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Texture Perception and the RADC/Hsu Texture Feature Extractor

Such analytic approaches can help provide clearer criteria for defining "Perceptually-based" automated analysis of remotely sensed data.

ABSTRACT: *Some of our initial attempts to relate digitized image measurements to the visual texture recognition process are presented. Specifically, we have compared human similarity/difference judgments of textural patterns based on real-world images with outcomes of the RADC/Hsu machine texture analysis which employs local statistics from small (e.g., 3 × 3) moving pixel windows. Such comparisons included the use of non-metric, multidimensional scaling techniques which allow the construction of stimulus dimension models for human and machine processes using micro-texturally common and specifiable image conditions. Our results indicate that such analytic approaches can help provide clearer criteria for defining "perceptually-based" automated analysis of remotely sensed data and the conditions which produce individual differences in the weighting of stimulus dimensions when judging differences among visual patterns. The data also corroborate the idea that a micro-texture approach to such problems is most appropriate since it can specify the building blocks which define complex configurations of given stimulus arrays "perceived" either by man or machine.*

INTRODUCTION

THE PERCEPTION of visual texture, though poorly understood, has long been recognized by aerial photo interpreters and psychologists as an important characteristic for the identification of objects and scenes (Avery, 1968; Gibson, 1950; Koffka, 1935; Reed, 1973). Recently, computer scientists, electrical engineers, geographers, and other scientists have vigorously engaged in physical/mathematical texture analyses of images. However, as shown in the literature reviewed by Rosenfeld (1975), Haralick (1975), Landgrebe (1978), and Hsu (1978), the bulk of the studies have centered on the development of texture measurements for mathematical discrimination of patterns. Few studies have attempted to relate these digitized image measurements to the visual texture recognition process (Mitchell *et al.*, 1977; Tamura *et al.*, 1978; Hsu, 1978; Hsu and Burright, 1979), although efforts have been made regarding texture perception by humans (Lipkin and Rosenfeld, 1970; Pickett, 1970; Ginsburg, 1973; Pollen and Taylor, 1974; Pribrum, 1974; Rosenfeld, 1975; Richards, 1978).

PHOTOGRAMMETRIC ENGINEERING AND REMOTE SENSING,
Vol. 46, No. 8, August 1980, pp. 1051-1058.

Using "random-dot" patterns, and a matching procedure analogous to that employed in human colorimetry, Richards (1978) has recently shown that visually equivalent textures (metamers) can be achieved by appropriate manipulations of a set of 3-5 "primaries". For instance, he has shown that the texture of a random-dot pattern with 63 greytone levels is not perceptually different from that of a pattern consisting of only three greytone levels. Obviously, the human visual system involves certain filtering processes. However, the generalizability of Richards' results to real-world pattern recognition and of machine texture analyses to human perception is poorly understood. This paper presents some of our initial attempts to address such questions more directly. Specifically, we have compared human similarity/difference judgments of textural patterns based on real-world images with machine texture measurement outcomes developed using local statistics from moving 3×3 and 5×5 pixel windows as employed in the RADC/Hsu texture analysis (Hsu, 1978). Such comparisons include the use of non-metric multi-dimensional scaling techniques (Takane *et al.*, 1977), which enable us to construct models for human and machine processes using microtexturally common and specifiable image conditions.

A Short Review of "Perceptually-Based" Texture Feature Extractors

Among the texture measures developed for image processing by machine, a few have been termed "perceptually-based"—but, for obvious reasons, such terminology certainly should be considered debatable at present. This section reviews briefly Mitchell/Myers/Boynes's Max-Min Descriptor (Mitchell *et al.*, 1977), Tamura/Mori/Yamawaki's texture feature extractor (Tamura *et al.*, 1978), and the RADC/Hsu texture measurement system (Hsu, 1978).

Based on Mitchell's earlier work (Mitchell, 1976), Mitchell *et al.* published their Max-Min Descriptor in 1977. Their texture parameters were obtained from the number of peaks (Max) and troughs (Min) along a scan line using several thresholds; e.g., given three threshold settings, three parameters based on the sum of peaks and troughs provided three texture measurements. This texture descriptor has been considered perceptually-based because it was inferred from the psychophysical literature that the human visual system tends to respond to local extremes. This texture feature extractor also has been tested against Haralick's grey-tone co-occurrence method (Haralick *et al.*, 1973), and shown to be equally effective for machine discrimination of patterns; however, the Max-Min descriptor is computationally much simpler.

Unlike Mitchell/Myers/Boyne's intuitive approach, Tamura *et al.* (1978) attempted to develop a set of complicated texture measurements from a relatively large group of pixels (128×128) which were supposedly visually identifiable texture features such as coarseness, contrast, directionality, line-likeness, regularity, and roughness—a macro-texture approach. Human experiments also were conducted with often used textural patterns produced in Brodatz' (1966) photographic album of textures. The authors indicated that their perceptually-based texture feature extractor did not perform well in similarity judgment tasks.

To investigate the relationship between the human performance and a machine solution regarding similarity judgments of texture patterns as revealed in choropleth maps, Hsu (1974) devised a ten-variable texture measure coupled with a normal model classifier to analyze differences (in terms of Mahalanobis D^2) among map surfaces. These variables were extracted from the wave-form parameters of both x and y axis scan lines, and involved (1) area above datum, (2) area below datum, (3) sum of the peak positions from origin, (4) sum of contrast values from peaks to troughs, and (5) sum of the number of peaks and troughs. Since a very high coefficient of correlation ($r = 0.97$) existed between the distances judged by human subjects and the machine solution (D^2), this ten-variable system was viewed as perceptually-based.

The RADC/Hsu Texture Feature Extractor/Classifier System

As reported in 1978, Hsu (under the sponsorship of U.S. Air Force/Rome Air Development Center) developed a new texture measure with 17 and 23 variables derived from a 3×3 and a 5×5 moving grid, respectively. The original (Hsu, 1974) five wave-form parameters were included in this sytem. This texture feature extractor has been shown to be highly effective; e.g., in reference to ground-truth information, a hit-rate of 85 to 90 percent has been obtained regarding land-use analysis from digitized, panchromatic images (Hsu, 1977).

The major difference between the ten-variable wave-form system (Hsu, 1974) and the 17-23 variable system (Hsu, 1978) is that the former was based on a concept of

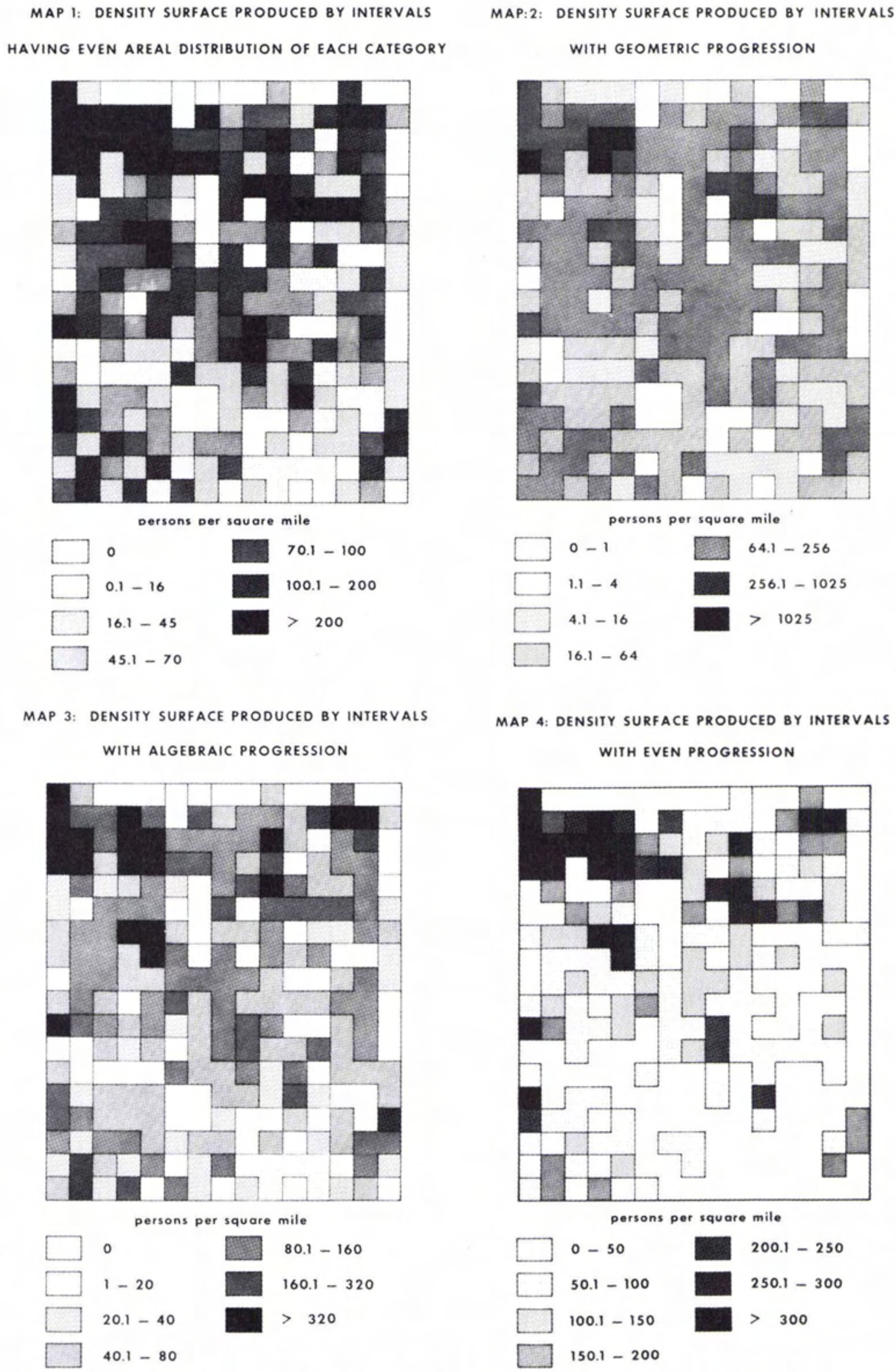

FIG. 1. The four choropleth maps which represent population density patterns as scaled by four different class-interval systems (see text).

macro-texture analysis, whereas the latter is derived from a micro-texture approach. That is, the latter system uses a moving grid (3×3 or 5×5 pixels) where the center-point is treated as the control point representing characteristics of the relatively small control (grid) area. With this control point/control area concept, we are able to generate a vector of texture variables for a single pixel, thus allowing us to perform a pixel-by-pixel classification task with black-and-white image data. Indeed, we believe that machine similarity measurements, especially if they are expected to relate generally to human perception, should be made on micro-textural features instead of visually apparent macro-textural features which already have been subjected to largely unknown and labile integrative processes (*cf.*, Kolers, 1972).

Perceptual Analyses of the RADC/Hsu Texture Measure

Experiment 1

To perform perceptual analyses with human subjects regarding similarity judgments, four choropleth maps were made showing population density patterns as scaled by four different class-interval systems (Figure 1). In the first experiment, ten naive human observers (cartography students) were asked to estimate the visual differences in all six of the possible double-map comparisons; e.g., map 1 vs map 2, map 1 vs map 3, etc. The allowable scale ranged from 0 (no perceptual difference) to 10 ("extremely different"). Table 1 summarizes the results in a symmetrical dissimilarity matrix of mean judged differences on the 10-point scale—standard deviations are given in parenthesis.

As indicated in Table 1, the map 1 vs 4 and map 2 vs 4 pairs were judged most different. The map pair judged least different, on average, was the map 1 vs 3 comparison.

Such a perceptual analysis of these map-similarity judgments is indeed an analog of a statistical classification logic utilizing a minimum distance criterion. Thus, a direct comparison between this perceptual analysis and a statistical discriminant analysis based on the machine feature extractor/classifier was attempted. To provide data for such a comparative analysis, the statistical distances between the same six pairs of maps were computed using the 10 waveform parameters as response variables. Here, the texture variables were obtained from scan lines on both the x and y axes. The macro-texture of these four maps were subsequently represented by four, 10×13 matrices, one for each map. The numbers of scan lines correspond with the rows and columns of the choropleth maps.

Discriminant analysis is precisely the statistical technique that can be used to assess the distances among these data matrices, and to determine whether the separation between two surfaces is statistically significant (Morrison, 1976). Table 2 shows results of this normal model machine solution in a symmetrical dissimilarity matrix analogous to the matrix of perceptual results given in Table 1.

The degree of correspondence between the average human perceptual judgments and the statistical discriminant analysis of the machine data was assessed by calculating a Pearson correlation coefficient. Using the data from Tables 1 and 2, the obtained coefficient is very high indeed ($r = 0.95$).

Experiment 2

Since we developed a texture feature extractor capable of analyzing the micro-texture of individual pixels using a 3×3 moving grid, we proceeded to determine how closely this 17-variable system correlated with human perceptual judgments. In addition, we wanted to know whether we could use only 3-5 of the 17 variables in this system to achieve a comparable level of performance. While such a 3-5 variable system would obviously result in reduced computer time (see below), it also is interesting

Table 1. Dissimilarity Matrix (Symmetrical) of Mean (and Standard Deviation) Perceptual Judgments

MAP	1	2	3	4
1		4.7 (1.6)	3.2 (2.4)	7.7 (2.2)
2			4.3 (1.1)	7.9 (1.3)
3				6.0 (1.8)
4				

Table 2. D^2 Distances Between Map Pairs—Normal Model Machine Solution (Macro-texture)

MAP	1	2	3	4
1		6.64*	1.05	13.45*
2			2.87	16.95*
3				12.84*
4				

[*$p < 0.01$—$F = 3.16$, df = 10, 25.]

to recall that Richards (1978) has reported that 3-5 "primaries" can produce texture metamers in visual matching of "random-dot" patterns by human observers.

To compensate for the potential loss of power in the feature extractor by using only 3-5 variables, and/or to better reflect the characteristics of the distributions of digitized image information (*cf.*, Hsu, 1978), we developed a non-linear classifier based on the stable distribution model which is still capable of ultimately employing the Mahalanobis D^2 as a quantitative distance measure (Hsu and Klimko, 1979). Compared with the normal distribution model, the stable distribution has four (instead of two) basic parameters, and is capable of handling both non-normal as well as normal distributions. Experiments with this stable model classifier have shown that the needed number of texture variables for a machine solution comparable to that obtained with the original, 17-variable normal model classifier is typically drastically reduced to about 3: e.g., stable distributions of the mean, first-neighbor contrast, and second-neighbor contrast. As a result, the data processing time for the same number of points (256×256) was reduced to 15 minutes from 90 minutes of CPU time using standard FORTRAN.

To assess the degree of correspondence between the human visual system and this newly developed machine processing system, we replicated the perceptual test discussed in Experiment 1 utilizing the same four choropleth maps, but ten different, naive observers (again, graduate and undergraduate Geography volunteers at SUNY-Binghamton). The judgments in replications 1 and 2 were quite comparable (r for first and second replications means = 0.92), and we pooled the set of 20 human observations to yield the mean (and standard deviation) results shown in Table 3, which is directly comparable to Table 1.

Comparable to Experiment 1, we also computed the distance between map pairs using the stable Mahalanobis classifier with only three tone-texture variables: mean brightness, 1st neighbor contrast, and 2nd neighbor contrast. Since individual matrices, instead of a pooled dispersion matrix, was utilized in the analysis, the Mahalanobis D^2 distances in the dissimilarity matrix are not symmetric (see Table 4). The upper diagonal D^2 values represent row to column comparisons and the lower diagonal D^2 values indicate column to row comparisons. The differences may be analogous to influences of orientation on human judgments, but these matters deserve further study. In these studies, the maps were oriented for human judgments as they are presented on these pages. However, to correlate this set of machine outcomes to the perceptually-judged scores, we initially employed the upper off-diagonal stable distribution solution. Other aspects of the asymmetric machine solution pattern will be considered below.

A product-moment correlation of $r = 0.96$ was obtained using the upper diagonal D^2 values in Table 4 and the average of the 20 human judgments (Table 3). Using the normal distribution machine solution for these four maps (Table 2), and the means of the 20 human judgments, the correlation is 0.98. The rank order correlation between the normal distribution machine solution and the human observations is perfect, as is the rank order correlation between the upper and lower diagonal stable distribution solutions. In terms of rank order, the normal solutions and the human judgments are very closely (but not perfectly) related to the upper and lower diagonal stable distribution solutions. Clearly, the outcome of our three-variable feature extractor/classifier also is highly correlated with human judgments, and provides another indication that our texture-tone machine analysis method may provide some insight into the intricate relationships between purely machine-based and perceptually-based pattern recognition systems.

TABLE 3. A SYMMETRIC DISSIMILARITY MATRIX FOR PERCEPTUAL JUDGMENTS BASED ON 20 HUMAN OBSERVERS (0–10 SCALE)

MAP	1	2	3	4
1		4.72	2.98	7.30
		(1.46)	(1.82)	(1.83)
2			4.12	7.70
			(1.68)	(1.37)
3				6.68
				(1.68)
4				

TABLE 4. THE ASYMMETRIC DISSIMILARITY MATRIX (D^2) FROM THE 3-VARIABLE FEATURE EXTRACTOR AND NON-LINEAR (STABLE) CLASSIFIER MACHINE SOLUTION

MAP	1	2	3	4
1	0	1.9	0.3	3.0
2	1.7	0	0.6	4.3
3	0.5	0.7	0	3.3
4	4.3	12.5	6.0	0.7

EXPERIMENT 3

In the next experiment, we decided to further examine the relationship between the 20 human perceptual judgments and the two machine solutions (normal and stable distribution models), and to determine how the machine solutions relate to a two-dimensional space derived by the non-metric individual difference scaling technique recently developed by the Psychometrics Laboratory at the University of North Carolina (*cf.*, Takane *et al.*, 1977).

First, we converted the entire three sets of data (human, normal model, and upper-diagonal stable model) into z-scores based on a common scale of 0-10 as used by the human observers. This was accomplished directly for the judgments of each individual human observer, and by considering the D (not D^2) values of each machine solution, and then assigning appropriate values relative to a maximum $D = 10$. These standardized dissimilarity scores are presented in Figure 2, with the x-coordinate as map pairs and y-coordinate as the z-scores. Standard errors for the mean human judgments ranged between 0.09 and 0.21 on this z-scale. The similarities among configurations of these standardized dissimilarity distances between map pairs by the three solutions, as expected by the correlations already reported, is quite striking.

To determine a framework in which the human and machine "judgments" of similarity among these map pairs might be viewed, we decided to use non-metric scaling procedures (*cf.* Hake and Rodman, 1966). Employing the multi-dimensional scaling technique developed by Takane *et al.* (1977), we obtained a two-dimensional model using the dissimilarity matrices generated by each of the 20 human subjects, plus those obtained from four machine solutions defined by the normal model as well as by the upper diagonal, lower diagonal, and upper plus lower averages of the stable model.

The two-dimensional model derived by this alternating least-squares method using the 24 dissimilarity matrices as defined above is presented in Figure 3. Dimension I (the x-axis) orders our four map stimuli as follows: Map 4, Map 1, Map 3, and finally Map 2. Since Map 4 is lightest, and the average greytone becomes darker following the map order along this dimension, it seems reasonable, at least tentatively, to label Dimension I as a "tone" dimension.

Dimension II (the y-axis) of the derived stimulus space orders our maps as Map 1, Map 3, Map 4, and finally Map 2. Since these maps were made from the same data set by systematically varying the class-interval used, we are able to describe the nature of each pattern quite accurately (*cf.* Hsu, 1974). For instance, Map 1 was produced by requiring that each class have the same areal distribution (equal area system); therefore, among all four maps, Map 1 should have the highest neighbor contrasts or the highest frequency of greytone changes between neighboring cells. In this regard, Map 3 is

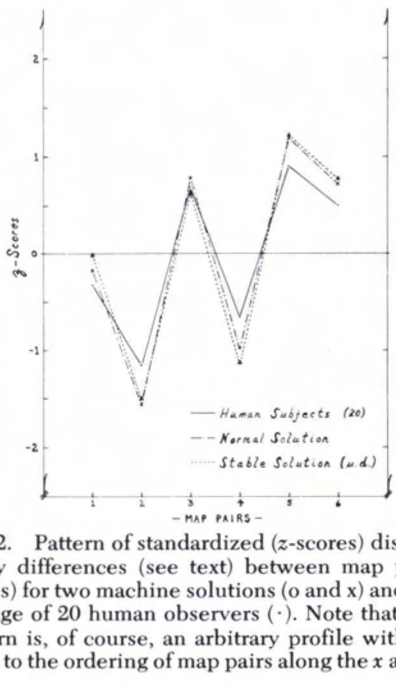

FIG. 2. Pattern of standardized (z-scores) dissimilarity differences (see text) between map pairs (x-axis) for two machine solutions (o and x) and the average of 20 human observers (·). Note that this pattern is, of course, an arbitrary profile with respect to the ordering of map pairs along the x axis.

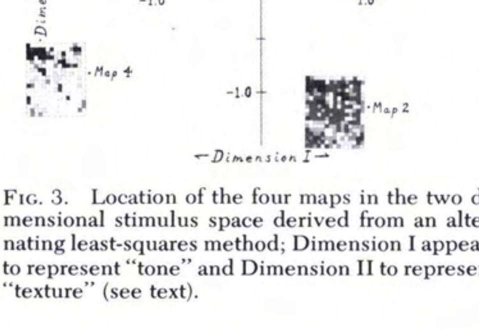

FIG. 3. Location of the four maps in the two dimensional stimulus space derived from an alternating least-squares method; Dimension I appears to represent "tone" and Dimension II to represent "texture" (see text).

almost the same as Map 1 since their class-interval systems vary only slightly. In contrast, Maps 2 and 4—at the "other end" of Dimension II *re* Maps 1 and 3—used class-interval systems which necessarily resulted in greytone patterns which produce relatively little contrast between and among *neighboring* cells. Thus, comparatively, the near neighbor contrasts in Maps 2 and 4 are considerably less than those displayed in Maps 1 and 3, and may be considered perceptually less "busy" or texturally less complex. Dimension II might reasonably be considered a "texture" dimension. However, it should be noted that it is doubtful that texture can be fully described, *in general,* along a single dimension (see above).

The individual differences scaling model employed enables us to examine how each of the 24 dissimilarity matrices (20 human observers, plus four machine solutions) weighted the importance of the two derived stimulus dimensions. All 24 of these weight vectors are plotted in Figure 4, with human observations depicted by dots, and the four machine solutions identified appropriately; the two coordinates represent the weights on Dimension I ("tone") and Dimension II ("texture"), respectively.

From Figure 4, it can be noted that all of the individual decisions are distributed very nearly along an arc of radius 1.0 in this weighting space. Any point on such an arc represents a perfect fit to the two-dimensional "tone-texture" model derived; the further a point is from this arc, the greater the stress (*cf.* Takane *et al.,* 1977) of that individual's judgment for the model. Clearly, there are distinct individual differences of the weightings in this model space,

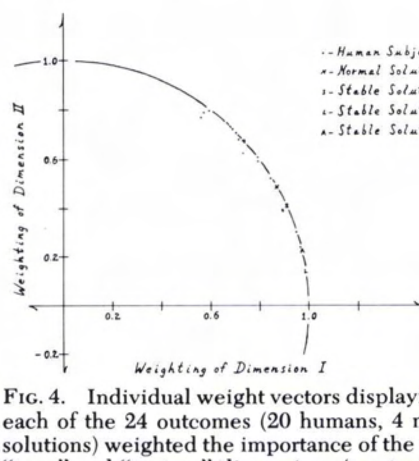

FIG. 4. Individual weight vectors displaying how each of the 24 outcomes (20 humans, 4 machine solutions) weighted the importance of the derived "tone" and "texture" dimensions (see text).

both among humans and among the machine solutions; but few of the points stress the model very highly. Interestingly, the bulk of the human observations tended to weigh the "texture" dimension somewhat more than the machine solutions did. Of the 20 human observers, six (almost 30 percent) yielded relatively high stress measures (>0.15) for the derived model, seven gave moderate stress scores (0.05 < stress < 0.15) and seven provided very little stress (≤0.01). Of the four machine solutions included in the creation of the "tone-texture" model presented, the stress value yielded by the normal distribution solution was 0.016, whereas all three of the stable distribution solutions had stress values of virtually zero.

Of course, ideally we would like to be able to establish *a priori* models of stimulus dimensions based strictly upon either human or machine solutions alone, and then determine how individual results relate to such models. Unfortunately, our present version of the Takane *et al.* scaling program does not perform this procedure appropriately, but such an approach is possible. In addition, the generality of our findings must be more fully explored. For instance, the four maps employed in these studies were created by varying class intervals and providing each stimulus with a total of seven greytone values. We are currently investigating patterns derived from real-world images which necessarily have different levels and numbers of levels of greytone values. Furthermore, additional human observers must be examined to determine if and how different perceptual models or dimensional weightings may appropriately characterize different sub-populations of subjects and/or viewing conditions.

Nonetheless, we believe that the types of analytical approaches utilized in the present series of experiments will continue to provide us with new and better insights regarding questions of what constitutes a perceptually-based system for automated analysis of remotely-sensed data, as well as a clearer appreciation of what human perception of visual textures involves, thus serving to bridge the gap between basic research with "random-dots" and the applied work of photo interpretation. Finally, we also wish to emphasize that a micro-texture approach to such problems is most appropriate since it can specify the basic building-blocks which define the complex structural configurations, in space (and time), of given physical stimulus arrays that are "perceived"—either visually, or by machine.

ACKNOWLEDGMENT

The authors wish to express their thanks to Ms. Jane Huang and Mr. Timothy Masters for their assistance in conducting the perceptual and computer analyses and the support from USAF/Rome Air Development Center, USAF Office of Scientific Research, and SUNY/Research Foundation.

REFERENCES

Avery, T. E., 1968. *Interpretation of Aerial Photography*, Burger's Publishing Company, Minneapolis, Minn.

Brodatz, P., 1966. *Textures: A Photographic Album for Artists and Designers*, Dover, New York.

Gibson, J. J., 1950. *The Perception of the Visual World*, Houghton, Mifflin, New York.

Ginsburg, A., 1973. Pattern Recognition Techniques Suggested From Psychological Correlates of a Model of Human Visual System, *IEEE Proceedings*. NABCON Record, Dayton, Ohio, pp. 309-316.

Hake, H. W., and A. S. Rodman, 1966. Perception and Recognition in J. B. Sidowski (ed), *Experimental Methods and Instrumentation in Psychology*, McGraw-Hill, New York.

Haralick, R. M., 1975. A Resolution Preserving Textural Transform for Images, *IEEE Proceedings of the Conference on Computer Graphics, Pattern Recognition and Data Structure*, pp. 51-61.

Haralick, R. M., K. Shanmagam, and I. Dinstein, 1973. Textural Features for Image Classification, *IEEE Transaction on Systems, Man, and Cybernetics*, Vol. SMC-3, No. 6, pp. 610-621.

Hsu, S., 1974. A Mathematical Method for Measuring Visual Differences in Multiple Map Comparisons, *Proceedings, American Congress on Surveying and Mapping*, pp. 116-132.

———, 1977. *Texture-Tone Feature extraction and Analysis*, Rome Air Development Center, Air Force Systems Command, Griffiss Air Force Base, Rome, New York. (RADC-TR-77-279)

———, 1978. Texture-tone Analysis for Automated Land-use Mapping, *Photogrammetric Engineering and Remote Sensing*, Vol. 44, No. 11, pp. 1393-1404.

Hsu, S., and R. Burright, 1979. A Perceptually-based Texture Feature Extraction for Similarity Measurements of Imagery Data, *Proceedings of American Society of Photogrammetry*, pp. 136-145.

Hsu, S., and E. Klimko, 1979. *Texture-tone Feature Extraction and Analysis: Phase II*, Rome Air Development Center, Air Force Systems Command, Griffiss Air Force Base, Rome, New York. (RADC-TR-79-37)

Koffka, K., 1935. *Principles of Gestalt Psychology*, Harcort, Brace, New York.

Kolers, P. A., 1972. Reading Pictures: Some Cognitive Aspect of Visual Perception, in T. S. Huang and O. J. Tretiak (eds), *Picture Bandwidth Compression*, Gorden and Bread, New York.

Landgrebe, 1978. "The Development of a Spectral/Spatial Classifier for Earth Observation Data," presented at IEEE Computer Society Conference in Pattern Recognition and Image Processing, Chicago, Ill., May 31-June 2.

Lipkin, B. S., and A. Rosenfeld (eds), 1970. *Picture Processing and Psychopictorics*, Academic Press, New York.

Mitchell, O. R., 1976. Texture Edge Detection and Classification Using Max-min Descriptors, *Image Understanding and Information Extraction*, T. S. Huang and K. S. Fu, Contract No. F3-6-2-75-C-0150, Griffiss Air Force Base, Rome, New York.

Mitchell, O. R., Myers, C. R. and W. Boyne, 1977. A Max-min Measure for Image Texture Analysis, *IEEE Transaction on Computers*, pp. 408-414.

Morrison, D., 1976. *Multivariate Statistical Methods*, McGraw Hill, (2nd Edition), New York.

Pickett, R. M., 1970. Visual Analysis of Texture in the Detection and Recognition of Objects, a chapter in Lipkin and Rosenfeld (eds), *Picture Processing and Psychopictorics*. Academic Press, pp. 289-308, New York.

Pollen, D. A., and J. H. Taylor, 1974. The Striate Cortex and Spatial Analysis, in *The Neurosciences 3rd Study Program*, Schmitt, F. O. and F. G. Worden (eds), MIT Press, Cambridge, Mass.

Pribrum, 1974. How is it That Sensing so Much We Can Do So Little? in *The Neurosciences 3rd Study Program*, Schmitt, F. O. and F. G. Worden (eds), MIT Press, Cambridge, Mass.

Reed, S. K., 1973. *Psychological Processes in Pattern Recognition*, Academic Press, New York.

Richards, W., 1978. *Experiments in Texture Perception*, Final report Contract F44620-74-0076, Air Force Office of Scientific Research, Bolling Air Force Base, Sponsor: ARPA Order No. 2765.

Rosenfeld, A., 1975. *Visual Texture Analysis: An Overview*, Technical Report Series (TR-406, F44620-72C-0062) University of Maryland, College Park, Maryland.

Tamura, A., S. Mori, and T. Yamawaki, 1978. Textural Feature Corresponding to Visual Perception, *IEEE Transaction on System, Man and Cybernetics*, Vol. SMC-8, No. 6, pp. 460-472.

Takane, Y., F. W. Young, and J. de Leeuw, 1977. Non-metric Individual Difference Multidimensional Scaling: An Alternating Least Square Method With Optimal Scaling Features, *Psychometrika*, 42, 7-67.

(Received 16 July 1979; accepted 19 February 1980)