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Resolving the Percentage of Component Terrains within Single Resolution Elements

The approximate maximum likelihood method proved to be equivalent or superior to weighted average and linear regression techniques and permitted estimation of the total area occupied by component terrains with ± 6 percent of the true area covered.

Introduction

Natural terrain variations within satellite resolution elements will commonly produce a pixel containing two or more spectrally distinguishable classes. As methods for the analysis of Landsat scanner data have become increasingly sophisticated and the general understanding of terrain spectra more detailed, the spectral differentiation of the components of these large resolution elements has become an important aspect in the practical application of satellite radiometric data.

The potential of subpixel resolution of terrains was investigated as part of a detailed program to establish the quantitative relationship between the surface spectral character and that sensed by the Landsat multispectral scanner (MSS). One of the results of the study was the development of a simple approximation to a maximum likelihood technique for resolving the percentage of component terrains within single resolution elements. Five test cases were examined in which the percentages of the components within pixels were known exactly. Four of the cases involved Landsat multispectral data; in the fifth case, an Exotech (designed to simulate the four Landsat MSS Bands) radiometer mounted on a tethered helium balloon platform, was used to record the spectral reflectance from a circular res-

Abstract: *An approximate maximum likelihood technique employing a widely available (BMD) discriminant analysis program has been developed for resolving the percentage of component terrains within single resolution elements. The method employs all four channels of Landsat data simultaneously and does not require prior knowledge of the percentage of components in mixed pixels. For five test cases, the method proved to be superior to single band weighted average and linear regression techniques and permitted an estimate of the total area occupied by component terrains to within ± 6 percent of the true area covered. It is believed this accuracy should be sufficient for many geologic applications of Landsat multispectral data.*

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olution cell approximately 40 metres in diameter.

Previous Methods of Resolving Mixtures

The development of procedures to estimate the percentages of mixed populations have previously been developed for, and applied to, agricultural applications. The agronomist's need for crop-yield information was aided by the typical homogeneity of cultivated fields.

The Agricultural Research Service of the USDA (Weigand *et al.*, 1974; Richardson *et al.*, 1975) developed linear regression models for extracting plant, soil, and shadow reflectance components of cropped fields from Landsat MSS data. They developed a model which related fractional plant and shadow cover and leaf area index (LAI) for 23 planted fields to Landsat brightness. The model gave regression lines with multiple correlation coefficients (in each band) statistically significant at the 0.01 probability level. These regression models could then be used to determine the component reflectance of the plant, soil, and shadowed areas.

The Environmental Research Institute of Michigan (Horwitz *et al.*, 1971, 1975) developed a series of probabilistic models to achieve subpixel resolution for analysis of crop acreage. A complex maximum likelihood algorithm was developed, and was based upon the weighted combinations of component class mean vectors and covariance matrices. The method proved promising for proportion estimates where spectral separation was high. Similar efforts based upon component class mean vectors were begun by Detchmendy and Pace (1972) and Hallum (1972). Work and Gilmer (1976) have used the maximum likelihood approach to help inventory prairie ponds and lakes with Landsat MSS data.

Development of an Approximate Maximum Likelihood Technique

Geologically significant (natural) terrain mixtures have not previously been explored. Therefore, a study was undertaken to develop a method for resolving subpixel mixtures of geological rather than agricultural terrain. Mixtures significant to the geologist will generally be combinations of rock and soil partially obscured by vegetation. These mixtures change frequently, both spatially and spectrally. Therefore, a fast and simple method which would incorporate the four-band Landsat data was sought. Ideally, the method should allow relatively small areas to be extracted from the Landsat spectral reflectance data and to be analyzed for the mixture components known to exist in that area.

The method developed approximates a maximum likelihood technique but requires much less computation. It employs linear discriminant analysis to incorporate the four channel data into a single variable for analysis. The function transforms a multivariate set of measurements on a sample into a single discriminant value, which is a transformed variable representing an oblique projection of the sample onto the linear discriminant line. This line in the original four-space joins the multivariate means for the two classes of homogeneous pixels. Squared distances measured along the discriminant function line are called Mahalanobis or M-distances.

The multivariate position of the spectral signatures of a mixed-pixel in relationship to the multivariate positions of the mean homogeneous components of the mixture produces an approximate maximum likelihood estimate of the proportions of the components under multivariate normality conditions. However, even under quite general conditions these proportion estimates may be very reasonable in the same sense that discriminant analysis is a useful tool under general conditions. The proportion estimates are given by the formula

$$P_y = 0.5 + 0.5\frac{d(m,x) - d(m,y)}{d(x,y)} \qquad (1)$$

where

- P_y = proportion of component Y in the mixed-pixel;
- $d(x,y)$ = the M-distance between the mean homogeneous components X and Y;
- $d(m,x)$ = the M-distance between the mixed-pixel (m) and the homogeneous component X;
- $d(m,y)$ = the M-distance between the mixed-pixel (m) and the homogeneous component Y;
- P_y = 0, if the estimate is negative; and
- P_y = 1, if the estimate is greater than 1.0.

Discriminant analysis programs are widely available. The stepwise discriminant analysis program (BMD-07M) developed by the Health Sciences Computing Facility at UCLA (Dixon, 1973) was used in the study. The developed technique involved extracting the four Landsat radiance values for pure-pixels (i.e., those made up entirely of one component of the mixed-pixels). These radiance values (DN) are converted to a satellite equivalent reflectance (R) based upon a standard light and dark target conversion (Lyon *et al.*, 1975). Generally, between 10 and 40 pure pixels were extracted for each component. The pure component data sets were used together with those of the mixed-pixels for which a proportion estimate was desired. The mixed-pixels are called "test cases" (Dixon, 1973), so that they are not involved in the calculation of the discriminant function but are subsequently used for validation purposes. The M-distance between the mean homogeneous components $(d,(x,y))$, and between each mixed-pixel and the homogeneous component X $(d(m,x))$, and between each mixed-pixel and the homogeneous component Y $(d(m,y))$ are calculated as standard products of BMD-07M. A simple program was then written to accept the M-distances as a data set and to calculate the percentage of a component in each mixed-pixel based upon Equation 1.

Comparison of Subpixel Resolution Methods

In order to assess the applicability and accuracy of the approximate maximum likelihood method (AML), the technique was compared to a solution of a general weighted average equation and to a linear regression technique.

The first method uses the mean satellite spectral reflectance for homogeneous pixels (representative of the components of the mixed-pixel and its reflectance) to solve the simple linear weighting equation for the proportion of one of the components. The weighted average equation for a two-component mixture for any single Landsat band can be written

$$M_i = (1 - P_y)\, X_i + (P_y)\, Y_i \qquad (2)$$

where

X_i = mean spectral reflectance for component X in band i;
Y_i = mean spectral reflectance for component Y in band i;
M_i = mean spectral reflectance for the mixed-pixel in band i; and
P_y = proportion of component Y in the mixed-pixel.

By rewritting Equation 2, we can solve for the proportion of a component within the mixed pixel

$$P_y = \frac{M_i - X_i}{Y_i - X_i}. \qquad (3)$$

This method will yield a proportion estimate for component X or Y in each of the four Landsat bands. Because the four bands may yield disparate proportion estimates, depending upon the spectral separability in each band, the method will indicate the best bands for resolving mixtures. However, deciding which band has the most accurate results is impossible when the correct proportion is unknown.

The weighted average equation can also be related to a linear regression model. Rewriting Equation 2,

$$M_i = (Y_i - X_i)\, P_y + X_i, \qquad (4)$$

we see that the equation is now a simple linear regression expression in which the slope is the difference between the component spectral signatures $(Y_i - X_i)$, and the intercept is the homogeneous reflectance component of one material, (X_i). Use of this method requires mixed-pixels in which the proportions of component terrains are known. The method also produces four equations, one for each Landsat band, which may give different results.

Landsat Data

To compare the maximum likelihood, weighted average, and linear regression methods, and to assess their accuracy, a Landsat data set was extracted in which the proportion of components within mixed-pixels could be determined exactly. The phreatophyte vegetation mounds present on Garfield Flat, a clay-silt playa (3.5 × 1.5 km) located about 35 km southeast of Hawthorne, Nevada, afforded ideal mixed-pixels for study. The width of these mounds is less than the width of a Landsat pixel. Thus, there are numerous pixels which exhibit varying percentages of playa and vegetation.

Low altitude aerial photography of the playa was used to create a 1:12,350 scale mosaic of the Flat. A Landsat orthogonal grey-scale representation (Dotprint) of the area was then created at this same scale (Ballew and Lyon, 1977) and overlain on the photomosaic. The area covered by vegetation in 17 mixed-pixels was outlined and enlarged to permit accurate planimetric measurements of the exact area covered. These known mixed-pixels were then used in the analysis.

The linear regression technique used a split mixed-pixel data set in which nine of the 17 mixed-pixels were used to calculate the regression equation to relate percent vegetation to satellite equivalent reflectance (R) in each band. The remaining eight mixed-pixels were then used to determine the accuracy of the regression for each band.

The approximate maximum likelihood method (AML) was performed by first finding pure-pixels of the homogeneous components of the mixture. Forty pixels of unvegetated playa from the areas surrounding the vegetation mounds, and 40 pixels of pure phreatophyte vegetation from that bordering the playa were extracted. The discriminant analysis was run and the percentage vegetation in each mixed-pixel was calculated with the program based upon Equation 1.

The simple weighted average equation (Equation 3), employing the mean spectral reflectance for the 40 pure vegetation and 40 pure playa pixels, was then used to calculate the percent vegetation for each mixed-pixel reflectance value in each band.

The results of the three methods for the remaining eight mixed-pixels available for comparison are given in Table 1, with the actual percent vegetation in each pixel. The root-mean-square error (RMSE) was employed to test and compare the accuracy of the three methods. The approximate maximum likelihood method gave the best overall results. A RMSE of approximately ±6 percent error in calculated vegetation percent cover was determined. The individual calculations based upon the weighted average equation in each band yield RMSE values slightly worse in bands 4, 5, and 6 and essentially equivalent in band 7 to the AML method. Overall, the linear regression method was the most inaccurate of the three techniques. Bands 4 and 5 are slightly inferior, while bands 6 and 7 are significantly inferior. These results, in general, conform to the expected accuracy based upon the correlation coefficients in these bands. The inferior results in bands 6 and 7 are due to the low spectral separability between the components in these wavelengths.

To further determine the accuracy of the methods, three additional Landsat data test cases were developed. The cases were chosen to represent mixtures of outcrop, soil, and vegetation which would typically be encountered in geologic studies with Landsat data. Successful resolution of rock outcrop from soil or vegetation within pixels would, for the first time, allow construction of outcrop area contours, as well as to serve as an aid to supervised classification schemes by establishing the reflectance characteristics of known-mixture classes. The three cases represent areas with mixtures of soil, vegetation—generally greasewood (Sarcobatus baileyi) and sagebrush (Artemisia tridentata)—and outcrop. The test cases in the Yerington area of western Nevada were

(1) Mason Butte—An area of pure- and mixed-pixels of granodiorite outcrop covered by vegetation, greasewood (S.b.) and shadscale (Atriplex confertifolia), and sedimentary soil covered by the same vegetation. A set of pure-pixels and four mixed-pixels were extracted for this site from a 1:24,000 Dotprint and orthophoto of the area.

(2) Wabuska Knob—Pixels of a vegetated (S.b., A.t.) alluvial soil and a similarly vegetated sedimentary soil with abundant talus of welded ash-flow tuff were extracted.

(3) Lincoln Flat—Here the components of

TABLE 1. COMPARISON OF METHODS FOR RESOLVING SUBPIXEL MIXTURES USING LANDSAT EQUIVALENT REFLECTANCE (R) DATA FOR CALCULATING THE PERCENTAGE OF VEGETATION WITHIN MIXED-PIXELS, GARFIELD FLAT, NEVADA

	weighted avg. eq. (%)				linear regression (%)					
Case	4	5	6	7	4 r0.79	5 0.77	6 0.50	7 0.82	AML (%)	Actual (%)
1	24.4	22.3	29.5	26.1	27.8	24.6	32.0	27.0	25.5	23.0
2	38.2	36.8	31.2	36.1	39.2	37.7	34.5	38.5	36.0	36.0
3	32.9	35.0	42.4	47.1	34.9	36.0	51.0	44.2	36.5	38.5
4	29.6	27.8	27.8	31.1	32.2	29.5	29.5	32.7	29.0	36.0
5	38.2	35.0	33.4	36.1	39.2	36.0	37.8	38.5	36.0	41.0
6	24.4	24.1	22.2	21.1	27.8	26.3	21.2	21.3	23.5	15.5
7	63.2	65.8	62.7	66.0	59.9	63.8	80.7	72.9	64.0	54.0
8	52.0	49.5	40.8	46.1	50.7	49.1	48.5	49.9	48.0	43.5
RMSE	6.79	7.23	6.90	6.69	6.57	6.79	12.3	8.48	6.19	

r: regression coefficient of determination
AML: Approximate Maximum Likelihood
RMSE: root-mean-square error

TABLE 2. RESULTS FOR THE APPROXIMATE MAXIMUM LIKELIHOOD METHOD (AML) FOR RESOLVING SUBPIXEL MIXTURES WITH LANDSAT REFLECTANCE DATA

	Garfield Flat % vegetation		Mason Butte % outcrop/veg		Wabuska Slope % talus soil		Lincoln Flat % tuff	
Case	Actual	AML	Actual	AML	Actual	AML	Actual	AML
1	36.0	32.0	10	2.0	50	53.5	35	28.0
2	23.0	25.5	30	29.5	45	40.0	50	51.0
3	38.5	38.0	40	42.8				
4	36.0	36.0	50	53.7				
5	31.0	29.0						
6	28.5	36.5						
7	33.5	23.0						
8	25.5	16.0						
9	36.0	29.0						
10	30.5	31.0						
11	41.0	36.0						
12	25.5	26.5						
13	15.5	23.5						
14	28.0	32.5						
15	41.0	40.5						
16	54.0	64.0						
17	43.5	48.0						
RMSE:	5.97		5.35		6.10		7.00	

the mixture are areas of white unwelded tuff with minor vegetation and a vegetated (S.b., A.t.) sedimentary soil derived from a local andesite.

The AML results for the three sites, along with the results for all 17 mixed-pixels at the Garfield Flat site, are given in Table 2. Remarkably, the root-mean-square error (RMSE) all fall within 5.5 to 7.0 percent.

The results appear to establish a relatively high degree of accuracy for the approximate maximum likelihood method for determining subpixel mixture proportions. To be within 6 percent of the actual percent of a component present within a pixel should be adequate for many geologic applications using Landsat spectral data. The degree of confidence in the actual percentages determined for the mixed-pixels is considered reasonable. However, because of some unavoidable spatial inaccuracies (approximately 1 percent) in the Dotprints, doubt could remain and the data may represent fortuitously successful results. To insure that the accuracy determined using the satellite data is genuine, the methods were tested on high resolution (balloon) data. Once the accuracy was firmly established, then the technique's application to Landsat data is unencumbered because the exact locations of only pure component pixels must be accurately determined.

BALLOON DATA

A tethered helium balloon system* (Marsh, 1978) mounted with an Exotech Landsat band radiometer (15° IFOV**) and automatic advance and exposure 35 mm camera was employed to measure near-surface reflectance of varying mixtures of terrain. A test site near the Stanford University campus at the Stanford Golf Course presented a viable terrain mixture of grass and sand. A sand trap surrounded by green grass was used as the target.

The procedure involved first recording a series of measurements of the pure components—sand and green grass. Four measurements of the sand and ten measurements of the surrounding grass were taken at an altitude of approximately 10 metres (FOV 2.5 m). Measurements of various

* The tethered helium balloon system, mounted with the Exotech radiometer and 35 mm automatic advance and exposure camera, was specially developed for the field research program by Julian H. Whittlesey (Whittlesey R & D, 29 Chicken Street, Wilton, Conn. 06897) under contract to the U.S. Geological Survey. Although originally designed for aerial photographic work over archaeological sites, the system worked well for acquisition of surface spectra.

** Instantaneous field-of-view.

mixtures of sand and grass were then recorded by successively raising the balloon from 50 to 200 metres and also by changing its position over the site. The color photographs taken simultaneously with the spectral readings were used to determine the exact height of the system from ground control points set at a known distance apart. Knowing the height of the radiometer-camera system, the exact area of the resolution cell could be calculated. The proportions of sand and grass within the field of view were then determined from planimetric measurements of the color photographs. The accuracy of this measurement was within 0.5 percent error.

The balloon flight produced a set of 16 mixed-pixels. The percent sand in these mixtures ranged from about 2 to 60 percent of the total area of the pixel. The data set was then divided in half, so that each subset represented the full range of sand percentages recorded. One set was used to calculate the linear regression equation in each band. The remaining eight mixture cells were used to compare the results of the linear regression equation with the 16 mixtures for the weighted average and maximum likelihood methods.

The linear regression calculation produced equations that had reasonable correlation coefficients in MSS bands 4 and 5. The correlation coefficients in MSS bands 6 and 7 were very poor. This was a result of the sand and grass having nearly the same spectral response in these wavelengths. Accurate percentages of sand or grass therefore could not be calculated in these bands. This same spectral similarity also precluded the determination of percent sand using the weighted average equation in bands 6 and 7.

The approximate maximum likelihood method was run employing just bands 4 and 5 (AML1) as well as all four bands (AML2); results for the three methods are compared in Table 3. These results indicated that the linear regression method in band 4 is slightly superior. However, the linear regression method in band 5 and the weighted average equations in bands 4 and 5 yield essentially equivalent results with the approximate maximum likelihood technique employing all four bands.

The spectral proximity of the sand and grass in bands 6 and 7 precluded their use in the weighted average and regression analysis. However, the maximum likelihood method is less accurate when these MSS bands are not incorporated in the analysis. The spectral data in bands 6 and 7 must contain information pertinent to resolving these mixtures. The AML method can exploit

TABLE 3. COMPARISON OF METHODS FOR RESOLVING SUBPIXEL MIXTURES USING BALLOON REFLECTANCE DATA FOR THE STANFORD GRASS AREA

Case	linear regression 4 r0.96	5 0.95	6 0.36	7 0.15	weighted avg. eq. 4	5	6	7	AML2	AML1	Actual
1	0	0	—	—	2.6	0	—	—	3.3	4.4	3.3
2	15.5	12.9	—	—	19.2	15.8	—	—	20.1	21.2	16.7
3	14.9	12.3	—	—	18.5	15.1	—	—	22.0	20.4	17.1
4	24.1	27.4	—	—	28.2	29.8	—	—	26.2	27.4	24.7
5	32.0	24.2	—	—	36.5	26.7	—	—	40.0	42.0	34.0
6	41.2	44.3	—	—	46.3	46.3	—	—	47.0	46.2	41.1
7	44.5	40.6	—	—	49.7	42.6	—	—	52.6	53.7	46.0
8	51.1	51.9	—	—	56.7	53.6	—	—	57.8	58.4	59.6
9					6.7	7.8	—	—	5.1	6.1	1.4
10					4.6	5.4	—	—	3.9	4.2	9.5
11					19.2	13.9	—	—	21.3	22.2	17.1
12					27.5	31.0	—	—	27.2	25.6	19.9
13					43.5	38.3	—	—	45.5	46.3	31.8
14					36.5	33.5	—	—	37.6	38.3	34.8
15					52.5	48.1	—	—	53.0	54.9	41.8
16					53.9	52.4	—	—	54.7	54.7	54.3
RMSE	3.70	5.98	—	—	5.45	5.47	—	—	6.20	6.79	

AML1: APPROXIMATE MAXIMUM LIKELIHOOD METHOD BANDS 4 and 5 ONLY
AML2: APPROXIMATE MAXIMUM LIKELIHOOD METHOD BANDS 4, 5, 6, 7
RMSE: root-mean-square error
r: regression coefficient of determination

TABLE 4. RESULTS FOR THE APPROXIMATE MAXIMUM LIKELIHOOD EXPERIMENT FOR MAPPING AREAS OF TWO-COMPONENT POPULATIONS

Total area of playa from map planimetric measurements	234,116 m^2
Maximum likelihood calculation:	
Total area classified pure playa	128,296 m^2
Total area classified playa from the mixed border pixels	101,885 m^2
Total area of playa	230,181 m^2
Discrepency	3,935 m^2 = 0.89 pixel

this information mathematically, while the other methods cannot make use of this information.

APPLICATION

The approximate maximum likelihood method can also be employed to perform more exact area determinations. A typical problem would be to define the exact area of a terrain feature with Landsat data. The AML method would allow for the percentage of the area of interest to be calculated within border pixels.

To test the reliability of the technique, the exact area of a small clay-silt playa in the Yerington Nevada area, which is surrounded by phreatophyte vegetation, was determined by planimetric measurements on the 1:24,000 orthophoto of the playa. The total number of pixels which are within and on the border of the playa, determined by overlaying the 1:24,000 Dotprint and orthophoto, were extracted. Thirty-five pure playa and 35 pure border vegetation pixels were then extracted and the discriminant analysis and percent calculations made for the area.

The total playa area determined from the maximum likelihood method is in error by only 3,935 m^2 (Table 4). This is an area of less than one pixel, or in terms of the total area of the playa, it is in error by less than 2 percent. This final example provides strong support for the viability and accuracy of the method.

CONCLUSIONS

The approximate maximum likelihood method proved to be equivalent or superior to weighted average and linear regression techniques and permitted estimation of the total area occupied by component terrains within ±6 percent of the true area covered. This accuracy should be sufficient for many geologic applications of Landsat multispectral data. The method employs existing and widely available discriminant analysis programs and, therefore, can be employed by any computer-based Landsat investigator.

A root-mean-square error of approximately 6 percent is considered valid justification for our asserting the accuracy of the method for resolving subpixel mixtures. The AML method must be considered logistically superior to a linear regression technique, because training sets of mixed resolution elements are not required. The AML method, by incorporating multiband data, also eliminates the uncertainty of determining which band yields the most accurate results, a problem inherent to a single-band weighted average technique.

The relationship of this error to a correct estimation of percentages for the Garfield Flat (Landsat) and Stanford Grass (balloon) test cases is given in Figure 1. The method, however, appears to consistently overestimate the pixel component percent of the darker materials (vegetation), and underestimate the pixel component percent of the

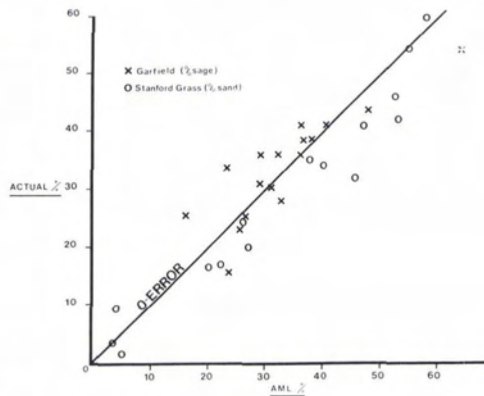

FIG. 1. Relationship of the error within the approximate maximum likelihood method (AML) to a correct estimation (actual) of terrain percentages for the Garfield Flat (Landsat) and Stanford Grass (Balloon) test cases.

brighter materials (sand). At this point we are uncertain if this is a true photometric effect or a mathematical bias in the approximate maximum likelihood method.

The success of the approximate maximum likelihood method has also indirectly provided insight into the performance of the multispectral scanner. Within the four Landsat test cases the mean contrast ratios between the spectral response of equally mixed-pixels and the spectral response of pure component pixels range from as low as 1.1 for the Wabuska Knob and Lincoln Flat sites to 2.0 for the Garfield Flat site. This indicates the AML method can resolve mixtures of similar spectral character, and most significantly, the scanner data is capable of portraying these mixtures within a single pixel.

As rock type, soil, and vegetation combinations change frequently, the simplicity of the AML method is particularly suited to geologic studies. However, employing existing discriminant analysis programming, the AML method can be used to resolve only two component mixtures. Modification of the discriminant analysis technique employing the Mahalanobis distances could be carried out to allow for several resolvable components.

The method, therefore, has applicability to two-component supervised classification schemes. By allowing the determination of the mixture of rock outcrop and/or soil and vegetation within a single pixel, the mean spectral signatures can be calculated for a variety of mixture classes. The spectral character of pixels with, say, 25, 50, and 75 percent vegetation could then be employed as training sets for detailed supervised multivariate discrimination algorithms.

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