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Direct Editing of Normal Equations of the Banded-Bordered Form

The influence of a particular object space point or image ray may be removed from a previously formed set of least-squares normal equations whose last relinearization has indicated convergence.

Abstract: *The development of an editing process by which the influence of a particular object space point or image ray may be removed from a previously formed set of least-squares normal equations whose last relinearization had indicated convergence is described. This process is an expansion of the sequential processing algorithm (Helmering, 1971). Whereas this previous algorithm was directed towards a strictly banded system of equations, the algorithm presented herein is developed for a banded-bordered system of equations. Additionally, this development is directed toward an off-line data reduction versus an on-line approach.*

Introduction

A currently accepted and adopted method of data reduction for photogrammetric triangulation is the bundle adjustment utilizing the Gaussian least-squares process (Brown, 1974). This process results in a method of iterative solutions converging to a point of minimum (near zero) parameter correction change. During each iteration, a new set of normal equations is formed, based on the most recent set of derived partial derivatives. Editing of erroneous data has previously been accomplished by one of two methods. First, the physical data pertaining to a point were deleted from the data set and the entire least-squares process begun again, including reformation of the normal equations. The second method is that of weighting factors, wherein the weight matrix for an erroneous point was altered, thereby *weighting out* the point by forcing a minimal contribution of that point. This method again required that the least-squares solution be initiated from the beginning.

In a banded-bordered system which contains photograph, internal camera, and ground point parameters, the formation of these equations is no trivial matter. Likewise, as data sets increase in volume and cameras become more and more sophisticated, the most time consuming process of a reduction becomes the normal equation formation. Thus, direct normal equation editing is particularly desirable and is designed to provide a vehicle by which editing can take place through normal equation altering and not by normal equation reformation.

Mathematical Development

Before discussing the editing process, a basic understanding of the initial formation of the normal equations shall be presented. The standard representation of a normal equation solution is expressed as

$$\boldsymbol{\delta} = \boldsymbol{N}^{-1}\boldsymbol{c}$$

where

$\boldsymbol{\delta}$ = vector of corrections,
$\boldsymbol{N}$ = normal equations, and
$\boldsymbol{c}$ = vector of constants.

Initial partitioning of these equations results in

$$\left[\begin{array}{c}\dot{\boldsymbol{\delta}}\\ \hline \ddot{\boldsymbol{\delta}}\end{array}\right] = \left[\begin{array}{c|c}\dot{\boldsymbol{N}} & \bar{\boldsymbol{N}}\\ \hline \bar{\boldsymbol{N}}^{\mathrm{T}} & \ddot{\boldsymbol{N}}\end{array}\right]^{-1}\left[\begin{array}{c}\dot{\boldsymbol{c}}\\ \hline \ddot{\boldsymbol{c}}\end{array}\right]$$

which defines the standard banded system of equations for which the initial sequential algorithm was developed (Helmering, 1971). Further par-

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TABLE 1. ELEMENT DEFINITION

Element	Accent	Parameter Set
$\dot{S}$	$\cdot$	Photo
$\mathring{S}$	$\circ$	Camera/System
$\ddot{N}$	$\cdot\cdot$	Ground Point
$\bar{N}$	$-$	Photo – Ground Point Correlation
$\breve{S}$	$\breve{}$	Photo – Camera/System Correlation
$\overset{\simeq}{N}$	$\simeq$	Camera/System – Ground Point Correlation

titioning of these equations defines the banded-bordered system for which this process was developed. This partitioning may be represented as

$$\begin{bmatrix} \dot{\delta} \\ \mathring{\delta} \\ \hline \ddot{\delta} \end{bmatrix} = \left[\begin{array}{ccc} \dot{S} & \breve{S} & \bar{N} \\ \breve{S}^{T} & \mathring{S} & \overset{\simeq}{N} \\ \hline \bar{N}^{T} & \overset{\simeq}{N}^{T} & \ddot{N} \end{array}\right]^{-1} \begin{bmatrix} \dot{c} \\ \mathring{c} \\ \hline \ddot{c} \end{bmatrix}$$

Table 1 further defines the elements of a banded-bordered system of normal equations with respect to parameter set contribution. To segregate the various parameter sets, a system of accents has been allocated. In order to expedite the normal equation formation and the editing process, a data stacking algorithm, *NO*n-*R*edundant *A*djustment (NORA), was implemented where the input data file was stacked by object point with each point record containing all basic information relative to that point. Principle parameters contained on a record are

- photos on which point appears;
- partial derivatives of image x and y with respect to photo parameters ($\dot{B}$);
- partial derivatives of image x and y with respect to camera parameters ($\mathring{B}$);
- partial derivatives of image x and y with respect to ground point parameters ($\ddot{B}$); and
- appropriate weight and epsilon factors.

Throughout this discussion, the subscript, i, will refer to the i^{th} photo and the subscript, j, will refer to the j^{th} point. Two figures are presented for clarification of the formation and folding of the normal equations. The figure legends define verbally the individual elements, and the basic derivations of the elements are given in the text. Figure 1 gives the basic structure of the formed normal equations of a banded-bordered system. The individual elements are defined mathematically as

$$\begin{aligned} \dot{S}_i &= \sum \dot{N}_i + \dot{w}_i = \sum \dot{B}_{ij} w_{ij} \dot{B}_{ij} + \dot{w}_i \\ \dot{c}_i &= \sum \dot{B}^{T}_{ij} w_{ij} \epsilon_j - \dot{w}_i \dot{e}_i \\ \breve{S}_i &= \sum \breve{N}_i = \sum \dot{B}^{T}_{ij} w_{ij} \mathring{B}_{ij} \\ \mathring{S} &= \sum \mathring{N} + \mathring{w} = \sum \mathring{B}^{T}_{ij} w_{ij} \mathring{B}_{ij} + \mathring{w} \\ \mathring{c} &= \sum \mathring{B}_{ij} w_{ij} \epsilon_{ij} - \mathring{w} \mathring{\epsilon} \end{aligned} \qquad (1)$$

$$\begin{aligned} \bar{N}_{ij} &= \dot{B}^{T}_{ij} w_{ij} \ddot{B}_{ij} \\ \overset{\simeq}{N}_j &= \sum \mathring{B}^{T}_{ij} w_{ij} \ddot{B}_{ij} \\ \ddot{N}_j &= \sum \ddot{B}^{T}_{ij} w_{ij} \ddot{B}_{ij} + \ddot{w}_j \\ \ddot{c}_j &= \sum \ddot{B}^{T}_{ij} w_{ij} \epsilon_{ij} - \ddot{w}_j \ddot{\epsilon}_j \end{aligned} \qquad (2)$$

Note: $\bar{N}_{ij}$ is by point, not a summation.

Figure 2 is a representation of the normal equations after the folding of the ground point data in-

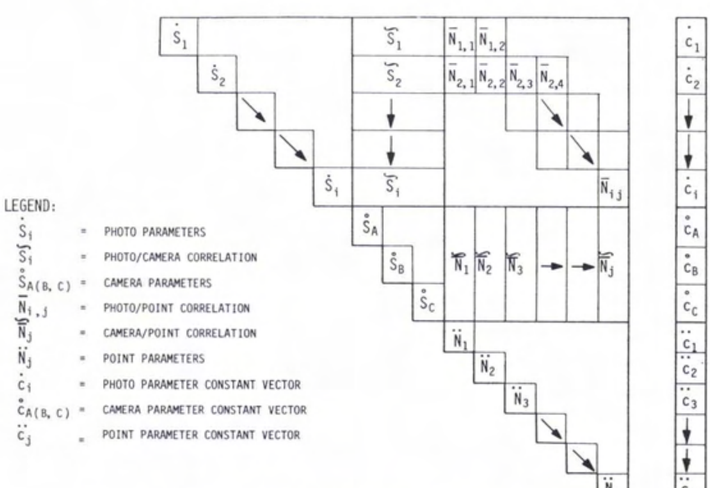

FIG. 1. General banded-bordered system of equations.

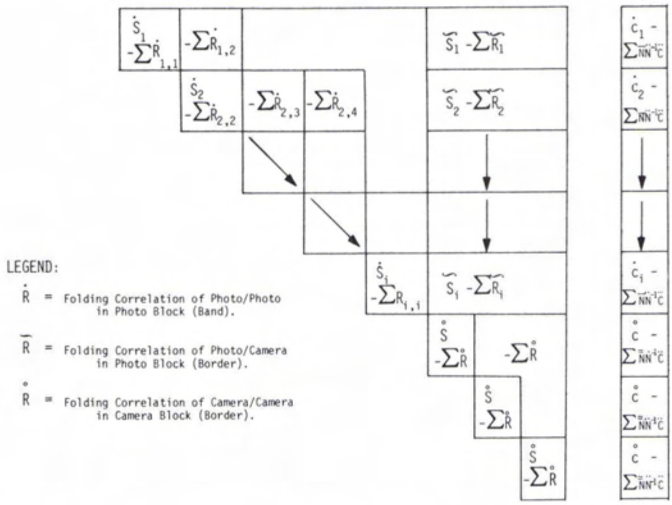

FIG. 2. Final system of equations with ground point data folded in.

to the elements. The folding elements are represented as:

$$\begin{aligned} \dot{\boldsymbol{R}}_i &= \bar{\boldsymbol{N}}_{ij}\ddot{\boldsymbol{N}}_j^{-1}\bar{\boldsymbol{N}}_{ij}^{\mathrm{T}} \\ \breve{\boldsymbol{R}}_i &= \bar{\boldsymbol{N}}_{ij}\ddot{\boldsymbol{N}}_j^{-1}\tilde{\bar{\boldsymbol{N}}}_j^{\mathrm{T}} \\ \mathring{\boldsymbol{R}} &= \tilde{\bar{\boldsymbol{N}}}_j\ddot{\boldsymbol{N}}_j^{-1}\tilde{\bar{\boldsymbol{N}}}_j^{\mathrm{T}} \end{aligned} \tag{3}$$

Thus, the final system of formed and folded normal equations may be represented by

(1) The Photo (banded portion)

$$\begin{aligned} \dot{\boldsymbol{S}}_i &= \sum (\dot{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{w}\dot{\boldsymbol{B}})_{ij} + \dot{\boldsymbol{w}}_i - \dot{\boldsymbol{R}}_i \\ &= \sum (\dot{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{w}\dot{\boldsymbol{B}})_{ij} + \dot{\boldsymbol{w}}_i - \bar{\boldsymbol{N}}_{ij}\ddot{\boldsymbol{N}}_j^{-1}\bar{\boldsymbol{N}}_{ij}^{\mathrm{T}} \\ \dot{\boldsymbol{c}}_i &= \sum \dot{\boldsymbol{B}}_{ij}^{\mathrm{T}}\boldsymbol{w}_{ij}\boldsymbol{\epsilon}_{ij} - \bar{\boldsymbol{N}}_{ij}\ddot{\boldsymbol{N}}_j^{-1}\ddot{\boldsymbol{c}}_j \end{aligned}$$

(2) The Camera/Photo (border portion)

$$\begin{aligned} \breve{\boldsymbol{S}}_i &= \sum (\dot{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{w}\mathring{\boldsymbol{B}})_{ij} - \breve{\boldsymbol{R}}_i \\ &= \sum (\dot{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{w}\mathring{\boldsymbol{B}})_{ij} - \bar{\boldsymbol{N}}_{ij}\ddot{\boldsymbol{N}}_j^{-1}\tilde{\bar{\boldsymbol{N}}}_j^{\mathrm{T}} \end{aligned} \tag{4}$$

(3) The Camera (border portion)

$$\begin{aligned} \mathring{\boldsymbol{S}} &= \sum (\mathring{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{w}\mathring{\boldsymbol{B}})_{ij} + \mathring{\boldsymbol{w}} - \mathring{\boldsymbol{R}} \\ &= \sum (\mathring{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{w}\mathring{\boldsymbol{B}})_{ij} + \mathring{\boldsymbol{w}} - \tilde{\bar{\boldsymbol{N}}}_j\ddot{\boldsymbol{N}}_j^{-1}\tilde{\bar{\boldsymbol{N}}}_j \\ \mathring{\boldsymbol{c}} &= \sum \mathring{\boldsymbol{B}}_{ij}^{\mathrm{T}}\boldsymbol{w}_{ij}\boldsymbol{\epsilon}_{ij} - \tilde{\bar{\boldsymbol{N}}}_j\ddot{\boldsymbol{N}}_j^{-1}\ddot{\boldsymbol{c}}_j \end{aligned}$$

Figure 3 illustrates those portions of the normal equations affected by the deletion of a single point ray. In this example, the ray to point five (5) which appears on photo three (3) is to be deleted.

It is necessary to retain the last set of partial

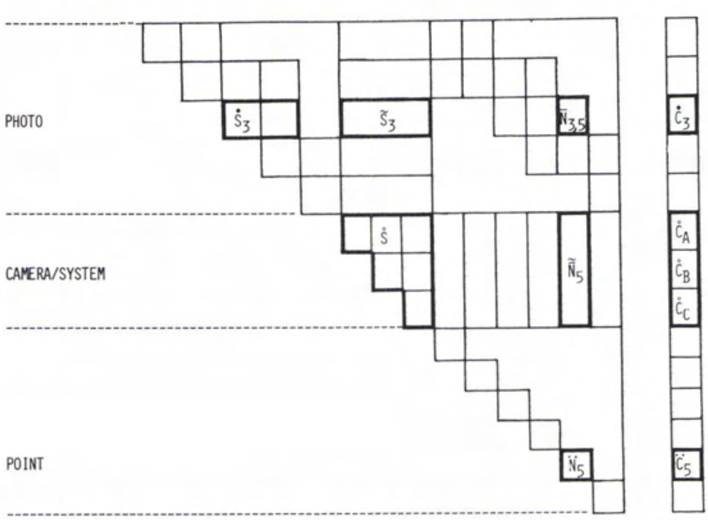

FIG. 3. Elements of normal equations influenced by single point ray (Example: Point 5 on Photo 3).

derivatives computed in addition to the normal equations. Because this derivation is for an off-line application in a production system environment, the following file types and recommended stacking orders must be created and maintained:

- the partial derivative file stacked by point;
- the photo-block file containing $\dot{\boldsymbol{S}}$, $\tilde{\boldsymbol{S}}$, and $\dot{\boldsymbol{c}}$ stacked by photo;
- the remaining border file containing $\hat{\boldsymbol{S}}$, $\hat{\boldsymbol{c}}$ in total; and
- the $\boldsymbol{N}$-double-dot file containing $\ddot{\boldsymbol{N}}^{-1}$, $\ddot{\boldsymbol{c}}$, $\tilde{\bar{\boldsymbol{N}}}$, $\bar{\boldsymbol{N}}$ stacked by point.

As the final folded elements of the normal equations ($\dot{\boldsymbol{S}}, \tilde{\boldsymbol{S}}, \hat{\boldsymbol{S}}, \dot{\boldsymbol{c}}, \hat{\boldsymbol{c}}$) are defined by summations over the number of points involved, the first impulse is to modify the existing elements through the elimination of single ray partials only. This editing procedure is a reversal of signs on elements summed during the formation and folding process. As an example, the apparent solution to deletion of a point's influence in the normal equations (see Equation 4) would be as follows (using $\dot{\boldsymbol{S}}$ only as an example):

$$\begin{aligned}\dot{\boldsymbol{S}}_i' &= \dot{\boldsymbol{S}}_i - \dot{\boldsymbol{B}}_{ij}^{\mathrm{T}}\boldsymbol{w}_{ij}\dot{\boldsymbol{B}}_{ij} + \bar{\boldsymbol{N}}_{ij}\ddot{\boldsymbol{N}}_j^{-1}\bar{\boldsymbol{N}}_{ij}^{\mathrm{T}} \\ &= \dot{\boldsymbol{S}}_i - (\dot{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{w}\dot{\boldsymbol{B}})_{ij} \\ &\quad + (\dot{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{w}\ddot{\boldsymbol{B}})_{ij}\,(\ddot{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{w}\ddot{\boldsymbol{B}})_{ij}^{-1}\,(\dot{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{w}\ddot{\boldsymbol{B}})_{ij}^{\mathrm{T}}\end{aligned} \tag{5}$$

Note that the signs on the second and third terms are reversed, and that $i = 3$ and $j = 5$ in Equations 5 through 8. However, the individually formed $\ddot{\boldsymbol{N}}$ terms for some point relationships cannot be inverted because they may be ill-conditioned. This problem makes it impossible to continue the solution. The individually formed $\ddot{\boldsymbol{N}}$ does not contain the initial $\ddot{\boldsymbol{w}}$. Thus, it was determined to handle the $\dot{\boldsymbol{S}}$ portion with individual formation, and the $\boldsymbol{R}$ portion (correlation) as a total entity.

The first step in the editing process is the deletion of the initial point influence, i.e.,

$$\begin{aligned}\dot{\boldsymbol{S}}_i' &= \dot{\boldsymbol{S}}_i - (\dot{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{w}\dot{\boldsymbol{B}})_{ij} \\ \tilde{\boldsymbol{S}}_i' &= \tilde{\boldsymbol{S}}_i - (\dot{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{w}\hat{\boldsymbol{B}})_{ij} \\ \hat{\boldsymbol{S}}' &= \hat{\boldsymbol{S}} - (\hat{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{w}\hat{\boldsymbol{B}})_{ij} \\ \dot{\boldsymbol{c}}_i' &= \dot{\boldsymbol{c}}_i - (\dot{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{w}\boldsymbol{\epsilon})_{ij} \\ \hat{\boldsymbol{c}}' &= \hat{\boldsymbol{c}} - (\hat{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{w}\boldsymbol{\epsilon})_{ij}\end{aligned} \tag{6}$$

Now one will select the $\bar{\boldsymbol{N}}$, $\tilde{\bar{\boldsymbol{N}}}$, and $\ddot{\boldsymbol{N}}$ elements from the $\ddot{\boldsymbol{N}}$ file and reform the $\boldsymbol{R}$ (correlation) portion deleting the total correlation influence from the equation as follows:

$$\begin{aligned}\dot{\boldsymbol{S}}_i'' &= \dot{\boldsymbol{S}}_i' + \dot{\boldsymbol{R}}_i = \dot{\boldsymbol{S}}_i' + (\bar{\boldsymbol{N}}_{ij}\ddot{\boldsymbol{N}}_j^{-1}\bar{\boldsymbol{N}}_{ij}^{\mathrm{T}}) \\ \dot{\boldsymbol{c}}_i'' &= \dot{\boldsymbol{c}}_i' + \bar{\boldsymbol{N}}_{ij}\ddot{\boldsymbol{N}}_j^{-1}\ddot{\boldsymbol{c}}_j \\ \tilde{\boldsymbol{S}}_i'' &= \tilde{\boldsymbol{S}}_i' + \tilde{\boldsymbol{R}}_i = \tilde{\boldsymbol{S}}_i' + (\bar{\boldsymbol{N}}_{ij}\ddot{\boldsymbol{N}}_j^{-1}\tilde{\bar{\boldsymbol{N}}}_j^{\mathrm{T}}) \\ \hat{\boldsymbol{S}}'' &= \hat{\boldsymbol{S}}' + \hat{\boldsymbol{R}} = \hat{\boldsymbol{S}} + (\tilde{\bar{\boldsymbol{N}}}_j\ddot{\boldsymbol{N}}_j^{-1}\tilde{\bar{\boldsymbol{N}}}_j^{\mathrm{T}}) \\ \hat{\boldsymbol{c}}'' &= \hat{\boldsymbol{c}}' + \tilde{\bar{\boldsymbol{N}}}_j\ddot{\boldsymbol{N}}_j^{-1}\ddot{\boldsymbol{c}}_j\end{aligned} \tag{7}$$

We have now deleted the point, j, influence created by the evaluation of Equations 1 and 4. The next

Table 2. Editing Steps in a Banded-Bordered System.

STEP	PHOTO	PHOTO/CAMERA	CAMERA	POINT	PHOTO/ POINT	CAMERA/ POINT
Deletion of Rays Initial Influence	$\dot{\boldsymbol{S}}' = \dot{\boldsymbol{S}} - \dot{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{w}\dot{\boldsymbol{B}}$ $\dot{\boldsymbol{c}}' = \dot{\boldsymbol{c}} - \dot{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{w}\boldsymbol{\epsilon}$	$\tilde{\boldsymbol{S}}' = \tilde{\boldsymbol{S}} - \dot{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{w}\hat{\boldsymbol{B}}$	$\hat{\boldsymbol{S}}' = \hat{\boldsymbol{S}} - \hat{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{w}\hat{\boldsymbol{B}}$ $\hat{\boldsymbol{c}}' = \hat{\boldsymbol{c}} - \hat{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{w}\boldsymbol{\epsilon}$			
Deletion of Correlation Terms	$\dot{\boldsymbol{S}}'' = \dot{\boldsymbol{S}}' + \bar{\boldsymbol{N}}\ddot{\boldsymbol{N}}^{-1}\bar{\boldsymbol{N}}^{\mathrm{T}}$ $\dot{\boldsymbol{c}}'' = \dot{\boldsymbol{c}}' + \bar{\boldsymbol{N}}\ddot{\boldsymbol{N}}^{-1}\ddot{\boldsymbol{c}}$	$\tilde{\boldsymbol{S}}'' = \tilde{\boldsymbol{S}}' + \bar{\boldsymbol{N}}\ddot{\boldsymbol{N}}^{-1}\tilde{\bar{\boldsymbol{N}}}^{\mathrm{T}}$	$\hat{\boldsymbol{S}}'' = \hat{\boldsymbol{S}}' + \tilde{\bar{\boldsymbol{N}}}\ddot{\boldsymbol{N}}^{-1}\tilde{\bar{\boldsymbol{N}}}^{\mathrm{T}}$ $\hat{\boldsymbol{c}}'' = \hat{\boldsymbol{c}}' + \tilde{\bar{\boldsymbol{N}}}\ddot{\boldsymbol{N}}^{-1}\ddot{\boldsymbol{c}}$			
Modification of Correlation Elements				$\ddot{\boldsymbol{N}}' = \ddot{\boldsymbol{N}} - \ddot{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{w}\ddot{\boldsymbol{B}}$ $\ddot{\boldsymbol{c}}' = \ddot{\boldsymbol{c}} - \ddot{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{w}\boldsymbol{\epsilon}$	$\bar{\boldsymbol{N}} \Rightarrow 0$	$\tilde{\bar{\boldsymbol{N}}}' = \tilde{\bar{\boldsymbol{N}}} - \dot{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{w}\ddot{\boldsymbol{B}}$
Inclusion of Modified Correlation Terms			$\hat{\boldsymbol{S}}''' = \hat{\boldsymbol{S}}'' - \tilde{\bar{\boldsymbol{N}}}'\ddot{\boldsymbol{N}}'^{-1}\tilde{\bar{\boldsymbol{N}}}'^{\mathrm{T}}$ $\hat{\boldsymbol{c}}''' = \hat{\boldsymbol{c}}'' - \tilde{\bar{\boldsymbol{N}}}'\ddot{\boldsymbol{N}}'^{-1}\ddot{\boldsymbol{c}}'$			

step is to modify the individual elements of the $\boldsymbol{R}$ terms and re-apply the modified terms to the equations.

It should be re-emphasized that the $\bar{\boldsymbol{N}}$ computation shown in Equations 2 is by point and reflects a particular point ray. Thus, $\bar{\boldsymbol{N}}$ for a deleted ray goes to zero and the edited point terms containing $\bar{\boldsymbol{N}}$ also go to zero. For this reason, only $\tilde{\bar{\boldsymbol{N}}}$, $\ddot{\boldsymbol{N}}$, and $\ddot{\boldsymbol{c}}$ need to be modified:

$$
\begin{aligned}
\tilde{\bar{\boldsymbol{N}}}_j' &= \tilde{\bar{\boldsymbol{N}}}_j - (\boldsymbol{B}^{\mathrm{T}}\boldsymbol{w}\ddot{\boldsymbol{B}})_{ij},\\
\ddot{\boldsymbol{N}}_j' &= \ddot{\boldsymbol{N}}_j - (\ddot{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{w}\ddot{\boldsymbol{B}})_{ij},\\
\text{and } \ddot{\boldsymbol{c}}_j' &= \ddot{\boldsymbol{c}}_j - (\ddot{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{w}\boldsymbol{\epsilon})_{ij}
\end{aligned}
\tag{8}
$$

$\ddot{\boldsymbol{N}}$ in this case contains both previous point influences and the initial $\ddot{\boldsymbol{w}}$ influence. In the event that a total point is to be deleted (i.e., all rays deleted) the $\ddot{\boldsymbol{N}}$ portion contains only the $\ddot{\boldsymbol{w}}$ elements, which are arbitrarily set to a very large constant. Thus, the inversion will result in a near-zero quantity and when inverted, the term does not become ill-conditioned, and the influence of the point disappears from the normal equations.

The only terms requiring further modification are $\mathring{\boldsymbol{S}}$ and $\mathring{\boldsymbol{c}}$, which are computed as

$$
\begin{aligned}
\mathring{\boldsymbol{S}}''' &= \mathring{\boldsymbol{S}}'' - \tilde{\bar{\boldsymbol{N}}}_j'\ddot{\boldsymbol{N}}_j'^{-1}\tilde{\bar{\boldsymbol{N}}}_j'^{\mathrm{T}}\\
\mathring{\boldsymbol{c}}''' &= \mathring{\boldsymbol{c}}'' + \tilde{\bar{\boldsymbol{N}}}_j'\ddot{\boldsymbol{N}}_j'^{-1}\ddot{\boldsymbol{c}}_j'
\end{aligned}
\tag{9}
$$

Summary

This summary is primarily intended as a review of the sequential steps in the editing process. Table 2, for generalization and clarity, presents this progression without the use of subscripts. Referring to this figure, the newly partitioned set of normal equations is

$$
\left[\begin{array}{c} \dot{\boldsymbol{\delta}}' \\ \mathring{\boldsymbol{\delta}}' \\ \hline \ddot{\boldsymbol{\delta}}' \end{array}\right]
=
\left[\begin{array}{cc|c} \dot{\boldsymbol{S}}'' & \tilde{\boldsymbol{S}}'' & \bar{\boldsymbol{N}}' \\ \tilde{\boldsymbol{S}}''^{\mathrm{T}} & \mathring{\boldsymbol{S}}''' & \tilde{\bar{\boldsymbol{N}}}' \\ \hline \bar{\boldsymbol{N}}'^{\mathrm{T}} & \tilde{\bar{\boldsymbol{N}}}' & \ddot{\boldsymbol{N}}'^{\mathrm{T}} \end{array}\right]^{-1}
\left[\begin{array}{c} \dot{\boldsymbol{c}}'' \\ \mathring{\boldsymbol{c}}''' \\ \hline \ddot{\boldsymbol{c}}' \end{array}\right]
$$

Care must be exercised with this technique so as not to employ it if a solution has not converged. In non-convergence, the most recent set of partial derivatives will not be representative of those used in the last formation of the normal equations.

Direct editing is currently designed as an experimental analytical tool. This should be understood until sufficient testing can be performed to answer the following questions:

- What percentage of rays/points can be deleted without accumulating spurious information?
- What magnitude limit on residuals can be accommodated before rendering the method useless?

An additional precaution to be considered when employing this technique is the fact that, by deleting the influence of a point employing partial derivatives formed during a relinearization, the problem which was initially defined has been altered. That is, the problem being solved with this editing technique is not the original one. For large data cases the significance of this point is most probably minimal. Additional analyses need be performed in order to truly define the direct editing effects and limitations.

Acknowledgment

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