10305.18	10305.177	0.003
(@ Cul de Sac)			
N COR LOTS 18/19	10293.36	10292.600	0.760
N COR LOTS 17/18	10343.05	10343.009	0.041
N COR LOTS 16/17	10342.58	10342.730	−0.150
E COR LOTS 38/39	10492.58	10492.583	−0.003
E COR LOTS 39/40	10582.58	10582.875	−0.295
E COR LOTS 40/41	10672.58	10672.766	−0.186
E COR LOTS 41/42	10762.58	10762.686	−0.106
E COR LOTS 42/43	10852.58	10852.605	−0.025
E COR LOTS 43/44	10942.58	10942.516	0.064
E COR LOTS 44/45	11032.58	11033.031	−0.541
LOTS 30/31/44/45	11033.28	11033.700	−0.420
LOTS 31/32/43/44	10943.28	10943.204	0.076
LOTS 32/33/42/43	10853.28	10858.188	−4.908
LOTS 33/34/41/42	10763.28	10763.384	−0.104
LOTS 34/35/40/41	10673.28	10673.669	−0.389
LOTS 35/36/39/40	10583.28	10583.768	−0.488
LOTS 36/37/38/39	10493.28	10493.662	−0.382
S COR LOTS 37/38	10403.28	10403.566	−0.286
SW COR LOT 37	10403.98	10404.068	−0.088
W COR LOTS 36/37	10493.98	10496.993	−3.013
W COR LOTS 35/36	10583.98	10584.27−	−0.290
W COR LOTS 34/35	10673.98	10674.376	−0.396
W COR LOTS 33/34	10763.98	10959.718	4.262
W COR LOTS 32/33	10853.98	10854.187	−0.207
W COR LOTS 31/32	10943.98	10944.292	−0.312
E COR LOTS 27/28	10944.33	10944.543	−0.213
E COR LOTS 26/27	10854.33	10854.447	−0.117
E COR LOTS 25/26	10764.33	10764.528	−0.198
E COR LOTS 24/25	10674.33	10674.431	−0.101
E COR LOTS 23/24	10584.33	10589.220	−4.890

TABLE 2(A)—(*Continued*)

Listing of Transformed Model and Ground Coordinates	Field Latitude	Model Latitude	Residual
E COR LOTS 22/23	10494.33	10499.124	−4.794
SE COR LOT 22	10404.33	10404.310	0.020
W COR LOTS 22/23	10495.03	10494.908	0.122
W COR LOTS 23/24	10585.03	10585.219	−0.189
W COR LOTS 24/25	10675.03	10675.324	−0.294
W COR OTS 25/26	10765.03	10765.225	−0.195
W COR LOTS 26/27	10855.03	10855.340	−0.310
W COR LOTS 27/28	10945.03	10945.232	−0.202
E COR LOTS 27/28	10944.33	10944.171	0.159
W COR LOTS 31/32	10943.98	10943.911	0.069
LOTS 31/32/43/44	10943.28	10943.409	−0.129
E COR LOTS 43/44	10942.58	10942.525	0.055
W COR LOTS 9/10	10942.24	10942.451	−0.211
W COR LOTS 10/11	10852.24	10852.364	−0.124
W COR LOTS 11/12	10762.24	10762.230	0.010
W COR LOTS 12/13	10672.24	10672.357	−0.117
W COR LOTS 28/29	11035.03	11034.965	0.065
E COR LOTS 28/29	11034.33	11034.268	0.062
W COR LOTS 30/31	11032.98	11033.830	−0.850
NW COR LOT 29	11125.03	11124.670	0.360
NE COR LOT 29	11124.33	11123.973	0.357
NW COR LOT 30	11123.98	11123.731	0.249
N COR LOTS 30/45	11123.28	11123.229	0.051
NE COR LOT 45	11122.58	11122.522	0.058
NW COR LOT 8	1082.24	11082.309	−0.069
W COR LOTS 7/8 (@ Cul de Sac)	11131.94	11131.731	0.209
E COR LOTS 7/8	1131.46	11131.407	0.053
SW COR LOTS 6/7 (@ Cul de Sac)	11164.11	11164.094	0.016
CR @ LOT 6	11182.24	11186.913	−4.673
S COR LOTS 5/6	11182.42	11182.154	0.266
S COR LOTS 4/5	11182.94	11182.716	0.224
S COR LOTS 3/4	11183.47	11183.093	0.377
S COR LOTS 2/3	11183.99	11183.665	0.325
S COR LOTS 1/2	11184.52	11188.927	−4.407
SW COR LOT 1	11185.04	11184.800	0.240
NW COR PROP (NW COR LOT 1)	11304.83	11304.491	0.339
N COR LOTS 1/2	11304.31	11303.919	0.391
N COR LOTS 2/3	11303.78	11303.347	0.433
N COR LOTS 4/5	11302.73	11302.970	−0.240
N COR LOTS 5/6	11302.21	11302.398	−0.188
N COR LOTS 4/5	11302.73	11302.970	−0.240
N COR LOTS 6/7	11303.68	11301.449	2.231
NE COR LOT 7	11301.26	11301.213	0.047

Calculation of standard error
sum of residuals = −3.955
residuals squared = 6.284

$$\text{Standard Error} = \pm\sqrt{\frac{6.284}{82}}$$

$$= 0.27$$

TABLE 2(B). DIFFERENCE BETWEEN PHOTOGRAMMETRIC AND GROUND COORDINATES (DEPARTURE)

	Listing of Transformed Model and Ground Coordinates Field Departure	Model Departure	Residual
SE COR PROP	10663.37	10663.692	
(SW COR PARK)			
SE COR LOTS 15/16	10663.06	10663.780	−0.726
E COR LOTS 14/15	10662.94	10663.497	−0.557
E COR LOTS 13/14	10662.84	10663.082	−0.242
E COR LOTS 12/13	10662.76	10662.863	−0.103
E COR LOTS 11/12	10662.68	10662.458	0.222
E COR LOTS 10/11	10662.59	10662.462	0.128
E COR LOTS 9/10	10662.51	10662.625	−0.115
E COR LOTS 8/9	10662.42	10662.220	0.200
W COR LOTS 9/10	10529.98	10529.588	0.392
W COR LOTS 10/11	10529.98	10529.798	0.182
W COR LOTS 11/12	10529.98	10529.235	0.745
W COR LOTS 12/13	10529.98	10529.631	0.349
W COR LOTS 13/14	10529.98	10529.850	0.130
W COR LOTS 14/15	10529.98	10529.864	0.116
SW COR LOT 15	10529.98	10510.186	19.794
W COR LOTS 15/16	10578.47	10578.162	0.308
(@ Cul de Sac)			
CR LOT 16	10529.71	10510.289	19.421
S COR LOTS 16/17	10508.86	10509.640	−0.780
W COR LOTS 11/12	10529.98	10529.235	0.745
W COR LOTS 12/13	10529.98	10530.031	−0.051
W COR LOTS 28/29	10050.00	10049.915	0.085
E COR LOTS 28/29	10170.00	10169.830	0.170
W COR LOTS 30/31	10230.00	10229.987	0.013
NW COR LOT 29	10050.00	10049.715	0.285
NE COR LOT 29	10170.00	10169.629	0.371
NW COR LOT 30	10230.00	10229.777	0.223
N COR LOT 30/45	10350.00	10349.683	0.317
NE COR LOT 45	10470.00	10469.402	0.598
NW COR LOT 8	10529.98	10529.318	0.662
W COR LOTS 7/8	10579.99	10579.438	0.552
(@ Cul de Sac)			
E COR LOTS 7/8	10662.34	10662.312	0.028
SW COR LOTS 6/7	10568.52	10568.087	0.433
(@ Cul de Sac)			
CR LOT 6	10530.28	10627.126	−96.846
S COR LOTS 5/6	10499.99	10499.639	0.351
S COR LOTS 4/5	10408.99	10409.505	−0.515
S COR LOTS 3/4	10319.99	10319.576	0.414
S COR LOTS 2/3	10229.99	10229.638	0.352
S COR LOTS 1/2	10139.99	10237.220	−97.230
SW COR LOT 1	10050.00	10049.565	0.435
NW COR PROP	10050.00	10049.686	0.314
(NW COR LOT 1)			
N COR LOTS 1/2	10140.00	10139.625	0.375
SW COR LOT 37	10230.00	10230.106	−0.106
W COR LOTS 36/37	10230.00	10288.712	−58.712
W COR LOTS 35/36	10230.00	10229.863	0.137
W COR LOTS 34/35	10230.00	10229.839	0.161
W COR LOTS 33/34	10230.00	10220.427	9.573
W COR LOTS 32/33	10230.00	10229.615	0.385
W COR LOTS 31/32	10230.00	10229.592	0.408
E COR LOTS 27/28	10170.00	10169.639	0.361
E COR LOTS 26/27	10170.00	10169.858	0.142
E COR LOTS 25/26	10170.00	10169.677	0.323
E COR LOTS 24/25	10170.00	10169.896	0.104

TABLE 2(B)—*(Continued)*

	Listing of Transformed Model and Ground Coordinates Field Departure	Model Departure	Residual
E COR LOTS 23/24	10170.00	10267.626	−97.626
E COR LOTS 22/23	10170.00	10267.845	−97.845
SE COR LOT 22	10170.00	10169.958	0.042
W COR LOTS 22/23	10050.00	10049.833	0.167
W COR LOTS 23/24	10050.00	10049.996	0.004
W COR LOTS 24/25	10050.00	10049.972	0.028
W COR LOTS 25/26	10050.00	10049.762	0.238
W COR LOTS 27/28	10050.00	10049.529	0.471
E COR LOTS 27/28	10170.00	10170.049	−0.049
W COR LOTS 31/32	10230.00	10229.806	0.194
LOTS 31/32/43/44	10230.00	10349.711	−119.711
E COR LOTS 43/44	10470.00	10469.831	0.169
W COR LOTS 9/10	10529.98	10529.383	0.597
W COR LOTS 10/11	10529.98	10529.798	0.182
S COR LOTS 17/18	10441.23	10500.389	−59.159
S COR LOTS 18/19	10335.18	10335.971	−0.791
W COR 19/PARK (@ Crime Lab)	10257.90	10258.763	−0.863
S COR LOTS 20/21	10170.00	10170.078	−0.078
NW COR LOT 21	10050.00	10050.192	−0.192
N COR LOTS 19/20 (@ Cul de Sac)	10307.70	10308.122	−0.422
N COR LOTS 18/19	10338.83	10319.500	19.330
N COR LOTS 17/18	10390.00	10390.141	−0.141
N COR LOTS 16/17	10470.00	10469.879	0.121
E COR LOTS 38/39	10470.00	10469.921	0.079
E COR LOTS 39/40	10470.00	10469.692	0.308
E COR LOTS 40/41	10470.00	10469.287	0.713
E COR LOTS 41/42	10470.00	10469.468	0.532
E COR LOTS 42/43	10470.00	10469.650	0.350
E COR LOTS 43/44	10470.00	10469.635	0.365
E COR LOTS 44/45	10470.00	10469.984	0.016
LOTS 30/31/44/45	10230.00	10349.483	−119.483
LOTS 31/32/43/44	10230.00	10349.525	−119.525
LOTS 32/33/42/43	10230.00	10447.246	−217.246
LOTS 33/34/41/42	10230.00	10349.554	−119.554
LOTS 34/35/40/41	10230.00	10349.559	−119.559
LOTS 35/36/39/40	10230.00	10349.768	−113.768
LOTS 36/37/38/39	10230.00	10349.792	−119.792
S COR LOTS 37/38	10350.00	10350.011	−0.011
N COR LOTS 2/3	10230.00	10229.563	0.437
N COR LOTS 4/5	10410.00	10319.492	90.508
N COR LOTS 5/6	10410.00	10409.430	0.570
N COR LOTS 4/5	10410.00	10519.492	90.508
N COR LOTS 6/7	10589.99	10589.298	0.692
NE COR LOT 7	10662.16	10661.786	0.374
W COR LOTS 26/27	10050.00	10049.934	0.066

Calculation of standard error,
sum of residuals = 12.494
residuals squared = 11.295

$$\text{Standard Error} = \pm \sqrt{\frac{11.295}{77}}$$

$$= 0.38$$

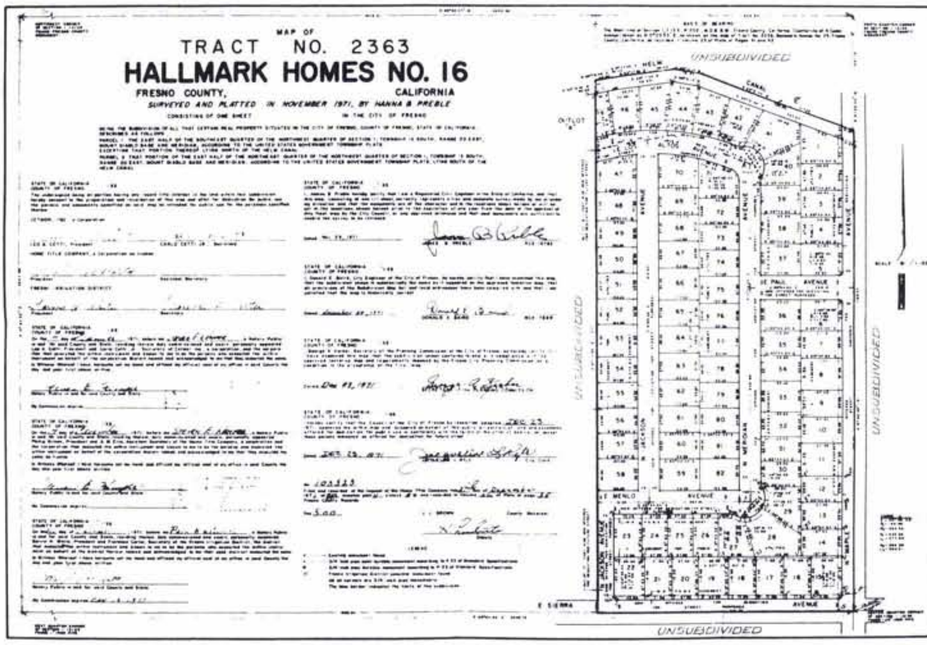

FIG. 6. Cadastral survey map of an urban parcel prepared by ground survey.

area, the position determination of about 15 lots in the tract Map 2363 of Fresno County was undertaken. Figure 6 shows the recorded tract map. The first problem was targeting the lot corners. There were two types of lot corners, one created by the intersection of fence corners and the other by the intersection of the fence lines which were extended to meet the road. Fence lines could be plotted using plotting instruments and large-scale photography. However, the intersection of fence lines may not be ideal for determining the position coordinates. Thus, black plastic targets with a white dot were placed near the fence lines and distances to the lot corners were measured and recorded in the field (Figure 7). On asphalt surfaces 6 inch white crosses were used as targets. Distances from at least two targets were measured to each lot corner. The photogrammetrically plotted fence intersection on the map could provide the necessary check on the position determined by the offsets from the targets or vice versa.

Photography at 1:7,200 and 1:3,000 scales were obtained. The 1:7,200 scale photo-

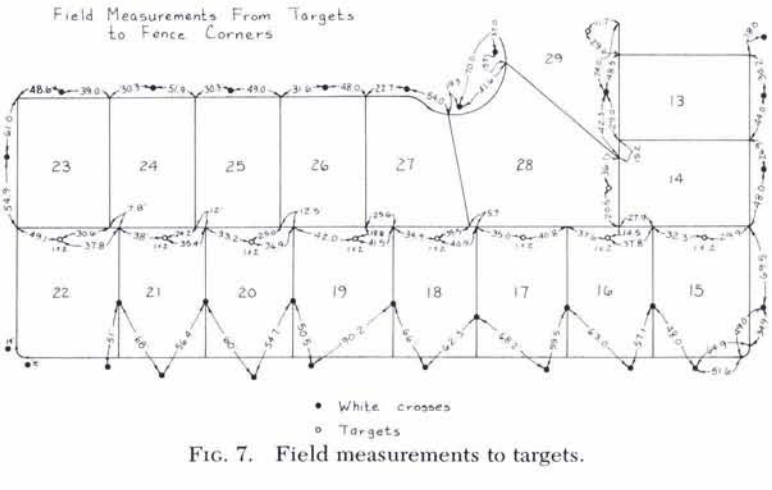

FIG. 7. Field measurements to targets.

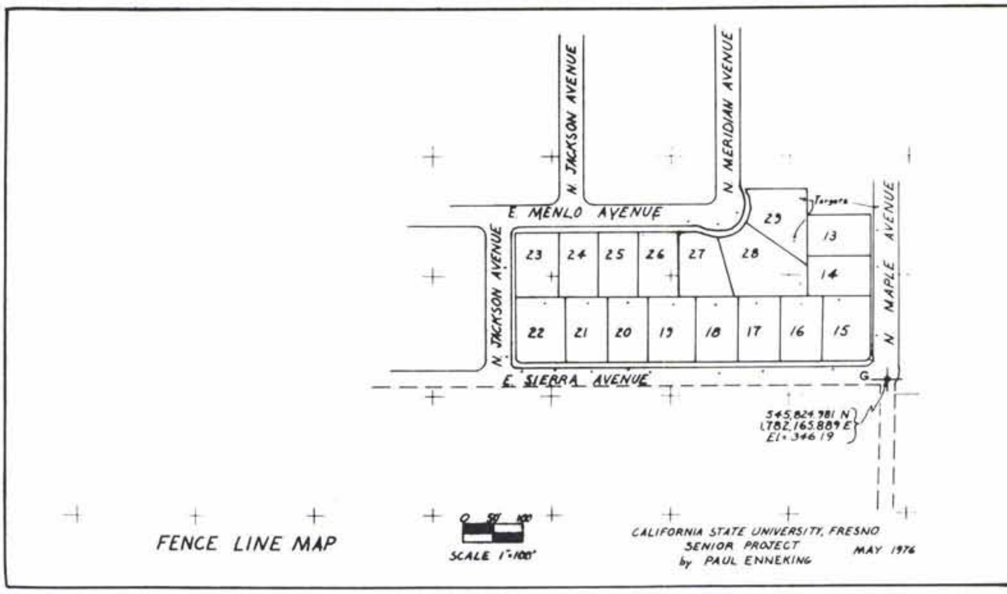

FIG. 8. Cadastral survey map prepared by photogrammetry (drawn at a scale of 1 in. = 100 ft using the Kern PG2).

graphs nearly cover the section and 1:3000 scale photographs cover the eighth of a section. Now using the Kern PG2 plotting instrument and 1:3000 scale photography, the fence line of the lots and position of the targets were plotted at a scale of 1 in. = 100 ft (Figure 8). The offsets from the targets and the intersection of the fence lines agreed within plotting accuracy. This fence line map showed agreement with the tract map.

Using the 1:7,200 scale photographs and the Kern PG2, the lot corners and the section corners were plotted at a scale of 1 in. = 200 ft (Figure 9). Then the coordinates of the lot corners were determined by digitizing and transforming. The coordinates were found to have an accuracy of about ±2 ft.

Increased accuracy of the position can be determined by using analytical photogrammetry, and the values obtained by plotting and digitizing can be used to check and eliminate blunders.

With the 1:7200 scale photography the coordinates of the lot corners were determined by analytical photogrammetric methods. The positions of well defined targets and points agreed well with the ground values but those of lot corners such as fence intersection, etc., which cannot be identified definitely because of the scale of the photograph, did not.

If 1:3000 scale photography had been used, then all the points could have been correctly identified and accurate coordinates obtained by analytical methods. In practice, a large project will involve the observation of many photos using stereo-comparators. But on the other hand if we use 1:7,200 scale photography then we will have to target every lot corner which is again costly and time-consuming.

STEREOSCOPIC DIGITIZING SYSTEM

In order to eliminate both problems, a new method of observing photo coordinates known as stereoscopic digitizing system was developed. This system consists of a Wang digitizer, a Wang calculator, and a mirror or zoom stereoscope. The mirror stereoscope is mounted on an aluminum framework in such a way that it is freely movable along the frame. Two spot light lamps are attached to the framework. These lamps can be slid on the frame. The whole frame is free to move in the y direction (Figure 10). Thus, a block of photographs can be laid on the digitizer and the points can be digitized, one photo at a time. These digitized coordinates can then be transformed to photo coordinates by the program in the Wang computer.

The procedure of observing points in photo 22, strip 2 (Figure 11) consists in taping photo 22 on the digitizer and laying the adjoining three photos from strip 1 and 3 and photos in strip 2 as shown. The fiducials marks in photo 22 are first digitized. Given

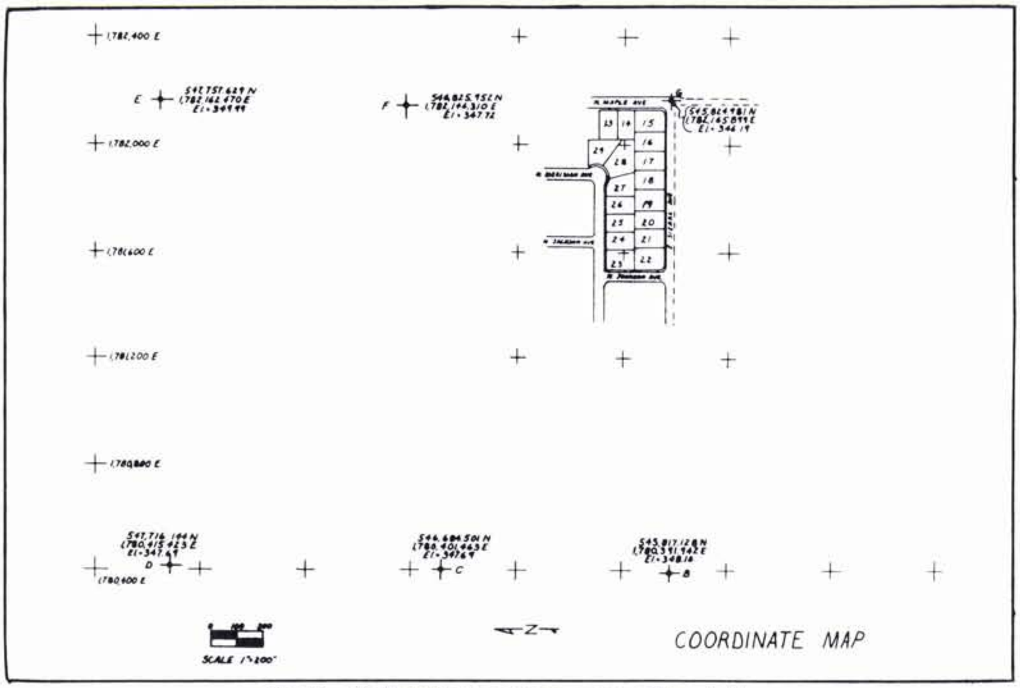

FIG. 9. Cadastral survey map prepared by photogrammetry (drawn at a scale of 1 in. = 200 ft using the Kern PG2).

the fiducial point coordinates, the program in the Wang computer determines the transformation parameters. Any points whose photo coordinates in photo 22 are required are identified under stereovision either using photo 21 or 23. The cursor is placed over the point and the reference mark on the cursor is then centered over the point using the attached x screw and y screw (Figure 12). Then, by placing the point transfer device on the other plate, the same point can be point transferred or vice versa. In practice one has to premark a point on one plate only, in which case the marker on the cursor can be placed on the exact point under stereoscopic observation. Once the reference mark on the cursor is placed exactly on the point, then it can be digitized and the program in

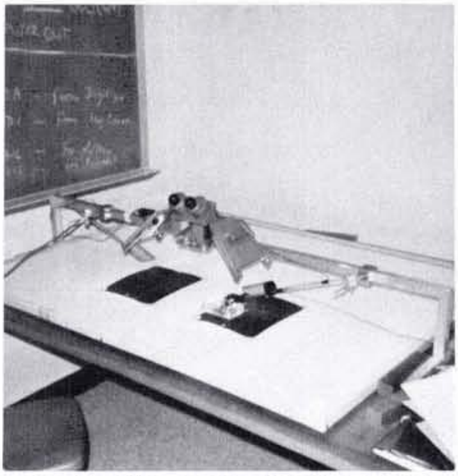

FIG. 10. Stereoscopic digitizing system.

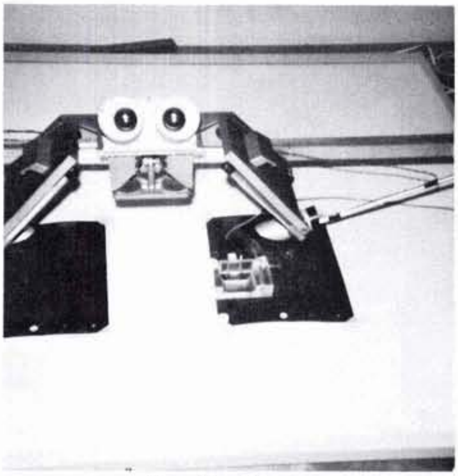

FIG. 11. Cursor.

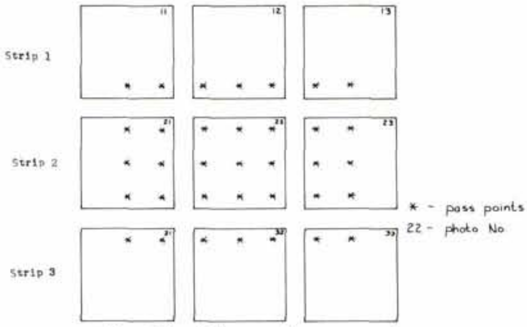

FIG. 12. Observation procedure.

the calculator will transform it to photo coordinates.

RESULTS FROM STEREO DIGITIZING SYSTEM (S.D.)

The 1:3,000 scale photography was digitized using the Wang digitizer. Since the least count of the digitizer was only 0.01 inches, a point was digitized four times and the mean coordinate was used in analytical triangulation. Again, since the digitizing system is only good to about 0.25 mm, a paper print was used instead of a glass diapositive. Table 3 shows the comparison between the results from monocomparator observations, PG2 observations, and the S.D. system. Table 4 gives the difference between lot corner coordinates obtained by analytical triangulation, and that obtained using the coordinates computed from nearby targets and distances measured to the lot corner. The coordinates of the targets were also obtained by analytical triangulation. The residuals in x, as well as in y, are given for both sets of data from the stereoscopic digitizing system and the monocomparator system.

An estimated standard error of ±0.2 ft at the control points using monocomparator data indicates that precision coordinates can be obtained by analytical photogrammetry. The large error of 4 ft at the lot corner, which is due to an identification problem, could be minimized, if necessary, by using large scale photography.

The stereoscopic digitizing system gives a standard error of ±1.5 ft at the control points. This compares well with the expected accuracy of ±1.3 ft for a measurement accuracy of 0.01 inches on the photograph. The standard error of ±2.8 ft for stereoscopic digitizing system against ±2.2 ft. for the monocomparator (Table 3), the standard error of ±1.8 ft. for stereoscopic digitizing system against ±1.1 ft for the monocomparator (Table 4), and the standard error of ±3.96 ft for stereoscopic digitizing system against ±2.02 ft for the monocomparator and of ±1.44 ft for the PG2 (Table 5) indicate that the stereoscopic digitizing system is a workable system. Its accuracy, of course, can be increased by using a more accurate digitizer and better viewing system such as a zoom stereoscope. It should be pointed out that

TABLE 3. COMPARISON OF COORDINATES FROM DIFFERENT METHODS

	S. digitized coordinate & sequential method with large-scale photograph (1:3000)	S. digitized coordinate & simultaneous method with large-scale photograph (1:3000)	Monocomparator coordinate & simultaneous method with small-scale photograph (1:6000)
Standard Error in comparison with PG2 (ft)	±3	±2.8	±2.2
	(estimated accuracy of Coordinate from PG2 is ±2 ft.)		
Estimated Standard Error at the control pts. (ft)	±4.5	±1.5	±0.2
Estimated Standard Error at the target pts. (69-100)	—	—	±2
Estimated Standard Error at the lot corners pt. (31-68)	—	—	±4

TABLE 4.

	S.D. System		Monocomparator System	
Point	Vx (ft)	Vy (ft)	Vx (ft)	Vy (ft)
54	−0.665	−0.694	0.044	0.224
55	−3.933	−0.944	−0.016	0.499
56	−4.092	0.541	−0.580	−0.056
64	2.052	0.012	0.416	−0.181
67	1.110	−1.069	1.071	0.040
68	−0.489	0.196	0.362	0.302
40	0.910	0.549	3.842	0.501
	$\sigma = \pm 1.804$ ft		$\sigma = 1.160$ ft	

the results of the PG2 are good mainly because of the resolution obtained in the viewing system.

Results in Tables 3 and 4 indicate that fence corners can be satisfactorily used for plotting and checking purposes; thus, targeting may not be required. Table 5 compares the lot sizes as given in the record of deeds to those obtained by inversing the coordinates of the lot corners obtained by all three methods.

CONCLUSION AND RECOMMENDATION

Photogrammetry can be used effectively in numerical cadastral survey if good control, targets, and large-scale photographs are used. The 1:6000 to 1:9,000 scale photographs can be used to determine the coordinates of the section, quarter, 1/16th corners, and center points together with the estimated positions of any lost corners or points. The 1:3000 scale photographs can be used to determine the coordinates of the lot corners and the positions of the lost corners. The lost corners can be restored by targeting the estimated point and an azimuth marker. In order to obtain maximum accuracy, not only the control points but also the corners and pass points should be targeted.

The 1:3000 scale photograph can be used effectively to check the accuracy of setting out a subdivision. This procedure may be used either by surveyors to check their work or by cities and counties to check the work of surveyors.

The stereoscopic digitizing system is very good to determine the coordinates of lot corners in an urban area. This is simple, inexpensive, and fast. The system does not require specialized operators.

Finally, it is necessary to emphasize that the coordinates of lots and section corners should be determined and stored so that they may be retrieved when required and plotted at any desired scale or used to compute area, etc. It is best if the responsibility to establish the coordinates of the section and quarter corners be assigned to the Bureau of Land Management or United States Geological Survey and the responsibility of establishing the coordinates of the lot corners be assigned to the cities and/or counties.

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TABLE 5. COMPARISON OF INVERSED DISTANCES

Line	Wang Digitizer Data		Comparator Data		PG2 Plotted Data		Record Distance in feet
	Dist. (ft)	Vd (ft)	Dist. (ft)	Vd (ft)	Dist. (ft)	Vd (ft)	
68-67	75.40	−3.40	77.45	−5.45	71.07	0.93	72.00
67-66	66.98	0.02	66.22	0.78	68.03	−1.03	67.00
66-65	69.92	−2.92	66.60	0.40	68.06	−1.06	67.00
65-64	65.31	1.69	66.48	0.52	67.04	−0.04	67.00
40-41	74.80	−2.80	76.17	−4.17	73.10	−1.10	72.00
41-43	67.54	−0.54	66.66	0.34	66.04	0.96	67.00
43-45	69.51	−2.51	67.21	−0.21	68.22	−1.22	67.00
53-54	78.21	1.79	81.58	−1.58	82.05	−2.05	80.00
37-36	66.45	1.21	67.77	−0.11	68.06	−0.40	67.66
36-35	89.25	−9.25	80.49	−0.49	90.87	−10.87	80.00
33-32	62.06	8.94	72.98	−1.98	71.04	−0.04	71.00
40-68	110.02	−0.02	109.30	0.70	108.10	1.90	110.00
41-67	109.44	0.56	109.30	0.70	107.06	2.94	110.00
34-50	113.48	−3.48	111.19	−1.19	108.07	1.93	110.00
54-55	68.87	0.13	69.14	−0.14	69.06	−0.06	69.00

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